Tension and Compression in Bars

1. Stress الإجهاد
2. Strain التشوه (الانفعال)
3. Constitutive Law قانون السلوك
4. Single Bar under Tension or Compression قضيب مفرد: شد أو ضغط
5. Systems of Bars جمل القضبان
6. Supplementary Examples أمثلة إضافية

Objectives: *Mechanics of Materials* investigates the stressing and the deformations of structures subjected to applied loads, starting by the simplest structural members, namely, bars in tension or compression.

يدرس ميكانيك المواد إجهادات وتشوهات الجمل الإنشائية (الهياكل الحاملة) الناتجة عن الحمولات الخارجية، مبتدأً بالعناصر الأبسط أي القضبان (العناصر الطولية) المشدودة أو المضغوطة.

In order to treat such problems, the kinematic relations and a constitutive law are needed to complement the equilibrium conditions which are known from Engineering Mechanics (Statics).

تقوم هذه الدراسة على:

(1) معادلات التوازن التي دُرست في الميكانيك الهندسي (علم السكون)

(2) العلاقات الكينماتيكية التي ستدرس وهي تصف التشوهات كمياً أي تحدد شكل ومقدار تغيرات الشكل الجيومتري.

(3) قوانين سلوك مادة الجملة وهي كما ستُعرض لاحقاً، قوانين تجريبية تعرّف السلوك الميكانيكي لمادة الهيكل الحامل.

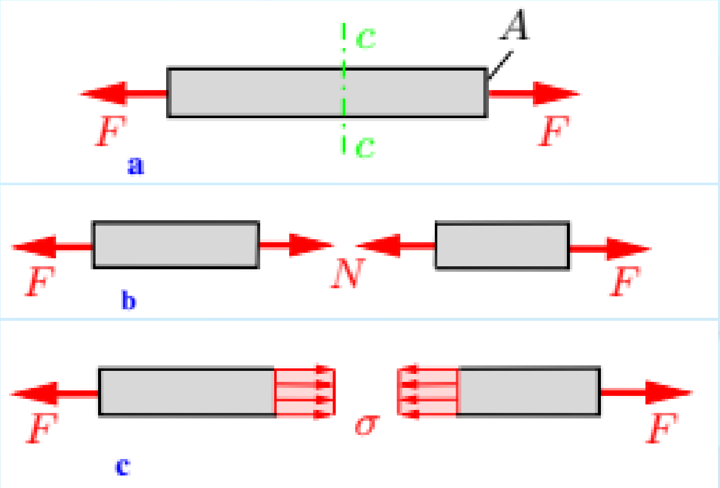
The kinematic relations represent the geometry of the deformation, whereas the behavior of the material is described by the constitutive law. The students will learn how to apply these equations and how to solve determinate as well as statically indeterminate problems. يعالج الطلبة مسائل مقررة سكونياً وأخرى غير مقررة سكونياً ؟؟

1. Stress
الإجهاد

Consider a straight bar with a constant cross-sectional area A.
Its *axis* is connecting the centroids of the cross sections.
Its ends are subjected to the forces *F* acting on the axis (Fig. a).

The *external* load causes *internal* forces, which can be visualized by an imaginary cut of the bar (Fig. b). They are distributed over the cross section and called *stresses* (Fig. c).

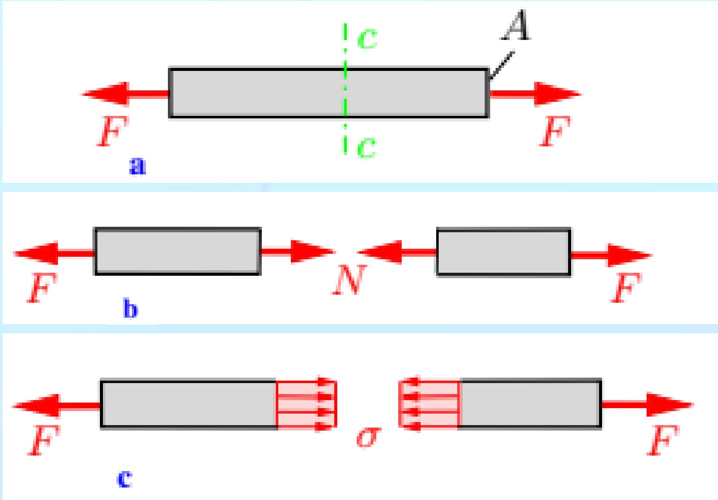
They have the dimension force/area, for example, as multiples of MPa (1MPa=1N/mm^2). The "Pascal" (1 Pa=1 N/m^2) after the mathematician & physicist Blaise Pascal (1623–1662). The notion of "stress" was introduced by Augustin Louis Cauchy (1789–1857).



In (Statics) we only dealt with the resultant of the stresses : The internal forces.

To determine the stresses we make an imaginary cut *c-c* perpendicular to the bar axis The stresses are shown in the free-body diagram (Fig. c); they are denoted by σ.

We assume that they act perpendicularly to the exposed surface A of the cross section and that they are uniformly distributed.



Since they are normal to the cross section they are called *normal stresses*. Their resultant is the normal force N shown in (Fig. b) .

Therefore we have $N=\sigma A$ and the stresses σ can be calculated from the normal force N:

$$\sigma = \frac{N}{A}$$

In the present example the normal force N is equal to the applied force F. Thus, we write the last equation as

$$\sigma = \frac{F}{A}$$

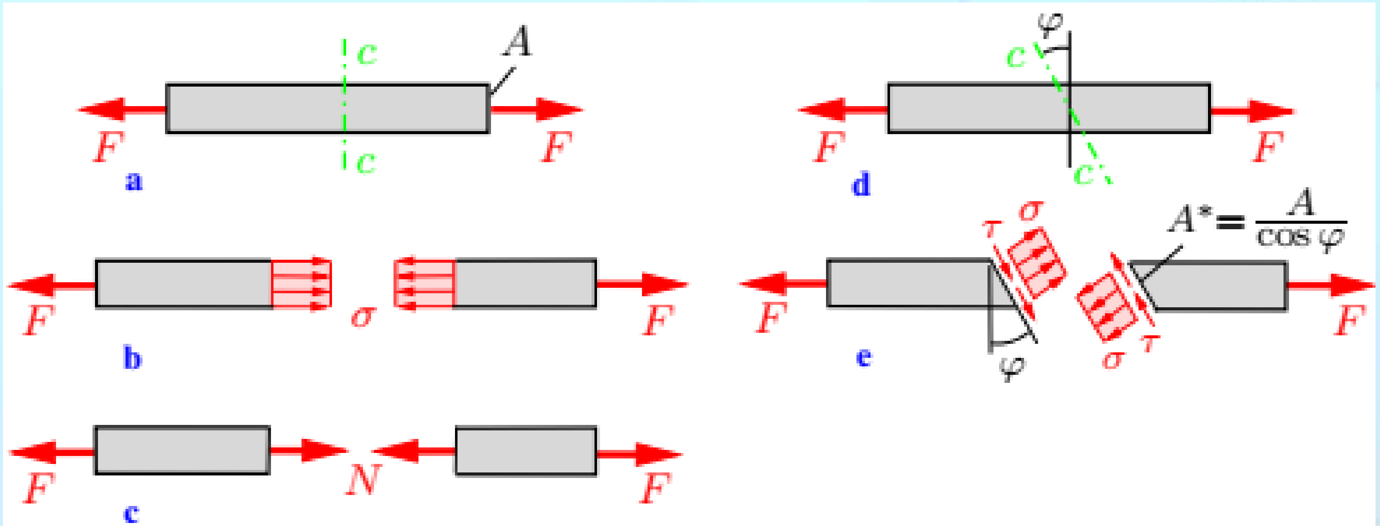
For a positive normal force N (tension) the stress σ is then positive (tensile stress). Reversely, if the normal force is negative (compression) the stress is also negative (compressive stress)

Let us now imagine the bar being sectioned by a cut which is not orthogonal to the axis of the bar so that its direction is given by the angle ϕ (Fig. d).

$$A^* = A/\cos\varphi.$$

Again we assume that they are uniformly distributed.

Resolve the stresses into a component σ perpendicular to the surface (normal stress) & a component τ tangential to the surface (shear stress) (Fig. e).



Equilibrium of the forces acting on the left portion of the bar (see Fig. e) yields:

$$\rightarrow: \ \sigma A^* \cos\varphi + \tau A^* \sin\varphi - F = 0$$
$$\uparrow: \qquad \sigma A^* \sin\varphi - \tau A^* \cos\varphi = 0$$

$\rightarrow$: $\sigma A^* \cos\varphi + \tau A^* \sin\varphi - F = 0$

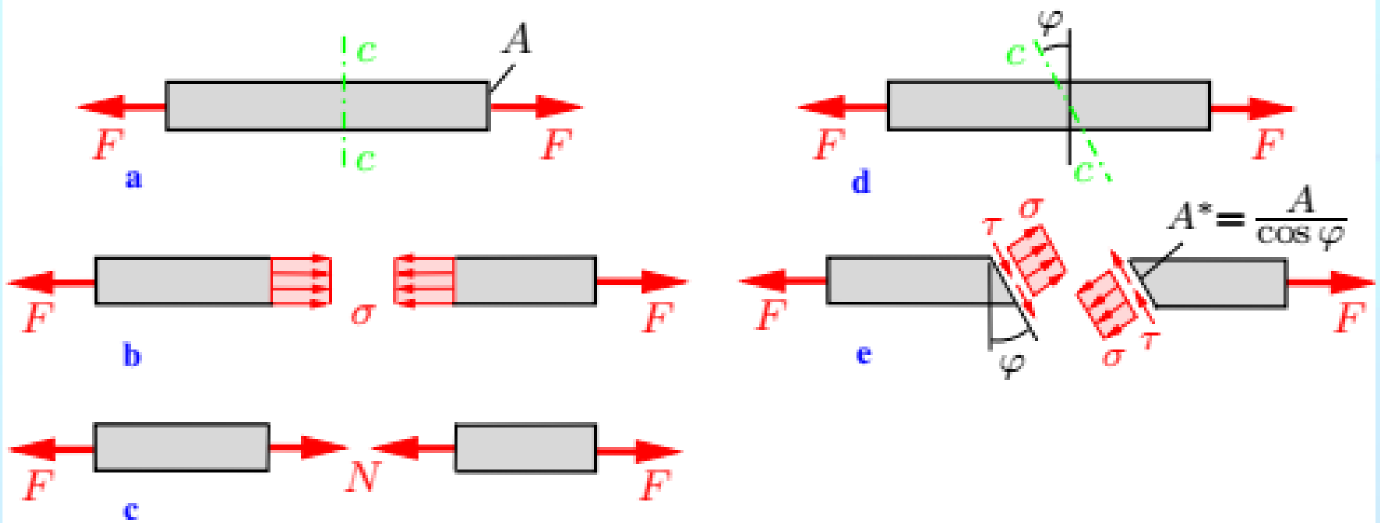
$\uparrow$: $\sigma A^* \sin\varphi + \tau A^* \cos\varphi = 0$

These Eq. Eqs. are written for the *forces*, *not* for the *stresses*. With $A^* = A/\cos\varphi$ we obtain

$$\begin{Bmatrix} \sigma + \tau \tan\varphi = \frac{F}{A} \\ \sigma \tan\varphi - \tau = 0 \end{Bmatrix}$$

Solving yields

$$\begin{Bmatrix} \sigma = \frac{1}{1+\tan^2\varphi}\frac{F}{A} \\ \tau = \frac{\tan\varphi}{1+\tan^2\varphi}\frac{F}{A} \end{Bmatrix}$$



It is practical to write these equations in a different form. Using the trigonometric relations

$$\frac{1}{1+\tan^2\varphi} = \cos^2\varphi = \tfrac{1}{2}(1+\cos 2\varphi), \quad \frac{\tan\varphi}{1+\tan^2\varphi} = \sin\varphi\cos\varphi, \quad \sin 2\varphi = \frac{2\tan\varphi}{1+\tan^2\varphi}, \quad \cos 2\varphi = \frac{1-\tan^2\varphi}{1+\tan^2\varphi}$$

and the abbreviation $\sigma_0 = F/A$ (normal stress in a section perpendicular to the axis) we get

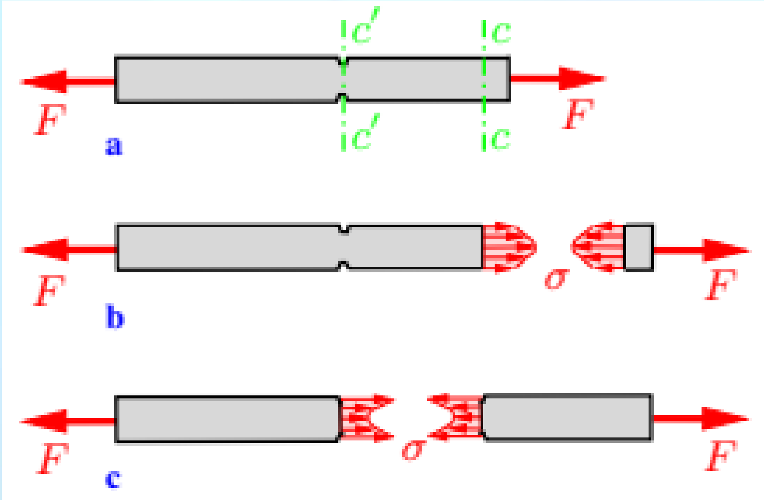
$$\sigma = \frac{\sigma_0}{2}(1+\cos 2\varphi), \qquad \tau = \frac{\sigma_0}{2}\sin 2\varphi$$

Stresses depend on the direction of the cut. If σ_0 is known, σ & τ can be calculated for any φ. The maximum value of σ is obtained for $\varphi = 0$, where $\sigma_{max} = \sigma_0$; the maximum value of τ is found for $\varphi = \pi/4$ where $\tau_{max} = \sigma_0/2$.

Two dangerous cuts:

If we section a bar near an end which is subjected to a concentrated force F (Fig. a, section *c-c*) we find that the normal stress is not distributed uniformly over area.

The concentrated force produces high stresses near it (Fig.b).



It can be shown that this *stress concentration* is restricted to sections close to the end concentrated force: the high stresses decay rapidly towards the average value σ_0 far from the end of the bar. This fact is referred to as *Saint-Venant's principle* (Adhémar Jean Claude Barré de Saint-Venant, 1797-1886).

The uniform distribution of the stress is also disturbed by holes, notches or any abrupt changes (discontinuities) of the geometry. If, for example, a bar has notches the remaining cross-sectional area (section *c-c*) is also subjected to a stress concentration (Fig. c). The determination of these stresses is not possible with the elementary analysis presented in this course.