Cyclic Loading

Engineering systems are often subjected to *cyclic loading*.
Automobiles & machines (millions of cycles).
Electronic components (fluctuating temperature).

Material degradation with time under *cyclic loading* is known as ***fatigue* (التعب)**.

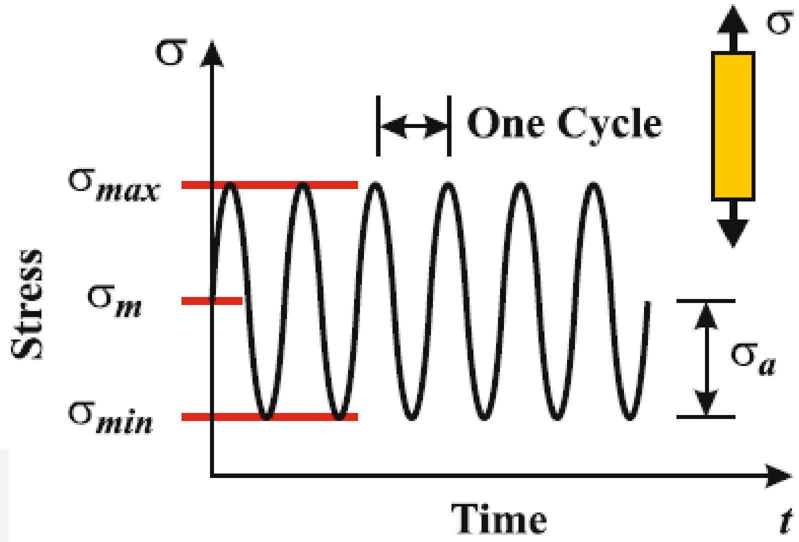
Fatigue can cause materials to fail at stress levels well below their uniaxial strength. In many steels, the *fatigue strength* is less than half of the *ultimate strength*.

A few definitions are useful to describe the nature of cyclic loading.

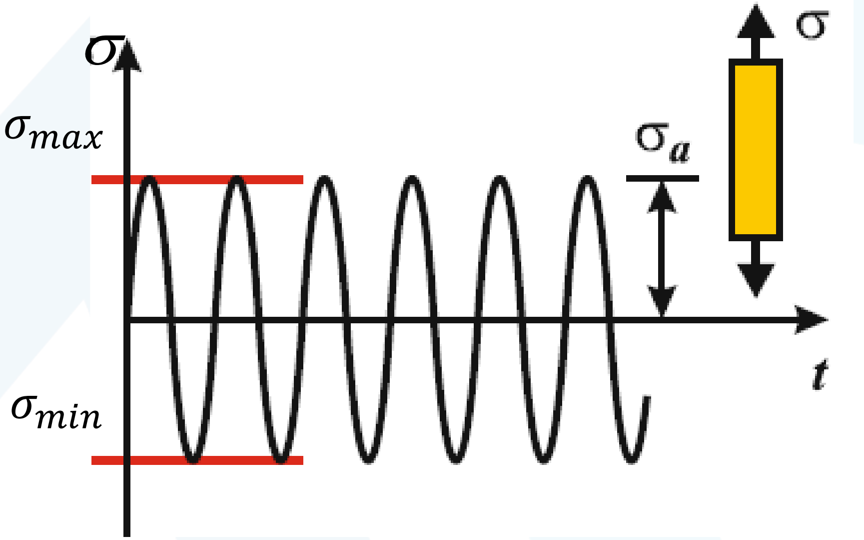
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$ Mean Stress الإجهاد المتوسط

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$ Stress amplitude مطال الإجهاد

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$ R-ratio

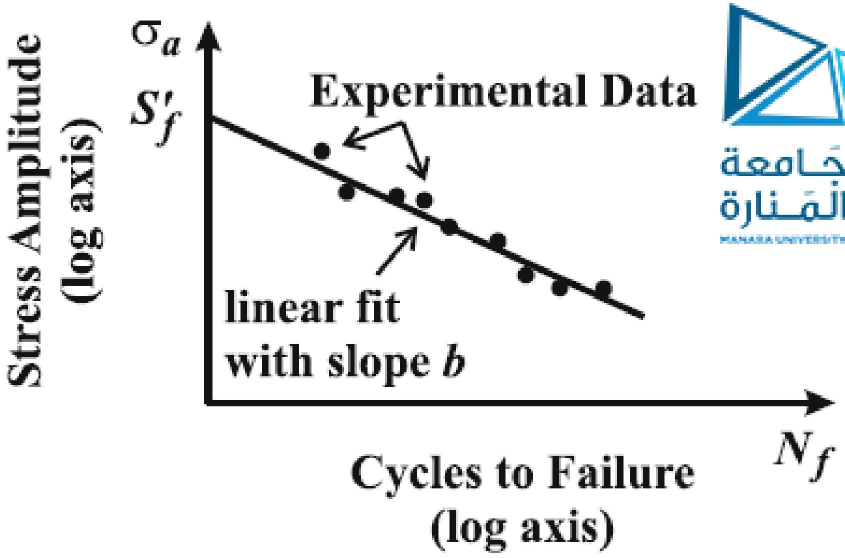


Typical $\sigma - t$ graph for cyclic loading.



$\sigma - t$ graph for standard fatigue test:
$\sigma_a = \sigma_{\text{max}} = |\sigma_{\text{min}}|, \sigma_m = 0, R = -1.0.$

When the stress amplitude is plotted against the number of cycles to failure on a log–log set of axes (or on a semi-log set of axes), the graph is known as the ***S–N curve***



The ***fatigue strength*** is the stress amplitude $\sigma_a = \sigma_{\max}$ corresponding to a specified number of cycles to failure.

When the data are plotted, the relationship between stress amplitude σ_a and cycles to failure N_f can be expressed in the following form:

$$\sigma_a = S'_f (N_f)^b$$

Where S'_f & b depend on the material, & are determined from a best-fit line of the material's ***S–N curve*** since:

$$\log(\sigma_a) = \log(S'_f) + b\log(N_f)$$

Quantity S'_f corresponds to the curve fit's intercept at $N_f = 1$ cycle, b is the slope of the curve in the log–log axes. Values for aluminum & steel are given

Property	Al 6061-T6	Steel A36
S_y	240 MPa	250 MPa
S_u	314 MPa	540 MPa
S_f'	505 MPa	1035 MPa
b	–0.082	–0.11
E	70 GPa	200 GPa

Ex. 1 Fatigue Strength of Structural Aluminum & Steel

Given: A tensile bar is subjected to cyclic loading, with zero mean stress

Required: Determine the fatigue strength, $\sigma_a = \sigma_{\max}$, (a) for an aluminum bar (Al 6061-T6) at 10^7 cycles and (b) for a steel bar (A36) at 10^6 cycles.

Solution: For aluminum, the fatigue strength corresponds to 10^7 cycles. From *Table* $S_f' = 505\ MPa$ & $b = -0.082$.

$\sigma_{a,al} = S_f'(N_f)^b = (505\ \text{MPa})(10^7)^{-0.082} = 135\ \text{MPa}.$ (56% S_y, 43% S_u)

For steel, the fatigue strength corresponds to 10^6 cycles. From *Table* $S_f' = 1035\ MPa$ & $b = -0.11$.

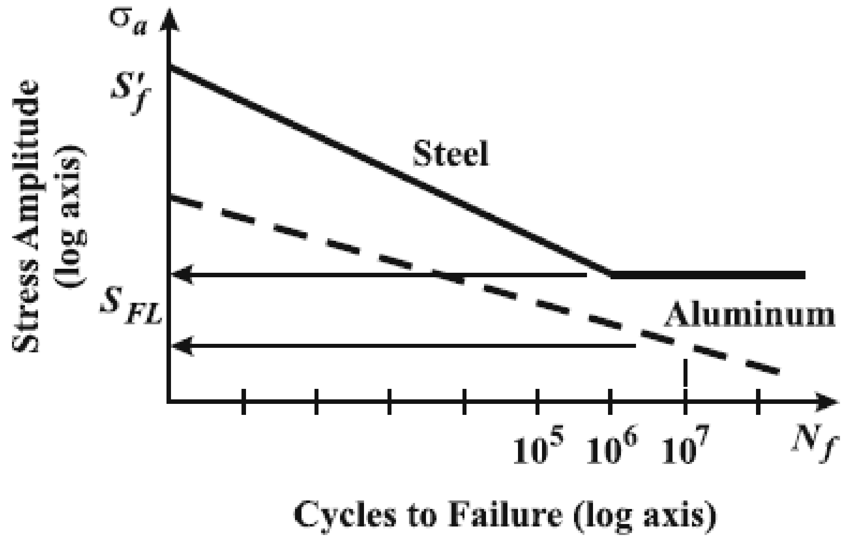
$\sigma_{a,st} = S_f'(N_f)^b = (1035\ \text{MPa})(10^6)^{-0.11} = 226\ \text{MPa}.$ (90% S_y, 42% S_u)

These values are also known as *fatigue limit* S_{FL} , as discussed below.

The fatigue limit for many steels is typically on the order of 35 - 50% of S_u.

Fatigue Limit

The ***S–N curve*** for most steels does't decrease to zero (*Fig.*). Below a certain value of σ_a, steel has essentially an infinite fatigue life. The stress amplitude below which fatigue failure does not occur is the *fatigue limit* S_{FL}, also known as the endurance limit حد التحمل.

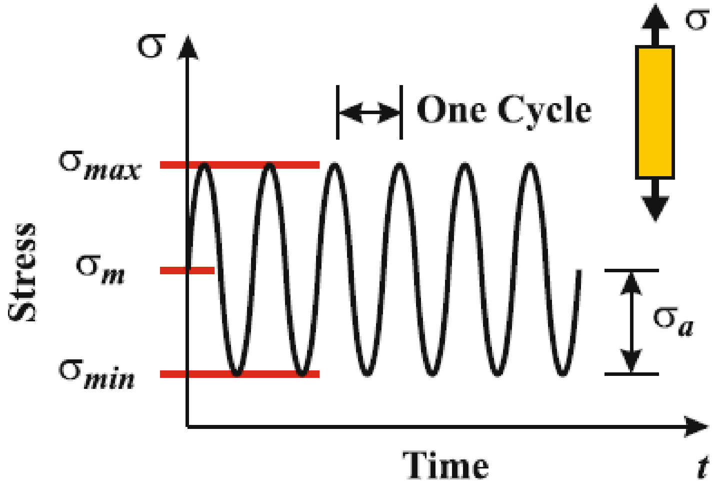


Type equation here.For steels, the *fatigue limit* S_{FL} typically corresponds to a fatigue life of about 10^6 cycles. The S–N curve exhibits no further reduction and becomes horizontal.

Aluminum does not exhibit such limiting behavior. No matter how small the stress amplitude σ_a, aluminum will eventually fail by fatigue. Accordingly, when designing with aluminum, the fatigue limit S_{FL} is generally taken to be the stress amplitude for $N_f = 10^7$ cycles.

Effect of Mean Stress on Fatigue Strength

In general, cyclic stresses are applied with a non-zero mean stress σ_m.

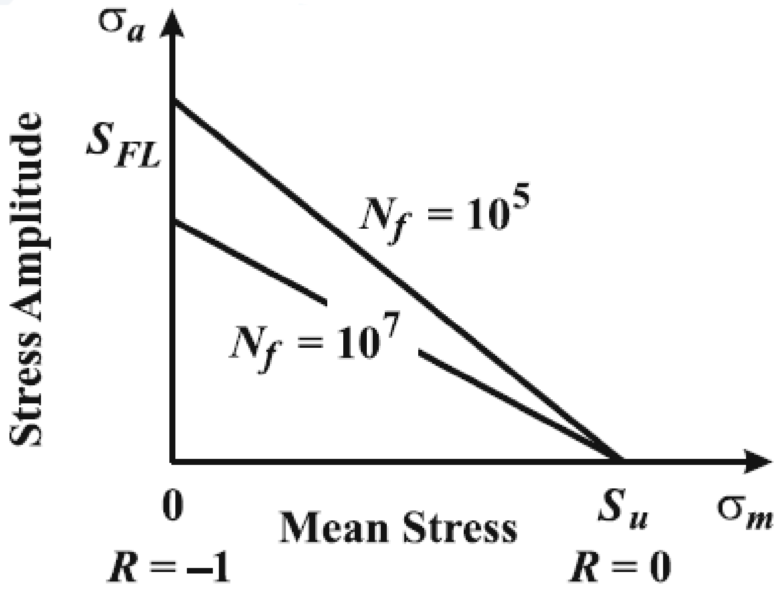


When this is the case, the fatigue strength for a given number of cycles to failure N_f is determined using the ***Goodman Diagram***

$$\frac{\sigma_a}{S_{FL}} + \frac{\sigma_m}{S_u} = 1$$

The new fatigue strength σ_a – the amplitude of the cyclic loading with mean stress σ_m for the specified cycles to failure N_f – is then:

$$\sigma_a = S_{FL}\left(1 - \frac{\sigma_m}{S_u}\right)$$



When $\sigma_m = S_u$, the materials breaks into two upon first loading; there can be no alternative stress σ_a ($\sigma_a = 0$). If the mean stress is zero, then the stress amplitude is the *fatigue limit* S_{FL} for the specified number of cycles to failure.

Ex. 2 Fatigue Strength With Non-Zero Mean Stress

Given: A tensile bar is subjected to cyclic loading with a mean stress of $\sigma_m = 100$ MPa.

Required: With the mean stress applied, determine the fatigue strength (a) for an Al 6061-T6 bar at 10^7 cycles and (b) for an A36 steel bar at 10^6 cycles.

Solution:

$$\sigma_{a,al} = S_{FL}\left(1 - \frac{\sigma_m}{S_u}\right) = 135\left(1 - \frac{100}{314}\right) = 91 \text{ MPa.}$$

$$\sigma_{a,st} = S_{FL}\left(1 - \frac{\sigma_m}{S_u}\right) = 226\left(1 - \frac{100}{540}\right) = 184 \text{ MPa.}$$