Structural Mechanics (1)

Week No-09

Analysis of Indeterminate Structures - Force Method

- Determinate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)

STRUCTURES

are classified from the analysis point of view to

DETERMINATE

INDETERMINATE

To predict the performance of a structure, its response elements such like sup. reactions, internal forces, stresses, deflections, strain, to external actions Loads, sup. settlement, temp. changes & fabric. errors, must be determined.

DETERMINATE	INDETERMINATE
Response elements are separable to	Response elements are not separable
Forces: support reactions & internal forces, then stresses, are determined by equilibrium equations	support reactions & internal forces number is greater than the available equilibrium equations
Deformations: deflections & Strains, are determined after knowing the first group	additional relationships based on the geometry of deformation of structures, <u>are needed</u>

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INDETERMINATE STRUCTURES

Advantages	Disadvantages
greater overall factor of safety	more sensitive to secondary effects
1. Smaller Stresses 2. Greater Stiffness 3. Redundancies	1. Support settlements 2. Temperature changes 3. Fabrication errors

ANALYSIS OF INDETERMINATE STRUCTURES

Three Pillars of Mechanics

Equilibrium Equations	Constitutive Equations or Behavior Lows	Compatibility Equations

the response elements
(support reactions, internal forces, stresses, deflections, strain)
are classified into primary & secondary unknowns

Force methods

Primary unknowns are **reactions & Internal forces** **Secondary unknowns are** **displacements: deflections & rotations (slopes)**

Displacement methods

Primary unknowns are **displacements: deflections & rotations (slopes)** **Secondary unknowns are** **reactions & Internal forces**

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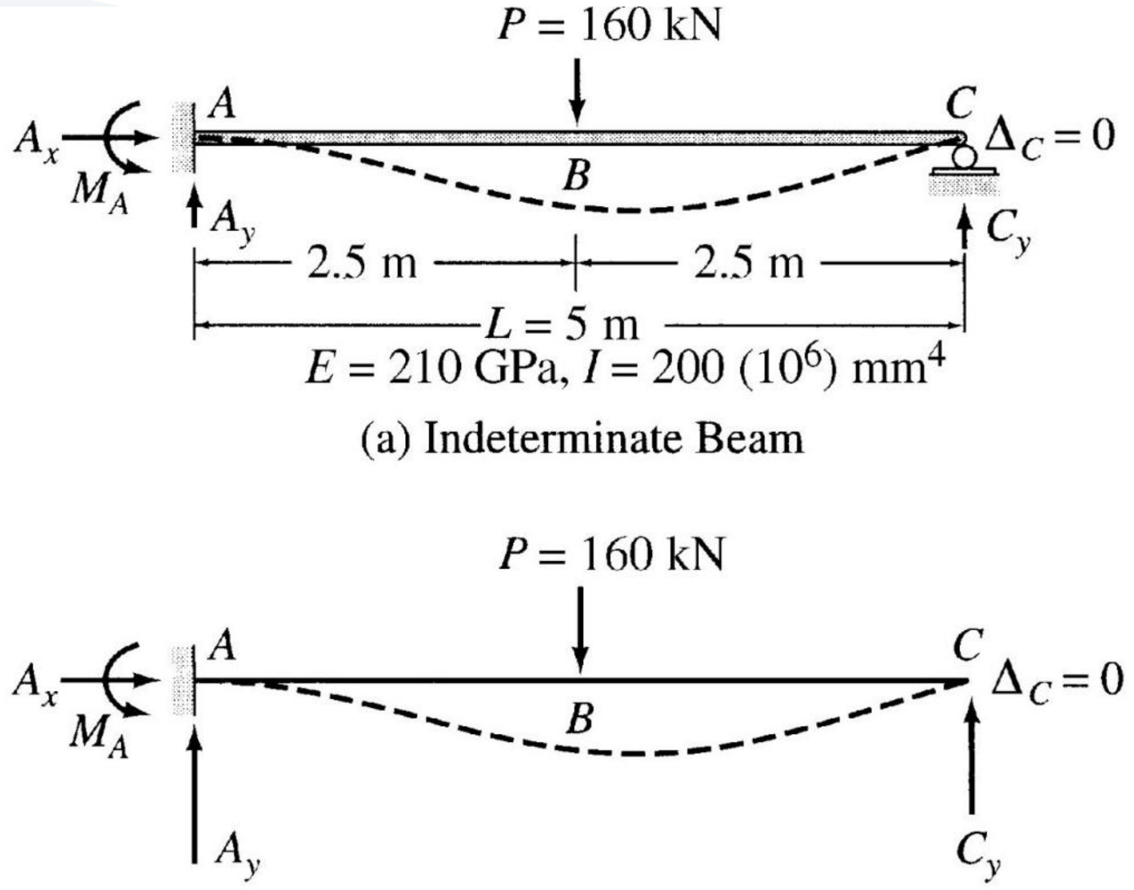
Force method: Beam with a single Degree of Indeterminacy

illustrative example



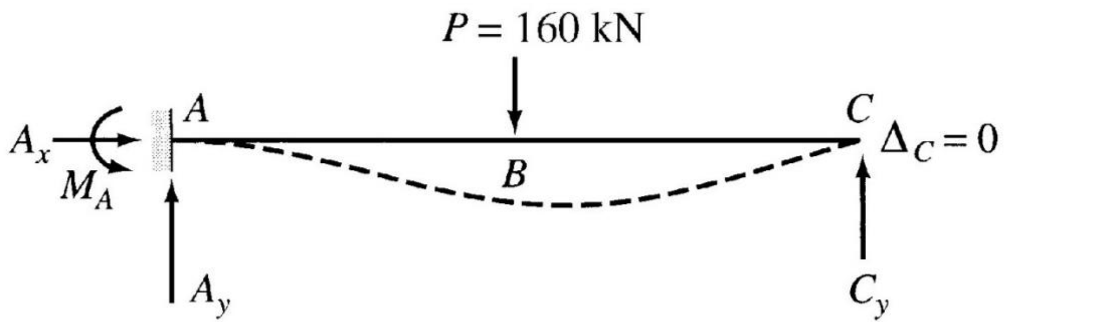
(a) Indeterminate Beam

(b) Primary Beam Subjected to External Loading and Redundant C_y

Force method: Beam with a single Degree of Indeterminacy

Compatibility Equation

$$\Delta_C = \Delta_{CO} + f_{CC}\, C_y = 0$$



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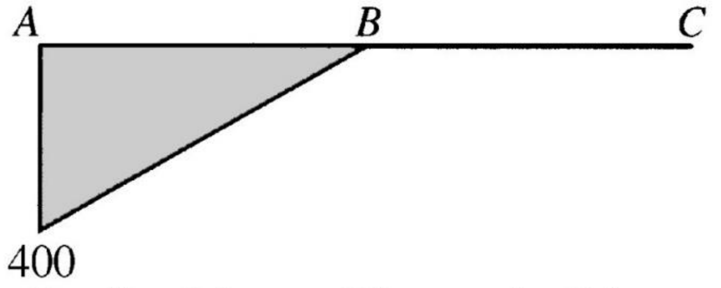
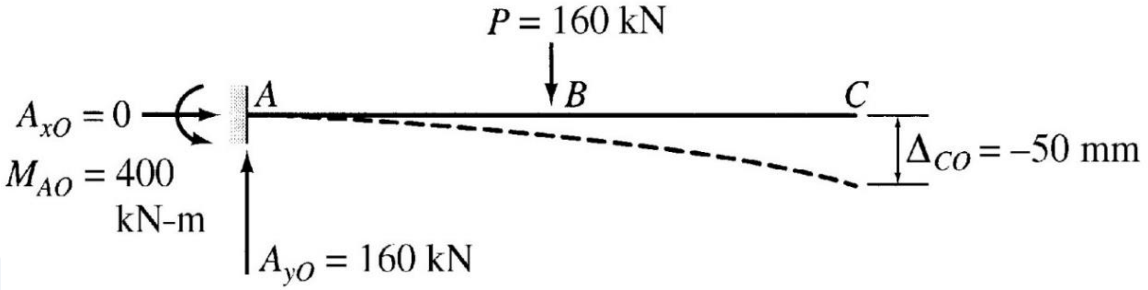


Bending Moment Diagram for Primary Beam Due to External Loading (kN-m)



(c) Primary Beam Subjected to External Loading

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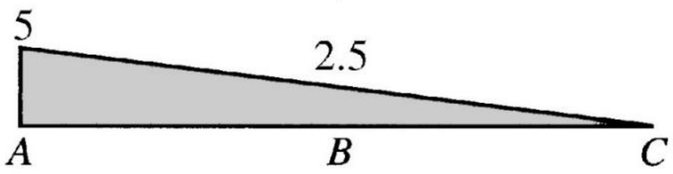

Bending Moment Diagram for Primary Beam Due to Unit Value of C_y (kN-m/kN

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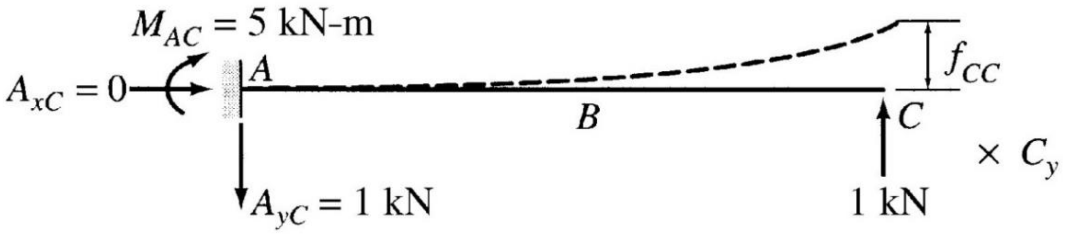
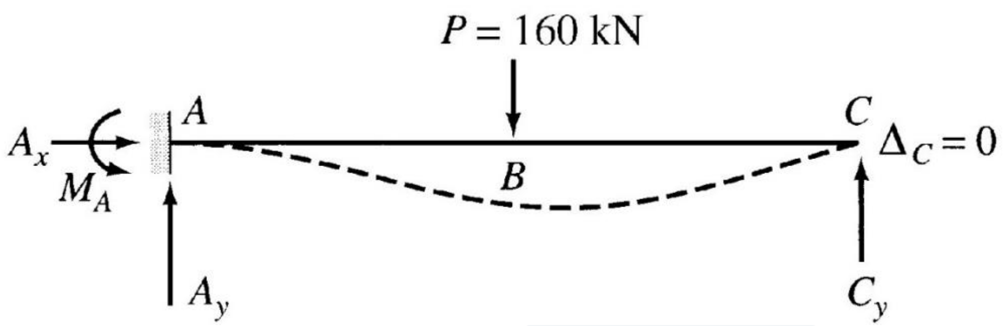


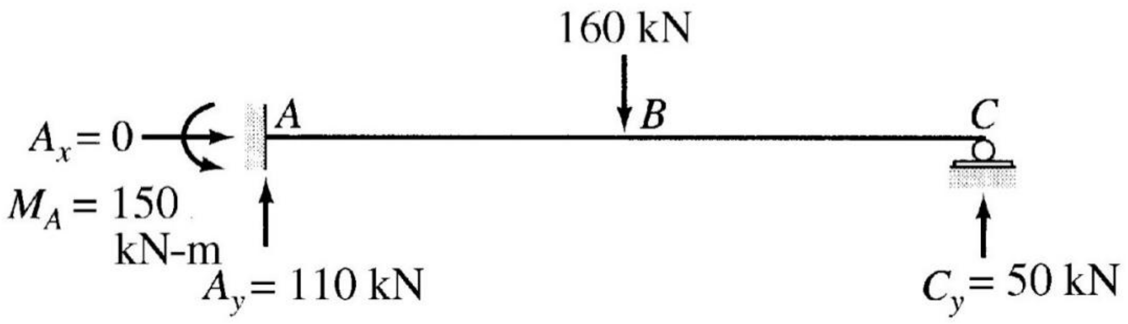
(d) Primary Beam Loaded with Redundant C_y

Force method: Beam with a single Degree of Indeterminacy

Compatibility Equation



$$C_y = -\Delta_{CO} / f_{CC} = 5P/16 = 50 \text{ kN}$$

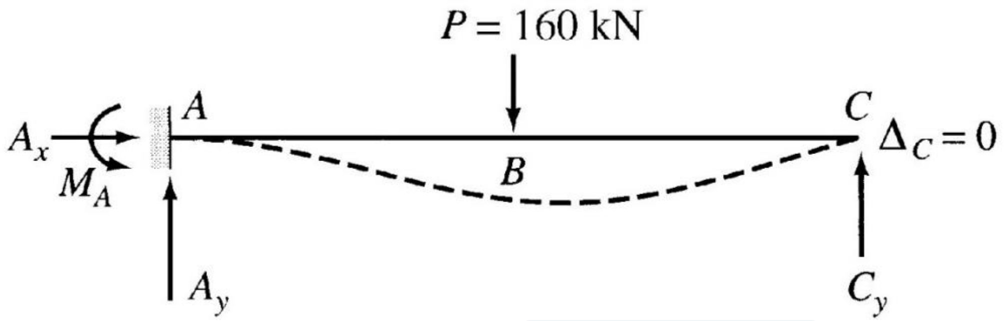


(e) Support Reactions for Indeterminate Beam

By the method of section & using the equilibrium equations, the BM & SF diagrams can be found

Force method: Beam with a single Degree of Indeterminacy

Compatibility Equation



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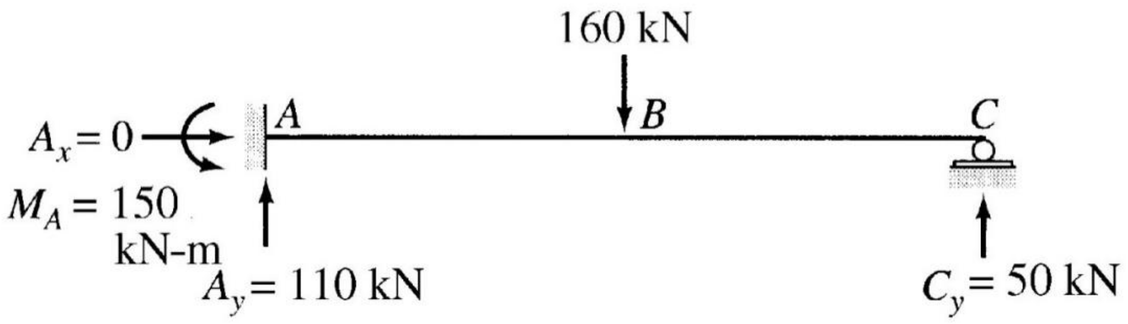
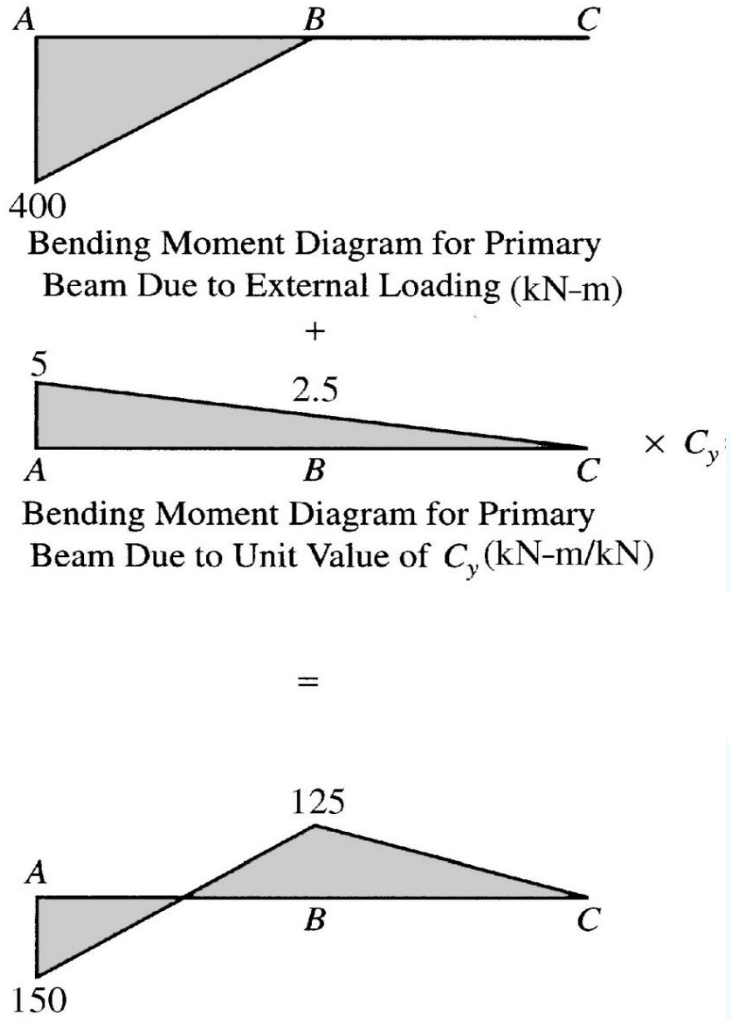


(e) Support Reactions for Indeterminate Beam

By the method of section & using the equilibrium equations, the BM & SF diagrams can be found

Force method: Beam with a single Degree of Indeterminacy

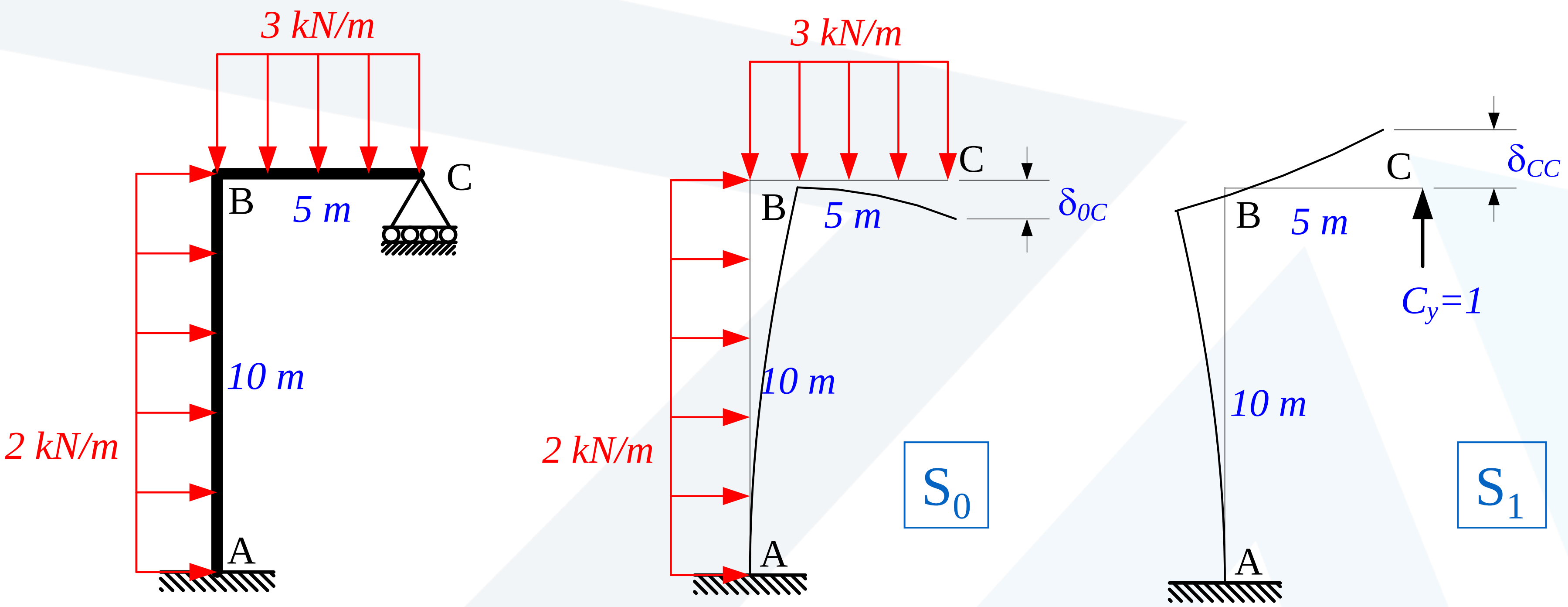
Or, the BM diagram can be found using the superposition principle as



(f) Bending Moment Diagram for Indeterminate Beam (kN-m)

Force method: Frame with a single Degree of Indeterminacy

Compute the support reactions in the frame. E= 200 GPa, I_c= 10^6 mm^4 for the column & $I_b=2I_c$, for the beam. Then draw the BM & SF diagrams

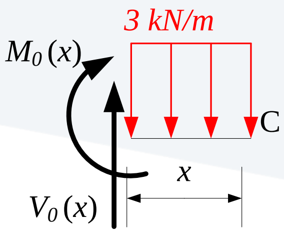
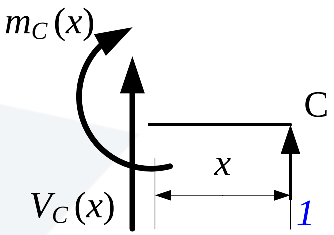
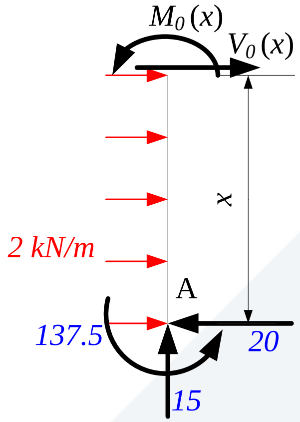
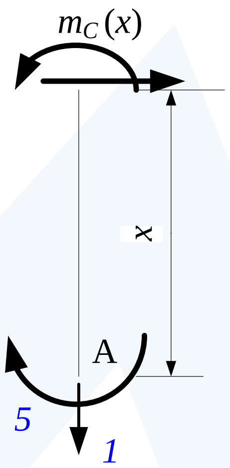


SOLUTION: The frame is statically indeterminate to degree one. Select C_y as the redundant. Draw the two determinate frames (S_0) & (S_1)

The compatibility equation is

$$\delta_{0C} + C_y \delta_{CC} = 0$$

Force method: Frame with a single Degree of Indeterminacy

Segment	FBD in S_0	$M_0(x)$	FBD in S_1	$m_C(x)$
CB $0 \le x \le 5$ *2EI* is constant		$-3x^2/2$		x
AB $0 \le x \le 10$ *EI* is constant		$-x^2+20x-137.5$		5

$$\delta_{0C} = \frac{1}{2EI}\int_0^5 \left(-\frac{3x^2}{2}\right)(x)dx + \frac{1}{EI}\int_0^{10}(-x^2+20x-137.5)(5)dx = -\frac{234.375}{2EI} - \frac{3541.67}{EI} = -\frac{3658.85}{EI}$$

$$\delta_{CC} = \frac{1}{2EI}\int_0^5 x^2 dx + \frac{1}{EI}\int_0^{10} 25 dx = \frac{270.833}{EI} \qquad \Rightarrow C_y = 13.5\,\text{kN}$$

Computing the reactions

$$\uparrow + \sum F_y = 0 \Rightarrow A_y + 13.5 - 15 = 0$$

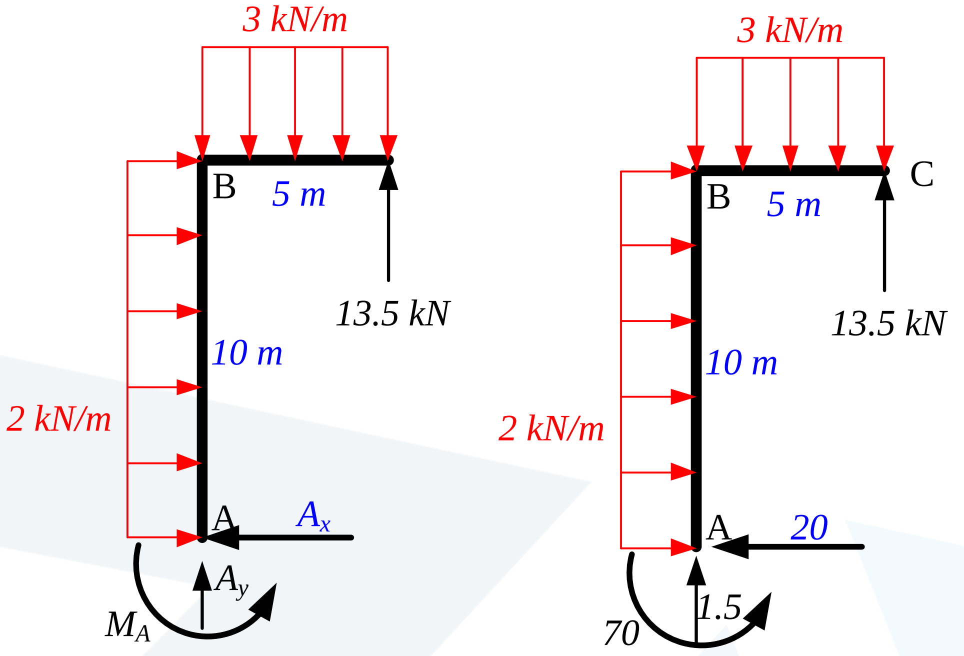
$$\Rightarrow A_y = 1.5\ kN$$

$$\overset{\rightarrow +}{\sum} F_x = 0 \Rightarrow A_x + 20 = 0 \Rightarrow$$

$$A_x = -20\ kN = 20\ kN\ (\leftarrow)$$

$$(\downarrow\uparrow +) \sum M_A = 0 \Rightarrow$$

$$M_A - (5)20 - (2.5)15 + (5)13.5 = 0 \Rightarrow M_A = 70\,\text{kN.m}$$



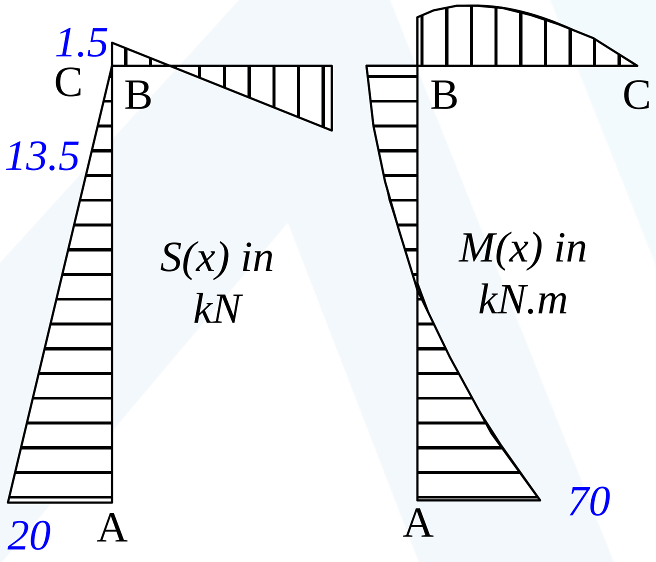
Drawing the BM & SF diagrams

In the segment AB: $0 < x < 10$,

$$M(x) = -x^2 + 20x - 70 \ \&\ S(x) = -2x + 20$$

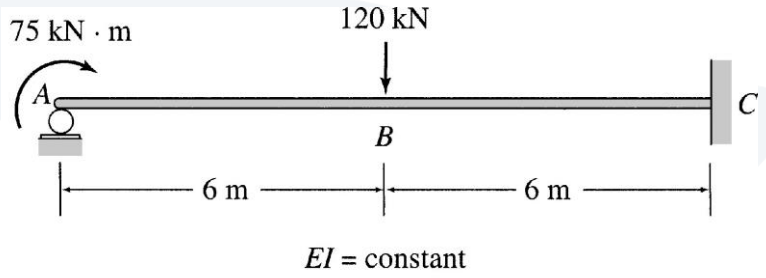
In Segment CB: $0 < x < 5$,

$$M(x) = -(3/2)x^2 + (27/2)x \ \&\ S(x) = 3x - 13.5$$



Homework

Pr.01: Determine the reactions and draw the shear and bending moment diagrams for the beam shown using force method. Select the reaction at the roller support to be the redundant.



Pr.02: Determine the reactions and draw the shear and bending moment diagrams for the frame shown using force method. Select the reaction at the roller support to be the redundant.