جامعة المنارة
كلية الهندسة

مقرر الرياضيات المتقطعة

جلسة العملي الثانية

Logical Equivalence:

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Other logical equivalences:

$p \rightarrow q \equiv \neg p \vee q$

$p \leftrightarrow q \equiv (p \longrightarrow q) \wedge (q \longrightarrow p)$

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Verify this logical equivalence

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg(p \vee (\neg p \wedge q)) \equiv (\neg p \wedge \neg(\neg p \wedge q))$ De Morgan law

$\equiv (\neg p \wedge (p \vee \neg q))$ De Morgan law

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ distributive law

$\equiv F \vee (\neg p \wedge \neg q)$ negation law

$\equiv \neg p \wedge \neg q$ identity law

Verify this logical equivalence:

$(p \longrightarrow r) \vee (q \longrightarrow r) \equiv (p \wedge q) \longrightarrow r$

$(p \longrightarrow r) \vee (q \longrightarrow r) \equiv (\neg p \vee r) \vee (\neg q \vee r)$ conditional law

$\equiv \neg p \vee r \vee \neg q \vee r$ associative law

$\equiv \neg p \vee \neg q \vee r \vee r$ commutative law

$\equiv (\neg p \vee \neg q) \vee (r \vee r)$ associative law

$\equiv (\neg p \vee \neg q) \vee r$ idempotent law

$\equiv \neg (p \wedge q) \vee r$ De Morgan law

$\equiv (p \wedge q) \longrightarrow r$ conditional law

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Verify this logical equivalence

$$(r \vee p) \longrightarrow (r \vee q) \equiv r \vee (p \longrightarrow q)$$

$(r \vee p) \longrightarrow (r \vee q) \equiv \neg(r \vee p) \vee (r \vee q)$ conditional law

$\equiv (\neg r \wedge \neg p) \vee (r \vee q)$ De Morgan law

$\equiv (\neg r \wedge \neg p) \vee r) \vee q$ associative law

$\equiv ((\neg r \vee r) \wedge (\neg p \vee r)) \vee q$ distributive law

$\equiv (T \wedge (\neg p \vee r)) \vee q$ negation law

$\equiv (\neg p \vee r) \vee q$ identity law

$\equiv \neg p \vee r \vee q$ associative law

$\equiv r \vee \neg p \vee q$ commutative law

$\equiv r \vee (\neg p \vee q)$ associative law

$\equiv r \vee (p \longrightarrow q)$ conditional law

Verify this logical equivalence:

$\neg q \longrightarrow (\neg p \lor r) \equiv p \longrightarrow (q \lor r)$

$\neg q \longrightarrow (\neg p \lor r) \equiv q \lor (\neg p \lor r)$ conditional law

$\equiv q \lor \neg p \lor r$ associative law

$\equiv \neg p \lor q \lor r$ commutative law

$\equiv \neg p \lor (q \lor r)$ associative law

$\equiv p \longrightarrow (q \lor r)$ conditional law

Verify this logical equivalence:

$\neg((\neg p \land q) \lor (\neg p \land \neg q)) \lor (p \land q) \equiv p$

$\neg((\neg p \land q) \lor (\neg p \land \neg q)) \lor (p \land q) \equiv \neg(\neg p \land (q \lor \neg q)) \lor (p \land q)$ distributive law

$\equiv \neg(\neg p \land T) \lor (p \land q)$ negation law

$\equiv \neg(\neg p) \lor (p \land q)$ identity law

$\equiv p \lor (p \land q)$ double negation law

$\equiv p$ absorption law

Show that this statement is Tautology

$(p \wedge (p \longrightarrow q)) \longrightarrow q$ (مصدوقة)

$(p \wedge (p \longrightarrow q)) \longrightarrow q \equiv (p \wedge (\neg p \vee q)) \longrightarrow q$	conditional law
$\equiv ((p \wedge \neg p) \vee (p \wedge q)) \longrightarrow q$	distributive law
$\equiv (F \vee (p \wedge q)) \longrightarrow q$	negation law
$\equiv (p \wedge q) \longrightarrow q$	identity law
$\equiv \neg(p \wedge q) \vee q$	conditional law
$\equiv (\neg p \vee \neg q) \vee q$	De Morgan law
$\equiv \neg p \vee (\neg q \vee q)$	associative law
$\equiv \neg p \vee T$	negation law
$\equiv T$ (tautology)	domination law

Show that this statement is Tautology

$(\neg q \wedge (p \vee q)) \longrightarrow p$ (مصدوقة)

$(\neg q \wedge (p \vee q)) \longrightarrow p \equiv ((\neg q \wedge p) \vee (\neg q \wedge q)) \longrightarrow p$ distributive law

$\equiv ((\neg q \wedge p) \vee F) \longrightarrow p$ negation law

$\equiv (\neg q \wedge p) \longrightarrow p$ identity law

$\equiv \neg(\neg q \wedge p) \vee p$ conditional law

$\equiv (q \vee \neg p) \vee p$ De Morgan law

$\equiv q \vee (\neg p \vee p)$ associative law

$\equiv q \vee T$ negation law

$\equiv T$ (tautology)

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Show that this statement is Contradiction

$(p \longrightarrow q) \wedge (\neg q \wedge p)$ (تناقض)

$(p \longrightarrow q) \wedge (\neg q \wedge p) \equiv (\neg p \vee q) \wedge (\neg q \wedge p)$ conditional law

$\equiv ((\neg p \vee q) \wedge \neg q) \wedge p$ associative law

$\equiv ((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \wedge p$ distributive law

$\equiv ((\neg p \wedge \neg q) \vee F) \wedge p$ negation law

$\equiv (\neg p \wedge \neg q) \wedge p$ identity law

$\equiv (\neg q \wedge \neg p) \wedge p$ commutative law

$\equiv \neg q \wedge (\neg p \wedge p)$ associative law

$\equiv \neg q \wedge F$

$\equiv F$ (contradiction)

Show that this statement is Contradiction

$(p \wedge q) \wedge \neg(p \vee q)$ (تناقض)

$(p \wedge q) \wedge \neg(p \vee q) \equiv (p \wedge q) \wedge (\neg p \wedge \neg q))$ De Morgan law

$\equiv p \wedge q \wedge \neg p \wedge \neg q$ distributive law

$\equiv p \wedge \neg p \wedge q \wedge \neg q$ commutative law

$\equiv (p \wedge \neg p) \wedge (q \wedge \neg q)$ distributive law

$\equiv F \wedge F$ negation law

$\equiv F$ (contradiction)

Determine if the following expressions are equivalent by using truth table

$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r) \equiv q \longrightarrow r$ (?)

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$p \rightarrow r$	$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r)$	$q \longrightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F
T	F	T	T	T	T	T	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	F
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

من جدول الحقيقة نجد عدم تطابق في قيم الحقيقة بين العبارتين في السطرين الرابع و السادس بالتالي العبارتين غير متكافئتين منطقياً

Determine if the following expressions are equivalent by using logical Equivalences

$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r) \equiv q \longrightarrow r$ (?)

$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r) \equiv (\neg p \vee (q \vee r)) \wedge (\neg p \vee r)$ conditional law

$\equiv (\neg p \vee ((q \vee r) \wedge r)$ distributive law

$\equiv \neg p \vee r$ absorption law

$\equiv p \longrightarrow r$ conditional law

$\not\equiv q \longrightarrow r$

العبارتين غير متكافئتين منطقياً

Use De Morgan 's law to write negations for the statements :

- Tom is 6 feet tall and he weighs at least 60 kg.
- Negation:Tom is not 6 feet tall or he weighs less than 60 kg.
- The bus was late or Tom's watch was slow.
- Negation:The bus was not late and Tom's watch was not slow.
- $1 < x \leq 4$
- Negation: $1 \geq x$ or $x > 4$

Writing logical formula for a truth table

formula	p	q	r	output
$p\wedge q \wedge r$	T	T	T	T
$p\wedge q \wedge\neg r$	T	T	F	T
$p\wedge \neg q \wedge r$	T	F	T	T
$p\wedge \neg q \wedge \neg r$	T	F	F	F
$\neg p\wedge q \wedge r$	F	T	T	T
$\neg p\wedge q \wedge \neg r$	F	T	F	F
$\neg p\wedge \neg q \wedge r$	F	F	T	F
$\neg p\wedge \neg q \wedge \neg r$	F	F	F	F

Idea 1: Look at the true rows and take the **"or"**.

$$(p\wedge q \wedge r)\vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$$

Idea 2: Look at the false rows, negate and take the **"and"**.

$$\neg(p \wedge \neg q \wedge \neg r)\wedge \neg(\neg p \wedge q \wedge \neg r) \wedge \neg(\neg p \wedge \neg q \wedge r) \wedge \neg(\neg p \wedge \neg q \wedge \neg r)$$

Write (inverse ,converse,contra-positive) for these conditional sentences:

- If $(x > 0)$ and $(y > 0)$ then $(x+y > 0)$
- **Inverse:** if $(x \leq 0)$ or $(y \leq 0)$ then $(x+y \leq 0)$
- **Converse:** if $(x+y > 0)$ then $(x > 0)$ and $(y > 0)$
- **Contra-positive:** if $(x+y \leq 0)$ then $(x \leq 0)$ or $(y \leq 0)$

Arguments
(الحجج)

Valid argument

Valid Argument Forms

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$		

Use truth table to prove that the following argument is valid:

1. $p \longrightarrow (q \vee r)$
2. $\neg q$

$\therefore\ p \longrightarrow r$

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$\neg q$	$p \longrightarrow r$
T	T	T	T	T	F	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	F	T	T	T

Argument is valid (الحجة صالحة)

Use truth table to prove that the following argument is invalid:

1. $p \lor r$
2. $(p \longrightarrow q) \land (q \longrightarrow r)$

$\therefore \neg q$

p	q	r	$p \lor r$	$p \to q$	$q \to r$	$(p \longrightarrow q) \land (q \longrightarrow r)$	$\neg q$
T	T	T	T	T	T	T	F
T	T	F	T	T	F	F	F
T	F	T	T	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	F
F	T	F	F	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

استناداً إلى السطر الأول أو السطر الخامس نجد أن الحجة غير صالحة (Argument is invalid)

Use truth table to prove that the following argument is valid:

1. p ∨ ¬q
2. ¬p ∨ q

∴ p ⟶ q

p	q	¬p	¬q	p∨¬q	¬p ∨ q	p ⟶ q
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Argument is valid (الحجة صالحة)

prove that the following argument is valid:

argument

a. $\neg p \wedge q$

b. $r \longrightarrow p$

c. $\neg r \longrightarrow s$

d. $s \longrightarrow t$

$\therefore t$

1.	$\neg p \wedge q$	from (a)
	$\therefore \neg p$	by specialization
2.	$r \longrightarrow p$	from (b)
	$\neg p$	from (1)
	$\therefore \neg r$	by modus tollens
3.	$\neg r \longrightarrow s$	from (c)
	$\neg r$	from (2)
	$\therefore s$	by modus ponens
4.	$s \longrightarrow t$	from (d)
	s	from (3)
	$\therefore t$	by modus ponens

Argument is valid