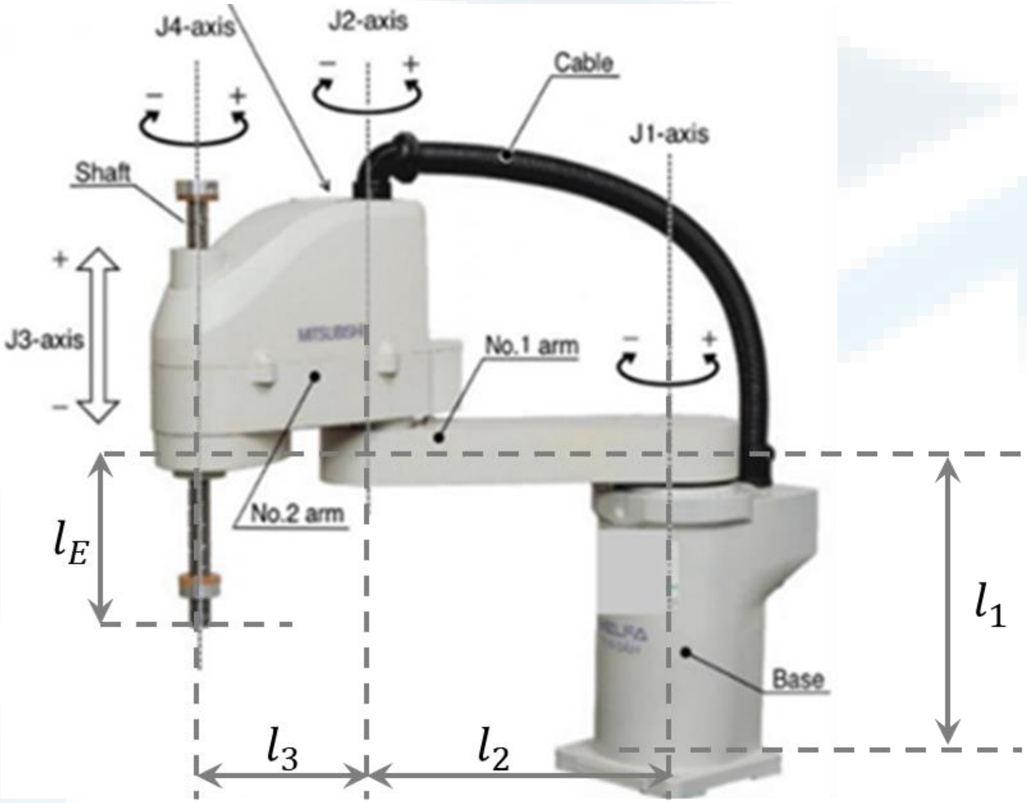
Inverse kinematic model

Study case SCARA Robot

SCARA RRPR



J4-axis
J2-axis
Cable
J1-axis
Shaft
J3-axis
No.1 arm
No.2 arm
Base
l_E
l_1
l_3
l_2

DH Table

Joint	a_i	α_i	d_i	θ_i
1	l_2	0	l_1	$\boldsymbol{\theta}_1$
2	l_3	0	0	$\boldsymbol{\theta}_2$
3	0	0	$-\boldsymbol{d}_3$	0
4	0	0	-l_E	$\boldsymbol{\theta}_4$

DKM

Find the actual parameters when :

$$\theta_1 = \frac{\pi}{3}, \theta_2 = \frac{\pi}{6}, d_3 = 100 \text{ mm}, \theta_4 = -\frac{\pi}{4}$$

Links length:

$$l_1 = 500, l_2 = 400, l_3 = 300, l_E = 50\ [mm]$$

DH Matrix:

$$T_{i-1}^{i} = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation from Base to End-Effector

```
Tr0E =

[ C124, -S124, 0, 400*C1 + 300*C12]
[ S124,  C124, 0, 400*S1 + 300*S12]
[    0,     0, 1,         450 - d3]
[    0,     0, 0,                1]
```

```
T0E =

    0.7071   -0.7071         0   200.0000
    0.7071    0.7071         0   646.4102
         0         0    1.0000   350.0000
         0         0         0     1.0000
```

Orientation Matrices

Rotation Matrix

R =

0.7071	-0.7071	0
0.7071	0.7071	0
0	0	1.0000

RPY

$$\begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\gamma + c_\alpha s_\beta s_\gamma & s_\alpha s_\gamma + c_\alpha s_\beta c_\gamma \\ s_\alpha c_\beta & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & -c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix}$$

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Solution

Position

- $E_x = 200\ mm$
- $E_y = 300 + 200\sqrt{3} = 646.41$
- $E_z = 350\ \text{mm}$

Orientation RPY angels

- $\alpha = \frac{\pi}{4}$
- $\beta = 0$
- $\gamma = 0$

IKM: Find the generalized parameters needed to put the end effector in the following configuration

Position

- $E_x = 200\ mm$
- $E_y = 300 + 200\sqrt{3}\ mm$
- $E_z = 350\ \text{mm}$

Orientation RPY angels

- $\alpha = \frac{\pi}{4}$
- $\beta = 0$
- $\gamma = 0$

Comparing two matrices

```
TrOE =

[ C124, -S124, 0, 400*C1 + 300*C12]
[ S124,  C124, 0, 400*S1 + 300*S12]
[    0,     0, 1,         450 - d3]
[    0,     0, 0,                1]
```

$$T_0^3 = \begin{pmatrix} 0.7071 & -0.7071 & 0 & E_x \\ 0.7071 & 0.7071 & 0 & E_y \\ 0 & 0 & 1.0000 & E_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

θ_2

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} C_1 L_2 + C_{12} L_3 \\ S_1 L_2 + S_{12} L_3 \\ L_1 - L_E - d_3 \end{bmatrix}$$

$$d_3 = L_1 - L_E - E_z = 100$$

$$\theta_2 = \pm acos(const) : const = \frac{{E_x}^2 + {E_y}^2 - {L_2}^2 - {L_3}^2}{2L_2L_3} = \frac{\sqrt{3}}{2} \Rightarrow \theta_2 = \pm\frac{\pi}{6} = \pm 30$$

First solution

$$\theta_2 = +\frac{\pi}{6} = 30$$

$$\left.\begin{aligned}(L_2 + L_3C_2) \times C_1 - L_3S_2 \times S_1 &= E_x \\ L_3S_2 \times C_1 + (L_2 + L_3C_2) \times S_1 &= E_y\end{aligned}\right\} \Rightarrow \begin{bmatrix} C_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \Rightarrow \theta_1 = \frac{\pi}{3} = 60$$

$$\theta_4 = atan2(0.7071, 0.7071) - \theta_1 - \theta_2 = -\frac{\pi}{4}$$

Comparing rotation matrix to RPY matrix

```
Rotation_Matrix =

[ C124, -S124, 0]
[ S124,  C124, 0]
[    0,     0, 1]
```

```
>> [ Orient ] = Roll_Pitch_Yaw_Matrix(pi/4,0,0)

Orient =

    0.7071   -0.7071         0
    0.7071    0.7071         0
         0         0    1.0000
```

Second solution

$$\theta_2 = -\frac{\pi}{6} = -30$$

$$\left.\begin{array}{l}(L_2 + L_3 C_2) \times C_1 - L_3 S_2 \times S_1 = E_x \\ L_3 S_2 \times C_1 + (L_2 + L_3 C_2) \times S_1 = E_y\end{array}\right\} \Rightarrow \theta_1 = 85.6 \Rightarrow \theta_4 = -10.6$$

Solutions

	$\boldsymbol{\theta}_1$	$\boldsymbol{\theta}_2$	$\boldsymbol{d}_3$	$\boldsymbol{\theta}_4$
Solution 1	60	30	100 mm	-45
Solution 2	85.6	-30	100 mm	-10.6

Thanks