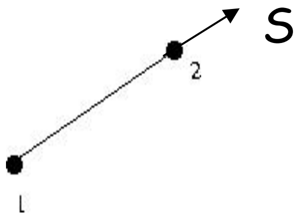
العناصر الحدية في الموديل ثنائي البعد



نستخدم عنصراً منتهياً في موديل أحادي البعد على شكل جائزٍ له عقدتان

شرط Neumann:

$$W^e_{Neu} = \int_0^{Le} \vec{q}.\vec{n}\, ds$$

$$\vec{q}.\vec{n} = q_0$$

$$N_1(s) = 1 - \frac{s}{L^e}, \quad N_2(s) = \frac{s}{L^e} \quad 0 \le s \le L^e$$

$$W^e_{Neu} = \int_0^{Le} \psi q_0\, ds = \langle \psi_1 \quad \psi_2 \rangle \frac{q_0 L^e}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = \langle \psi_1 \quad \psi_2 \rangle \{F^e_{Neu}\}$$

شرط Cauchy:

$$W^e_{cauchy} = \int_0^{Le} \vec{q}.\vec{n}\, ds$$

$$\vec{q}.\vec{n} = \psi h (T - T_{ext})$$

$$W^e_{Cau} = \int_0^{Le} \psi h (T - T_{ext}) ds$$

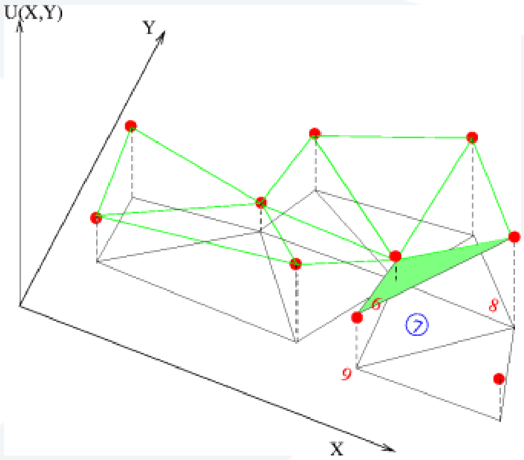
$$W^e_{Cau} = \int_0^{Le} \psi h (T(s) - T_{ext}) ds = \langle \psi_1 \quad \psi_2 \rangle \frac{hL^e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} - \langle \psi_1 \quad \psi_2 \rangle \frac{hT_{ext} L^e}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \langle \psi_1 \quad \psi_2 \rangle \left(\left[K^e_{Cau} \right] \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} - \{F^e_{Cau}\} \right)$$

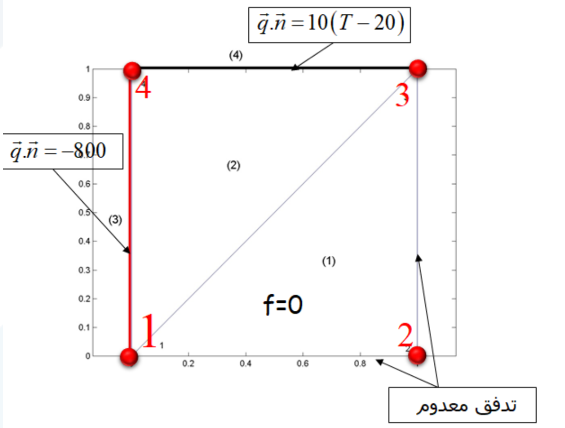
شرط Dirichlet

هذا الشرط يتم إدخاله في جملة المعادلات النهائية في آخر خطوة

الحل التقريبي على كل عنصر ممثل في الشكل التالي:



تطبيق:



إحداثيات النقاط

$$vcorg = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

مصفوفة الارتباط:

$$conec = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 0 \\ 4 & 1 & 0 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

حساب المصفوفات والأشعة العنصرية

العنصر الأول:

$$vcore_{el1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$2 \times A^e = (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)$$

$$2 \times A^e = (1-0) \times (1-0) - (1-0) \times (0-0)$$

$$2 \times A^e = 1 \times 1 - 1 \times 0 = 1 \Rightarrow A^e = 0.5$$

$$[B]^{el1} = \frac{1}{2A^e} \begin{bmatrix} y_{23} & y_{31} & y_{12} \\ x_{32} & x_{13} & x_{21} \end{bmatrix} = \frac{1}{2 \times 0.5} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \Rightarrow [B]^{el1} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$vk^{el1} = kA^{el1} \left([B]^{el1}\right)^T [B]^{el1} =$$

$$105 \times \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} +105 & -105 & 0 \\ -105 & 210 & -105 \\ 0 & -105 & 105 \end{bmatrix}$$

العنصر الثاني:

$$vcore_{el2} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$2 \times A^e = (1-0)(1-0) - (0-0)(1-0)$$

$$2 \times A^e = 1 \times 1 - 0 \times 1 = 1 \Rightarrow A^e = 0.5$$

$$[B]^{el2} = \frac{1}{2A^e} \begin{bmatrix} y_{23} & y_{31} & y_{12} \\ x_{32} & x_{13} & x_{21} \end{bmatrix} = \frac{1}{2 \times 0.5} \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow [B]^{el2} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$vk^{el2} = kA^{el2}\left([B]^{el2}\right)^T [B]^{el2} =$$

$$105 \times \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 105 & 0 & -105 \\ 0 & 105 & -105 \\ -105 & -105 & 210 \end{bmatrix}$$

حساب المصفوفات والأشعة العنصرية على حدود المجال:

$$Wcl = W_{Dir} + \sum_e W_{Neu}^e + \sum_e W_{Cau}^e$$

$$W_{Neu}^e = \int_0^{Le} \psi q_0 \, ds = \int_0^{Le} \langle \psi_4 \quad \psi_1 \rangle \begin{Bmatrix} 1 - \frac{s}{L_e} \\ \frac{s}{L_e} \end{Bmatrix} q_0 \, ds = \langle \psi_4 \quad \psi_1 \rangle q_0 \begin{bmatrix} s - \frac{s^2}{2L_e} \\ \frac{s^2}{2L_e} \end{bmatrix}_0^{L_e}$$

$$W_{Neu}^e = \langle \psi_4 \quad \psi_1 \rangle q_0 \begin{bmatrix} L_e - \frac{L_e}{2} \\ \frac{L_e}{2} \end{bmatrix} = \langle \psi_4 \quad \psi_1 \rangle q_0 \frac{L_e}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$W_{Neu}^{e}=-\langle\psi_4 \quad \psi_1\rangle 800\frac{L_e^{(neumann)}}{2}\begin{Bmatrix}1\\1\end{Bmatrix}=-\langle\psi_4 \quad \psi_1\rangle\begin{Bmatrix}400\\400\end{Bmatrix}$$

$$\Rightarrow vf_e^{(Neumann)}=\begin{Bmatrix}400\\400\end{Bmatrix}$$

$$W_{Cau}^{e}=\int_{S_{Cau}}\langle\psi_3 \quad \psi_4\rangle\begin{Bmatrix}1-\frac{s}{L_e}\\ \frac{s}{L_e}\end{Bmatrix}h\left\langle 1-\frac{s}{L_e} \quad \frac{s}{L_e}\right\rangle\begin{Bmatrix}T_1\\T_2\end{Bmatrix}ds-\int_{S_{Cau}}\langle\psi_3 \quad \psi_4\rangle\begin{Bmatrix}1-\frac{s}{L_e}\\ \frac{s}{L_e}\end{Bmatrix}h\ T_{ext}\,ds$$

$$W_{Cau}^{e}=\int_{S_{Cau}}\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}1-2\frac{s}{L_{el_{cauchy}}}+\frac{s^2}{L_{el_{cauchy}}^2} & \frac{s}{L_{el_{cauchy}}}-\frac{s^2}{L_{el_{cauchy}}^2}\\ \frac{s}{L_{el_{cauchy}}}-\frac{s^2}{L_{el_{cauchy}}^2} & \frac{s^2}{L_{el_{cauchy}}^2}\end{bmatrix}_0^{L_{el\,4}}h\begin{Bmatrix}T_3\\T_4\end{Bmatrix}ds-\int_{S_{Cau}}\langle\psi_3 \quad \psi_4\rangle\begin{Bmatrix}1-\frac{s}{L_{el_{cauchy}}}\\ \frac{s}{L_{el_{cauchy}}}\end{Bmatrix}h\ T_{ext}\,ds\Rightarrow$$

$$W_{Cau}^{e}=\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}s-2\frac{s^2}{2L_{el_{cauchy}}}+\frac{s^3}{3L_{el_{cauchy}}^2} & \frac{s^2}{2L_{el_{cauchy}}}-\frac{s^3}{3L_{el_{cauchy}}^2}\\ \frac{s^2}{2L_{el_{cauchy}}}-\frac{s^3}{3L_{el_{cauchy}}^2} & \frac{s^3}{3L_{el_{cauchy}}^2}\end{bmatrix}_0^{L_{el\,4}}h\begin{Bmatrix}T_3\\T_4\end{Bmatrix}-\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}L_{el_{cauchy}}-\frac{L_{el_{cauchy}}}{2}\\ \frac{L_{el_{cauchy}}}{2}\end{bmatrix}h\ T_{ext}\Rightarrow$$

$$W_{Cau}^{e}=\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}L_{el_{cauchy}}-L_{el_{cauchy}}+\frac{L_{el_{cauchy}}}{3} & \frac{L_{el_{cauchy}}}{2}-\frac{L_{el_{cauchy}}}{3}\\ \frac{L_{el_{cauchy}}}{2}-\frac{L_{el_{cauchy}}}{3} & \frac{L_{el_{cauchy}}}{3}\end{bmatrix}h\begin{Bmatrix}T_3\\T_4\end{Bmatrix}-\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}L_{el_{cauchy}}-\frac{L_{el_{cauchy}}}{2}\\ \frac{L_{el_{cauchy}}}{2}\end{bmatrix}h\ T_{ext}\Rightarrow$$

$$W_{Cau}^{e}=\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}\frac{L_{el_{cauchy}}}{3} & \frac{L_{el_{cauchy}}}{6}\\ \frac{L_{el_{cauchy}}}{6} & \frac{L_{el_{cauchy}}}{3}\end{bmatrix}h\begin{Bmatrix}T_3\\T_4\end{Bmatrix}-\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}\frac{L_{el_{cauchy}}}{2}\\ \frac{L_{el_{cauchy}}}{2}\end{bmatrix}h\ T_{ext}\Rightarrow \boxed{W_{Cau}^{e}=\langle\psi_3 \quad \psi_4\rangle\frac{L_{el_{cauchy}}}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}h\begin{Bmatrix}T_3\\T_4\end{Bmatrix}-\langle\psi_3 \quad \psi_4\rangle\frac{L_{el_{cauchy}}}{2}\begin{bmatrix}1\\1\end{bmatrix}h\ T_{ext}}$$

$$W_{Cau}^{el}=\langle\psi_3 \quad \psi_4\rangle\begin{bmatrix}3.33.. & 1.66..\\1.66.. & 3.33..\end{bmatrix}\begin{Bmatrix}T_3\\T_4\end{Bmatrix}-\langle\psi_3 \quad \psi_4\rangle 10\times 20\,\frac{1}{2}\begin{Bmatrix}1\\1\end{Bmatrix}\Rightarrow$$

$$W_{Cau}^{el}=\langle\psi_3 \quad \psi_4\rangle\left(\begin{bmatrix}3.33.. & 1.66..\\1.66.. & 3.33..\end{bmatrix}\begin{Bmatrix}T_3\\T_4\end{Bmatrix}-\begin{Bmatrix}100\\100\end{Bmatrix}\right)$$

$$vk_e^{(cauchy)} = \begin{bmatrix} 3.33.. & 1.66.. \\ 1.66.. & 3.33.. \end{bmatrix}, \quad vf_e^{(cauchy)} = \begin{Bmatrix} 100 \\ 100 \end{Bmatrix}$$

<u>مرحلة التجميع:</u>

<u>أولاً: تجميع مصفوفات الصلابة العنصرية</u>

$$vk^{el1} = \begin{bmatrix} +105 & -105 & 0 \\ -105 & 210 & -105 \\ 0 & -105 & 105 \end{bmatrix}$$

$$vk^{el2} = \begin{bmatrix} 105 & 0 & -105 \\ 0 & 105 & -105 \\ -105 & -105 & 210 \end{bmatrix}$$

$$vk_e^{(cauchy)} = \begin{bmatrix} 3.33.. & 1.66.. \\ 1.66.. & 3.33.. \end{bmatrix}$$

$$vkg = \begin{bmatrix} 210 & -105 & 0 & -105 \\ -105 & 210 & -105 & 0 \\ 0 & -105 & 213.33 & -103.34 \\ -105 & 0 & -103.34 & 213.33 \end{bmatrix}$$

ثانياً : تجميع أشعة التحريض العنصرية:

$$\left\{F^{(el\,3)}\right\}=\begin{Bmatrix}400\\400\end{Bmatrix}$$

$$vf_e^{(cauchy)}=\begin{Bmatrix}100\\100\end{Bmatrix}$$

$$\{F\}=\begin{Bmatrix}400\\0\\100\\400+100\end{Bmatrix}=\begin{Bmatrix}400\\0\\100\\500\end{Bmatrix}$$

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T1=103.0557
T2=101.1547
T3= 99.2537
T4=101.1471
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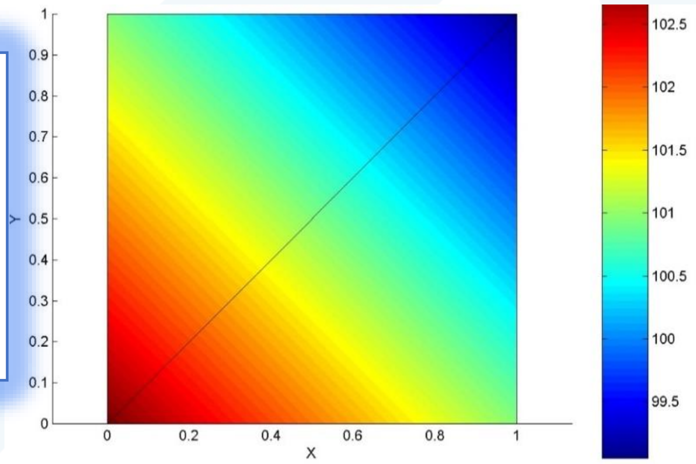
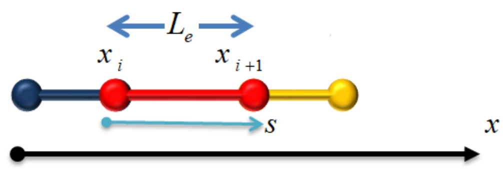


العنصر المرجعي أحادي البعد:

- *تغيير المتحول*



$$x(s) = s + x_i$$

$$\frac{dx}{ds} = 1$$

$$x \in [x_i, x_{i+1}] \rightarrow s \in [0, L^e], L^e = x_{i+1} - x_i$$

$$\int_{x_i}^{x_{i+1}} (\dots) dx = \int_0^{L^e} (\dots) ds$$

$$\sum_{i=1}^{N} \int_{x_i}^{x_{i+1}} (\dots) dx = \sum_{e=1}^{nelt} \int_0^{L^e} (\dots) ds$$

توابع التقريب على العنصر المرجعي خطية وتعطى بالعلاقات التالية:

$$N_1(x) = \frac{x_{i+1} - x}{L_e}$$

$$N_2(x) = \frac{x - x_i}{L_e}$$

$$N_1(s) = 1 - \frac{s}{L^e}, N_2(s) = \frac{s}{L^e}$$