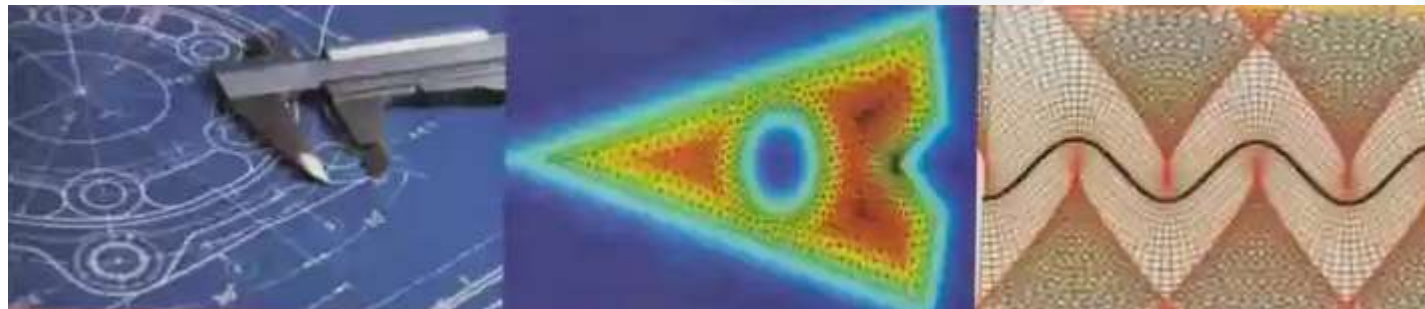


CEDC301: Mathematics Engineering

Exercises 1: Functions of a Complex Variable: Part A



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1. If $f(z) = z^{24} - 3z^{20} + 4z^{12} - 5z^6$, find $f\left(\frac{1+i}{\sqrt{2}}\right)$

2. Let z and w be complex numbers such that $|z| = 1$ and $|w| \neq 1$. Prove that

$$\left| \frac{z - w}{1 - z\bar{w}} \right| = 1$$

3. Using the epsilon-delta definition, Prove that

$$\lim_{z \rightarrow 1+i} (2+i)z = 1 + 3i$$

4. Using the epsilon-delta definition, Prove that

$$\lim_{z \rightarrow 1+i} [(1-i)z + 2i] = 2 + 2i$$

5. Show that the given limit does not exist

$$\lim_{z \rightarrow 0} \left(\frac{2xy}{x^2 + y^2} - \frac{y^2}{x^2} i \right)$$

6. Show that the given limit does not exist

$$\lim_{z \rightarrow 0} \left(\frac{z}{\bar{z}} \right)^2$$

7. Determine whether or not the limit exists

$$\lim_{z \rightarrow 0} \left(\frac{2y^2}{x^2} - \frac{x^2 - y^2}{y^2} i \right)$$

8. Show that the function $f(z) = -(2xy + 5x) + i(x^2 - 5y - y^2)$ is analytic for all z . Find $f'(z)$.
9. Determine whether the function $f(z) = x^3 + xy^2 - 4x + i(4y - y^3 - x^2y)$ is differentiable. Is it analytic?