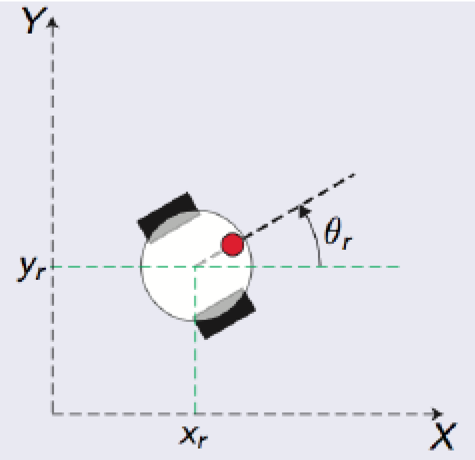
WMR Kinematic model

Unicycle & Bicycle

Unicycle Model

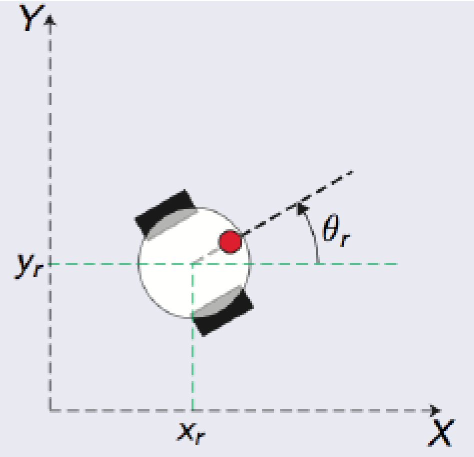
- The configuration is described by $q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$
- Constraint: $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$
- Pfaffian Form: $A(q)\dot{q} = 0$ with:

$$\begin{cases} A(q) = [\sin\theta, -\cos\theta, 0] \\ q = [x, y, \theta]^T \end{cases}$$



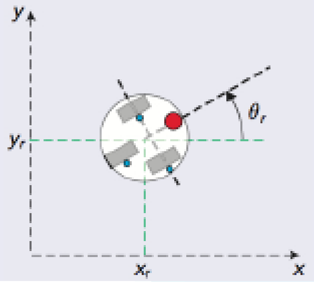
$$\mathrm{Ker}(A(q)) = span\left(\begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \mathrm{Im}(G(q))$$

What is the difference between…..



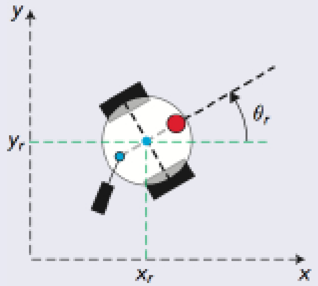
Synchronized Drive Model

- Adjustable parallel wheels
- Same inputs $[v, \omega]^T$
- $[x, y, \theta]^T$ position of any chones point of the robot, robot orientation



Differential Drive Model

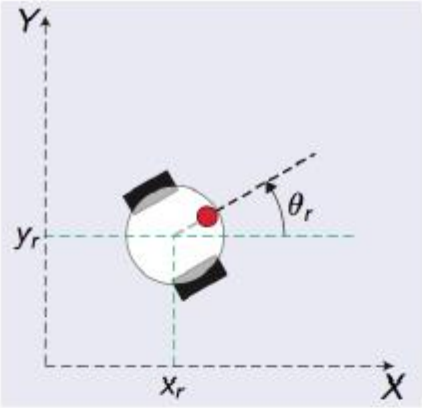
- Two wheels separately controlled
- A passive wheel for static support
- $[x, y, \theta]^T$ position of the wheelbase midpoint, robot orientation



Differential drive model

Remarks

- Most popular unicycle type (kinematically equivalent)
- Two independent coaxial wheels
- One or more passive castor wheels added for static stability
- It can rotate on the spot if $\omega_R = -\omega_L$ are set



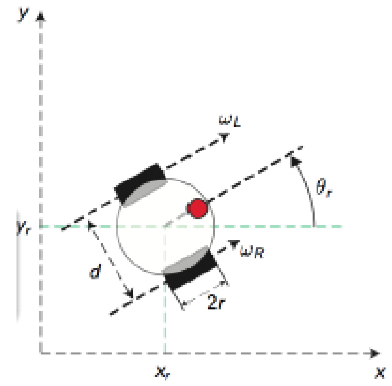
Khepera III
(K-Team, EPFL)

Sbot (EPFL)

Sentinel (iRobot)

Differential drive final model

- Defining $\begin{cases} \omega_R = \text{right wheel speed} \\ \omega_L = \text{left wheel speed} \\ d = \text{wheelbase} \\ r = \text{wheel radius} \end{cases}$



Model with inputs ω_R, ω_L

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{d} & -\frac{r}{d} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}$$

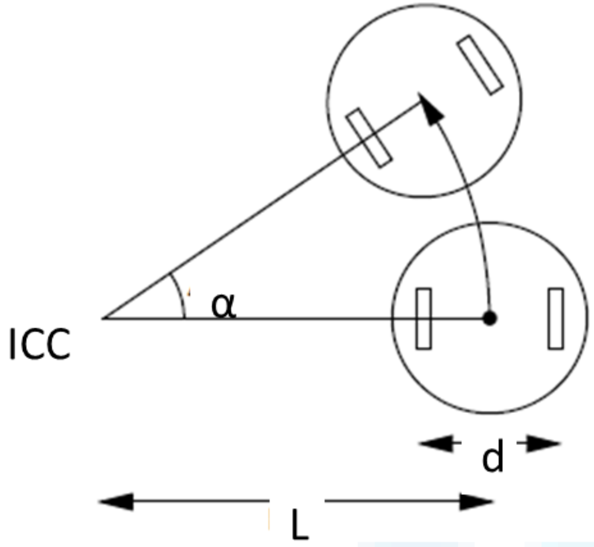
In the State Space

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{d} & -\frac{r}{d} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} =$$

$$\begin{bmatrix} \frac{r\cos\theta}{2} & \frac{r\cos\theta}{2} \\ \frac{r\sin\theta}{d} & \frac{r\sin\theta}{d} \\ \frac{r}{d} & -\frac{r}{d} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}$$

Circular path of differential drive robot

- $\omega = \frac{v_r}{L+\frac{d}{2}} = \frac{v_l}{L-\frac{d}{2}}$
- $\omega = \frac{v_r - v_l}{d}$
- $v = \frac{v_r + v_l}{2}$
- $L = \frac{d}{2}\left(\frac{v_r + v_l}{v_r - v_l}\right)$



Bicycle Model

Definition

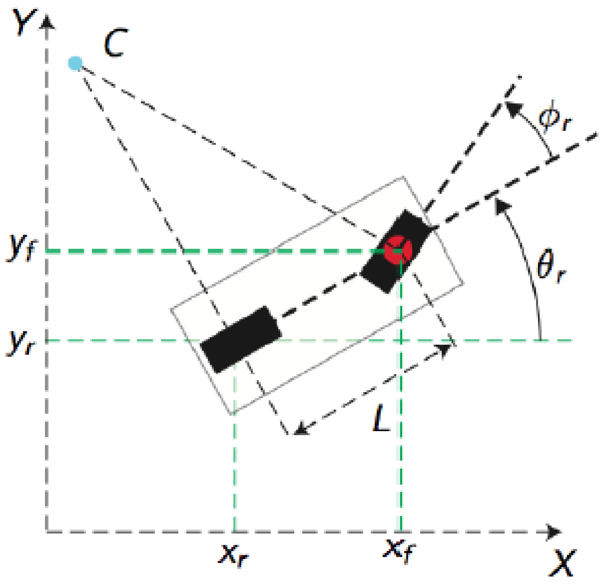
A bicycle is a vehicle having a caster (adjustable wheel) and a fixed wheel with their rotation axes perpendicular to the longitudinal plane.

- The configuration is described by:

$$q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix}$$

- C = instantaneous center of rotation

We consider only the case with front-wheel drive



Bicycle kinematic Model

System subject to two constraints, one for each wheel.

$$
\begin{cases}
\dot{x}_f \sin(\theta_r + \phi_r) - \dot{y}_f \cos(\theta_r + \phi_r) & = 0 \\
\dot{x}_r \sin(\theta_r) - \dot{y}_r \cos(\theta_r) & = 0
\end{cases}
$$

The point (x_f, y_f) represents the Cartesian position of the contact point between the front wheel and the ground.

The points (x_r, y_r), (x_f, y_f) are related one to the other. Indeed:

$$
\begin{aligned}
x_f &= x_r + L\cos\theta_r \\
y_f &= y_r + L\sin\theta_r
\end{aligned}
$$

$$
\begin{aligned}
x_r &= x_f - L\cos\theta_r \\
y_r &= y_f - L\sin\theta_r
\end{aligned}
$$

$$
\dot{x}_f \sin(\theta_r + \phi_r) - \dot{y}_f \cos(\theta_r + \phi_r) = 0
$$

$$\Downarrow$$

$$
\dot{x}_r \sin(\theta_r + \phi_r) - \dot{y}_r \cos(\theta_r + \phi_r) - L\dot{\theta}_r \cos\phi_r = 0
$$

Bicycle kinematic constraints

- Then, the kinematic constraints of the bicycle are:

$$\begin{cases} \dot{x}_r \sin(\theta_r + \phi_r) - \dot{y}_r \cos(\theta_r + \phi_r) - \dot{\theta}_r \cos\phi_r &= 0 \\ \dot{x}_r \sin(\theta_r) - \dot{y}_r \cos(\theta_r) &= 0 \end{cases}$$

In Pfaffian form

$$A(q) = \begin{bmatrix} \sin\theta_r & -\cos\theta_r & 0 & 0 \\ \sin(\theta_r + \phi_r) & -\cos(\theta_r + \phi_r) & -L\cos\phi_r & 0 \end{bmatrix}$$

- Then:

$$\text{Ker}(A(q)) = span\left(\begin{bmatrix} \cos\theta_r \cos\phi_r \\ \sin\theta_r \cos\phi_r \\ \frac{1}{L}\sin\phi_r \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \right) = \text{Im}(G(q))$$

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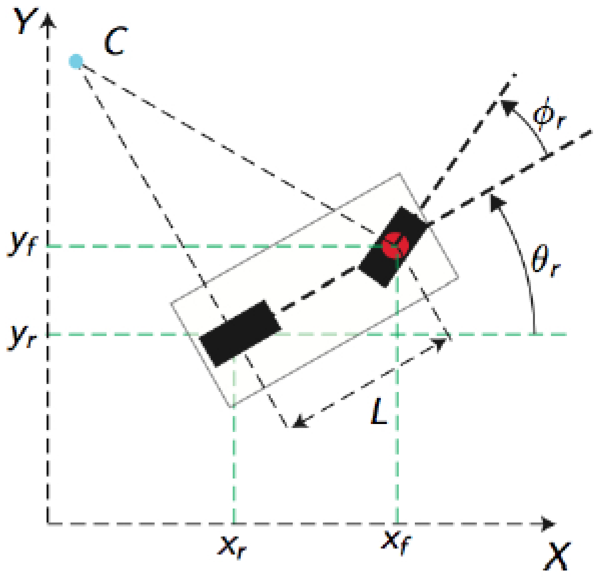
Bicycle kinematic Model

- v: linear traction velocity
- ω: angular velocity of the vehicle



$$\dot{q} = \begin{bmatrix} \cos\theta_r \cos\phi_r \\ \sin\theta_r \cos\phi_r \\ \frac{1}{L}\sin\phi_r \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega = \begin{bmatrix} \cos\theta_r \cos\phi_r & 0 \\ \sin\theta_r \cos\phi_r & 0 \\ \frac{1}{L}\sin\phi_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

Unicycle and Bicycle

- The kinematic structure unicycle and bicycle are the most used and widespread applications (especially industrial)
- Other kinematic structures are used for particular applications
- Kinematic models of more complex structures are obtained taking into account the constraints introduced by each wheel

Thanks

Think about wheels (number and type) you want to use when designing a WMR.....