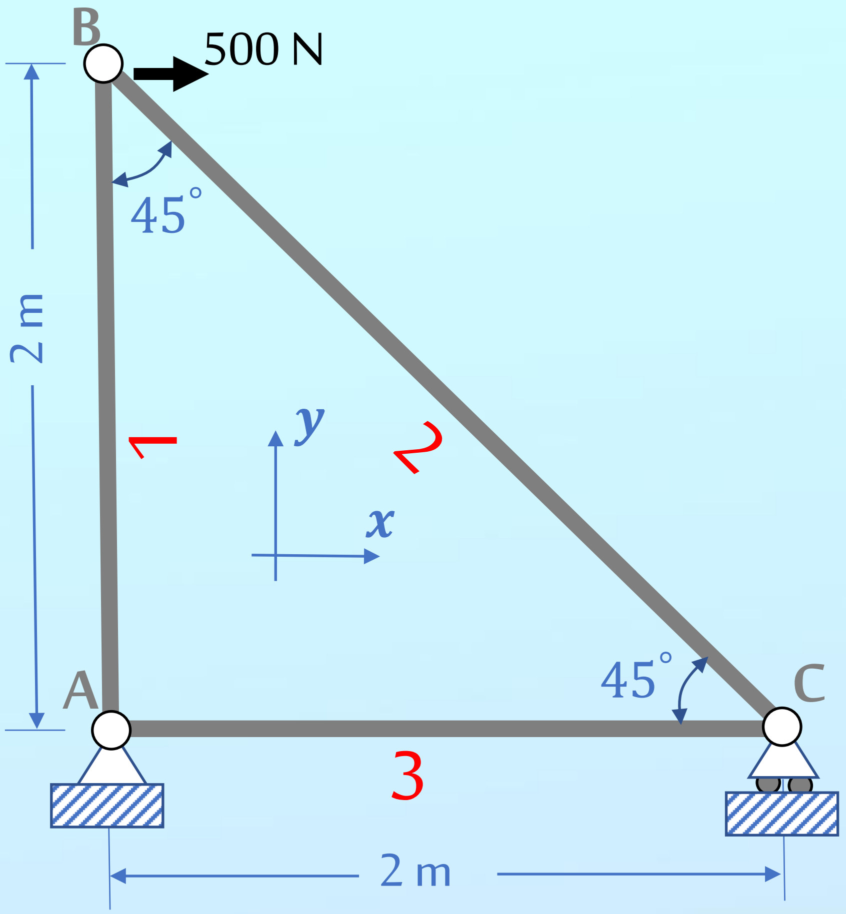


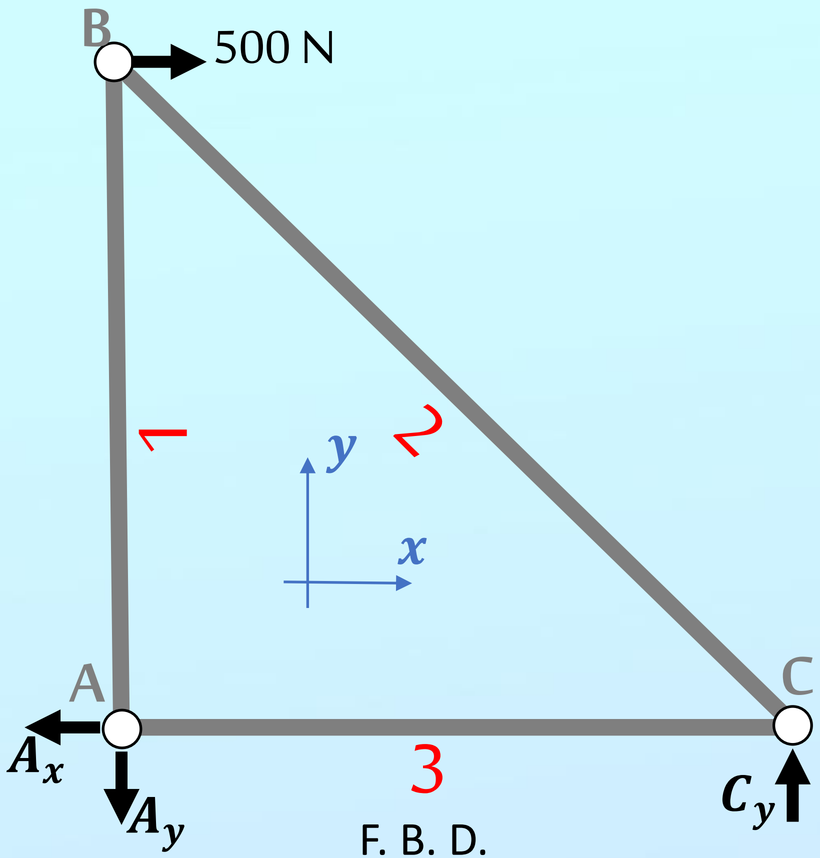
Review & Illustrative Example 1.

Determine the force in each member of the truss and indicate whether the members are in tension (T) or compression (C).

Solution:

0) Preliminary: تمهيد

- Elements and Joints numbering ترقيم العناصر وتسمية العقد
- Choosing Coordinate Systems اختيار جملة إحداثيات
- Completing data dimensions & angles استكمال الأبعاد والزوايا



F. B. D.

Review & Illustrative Example 1.

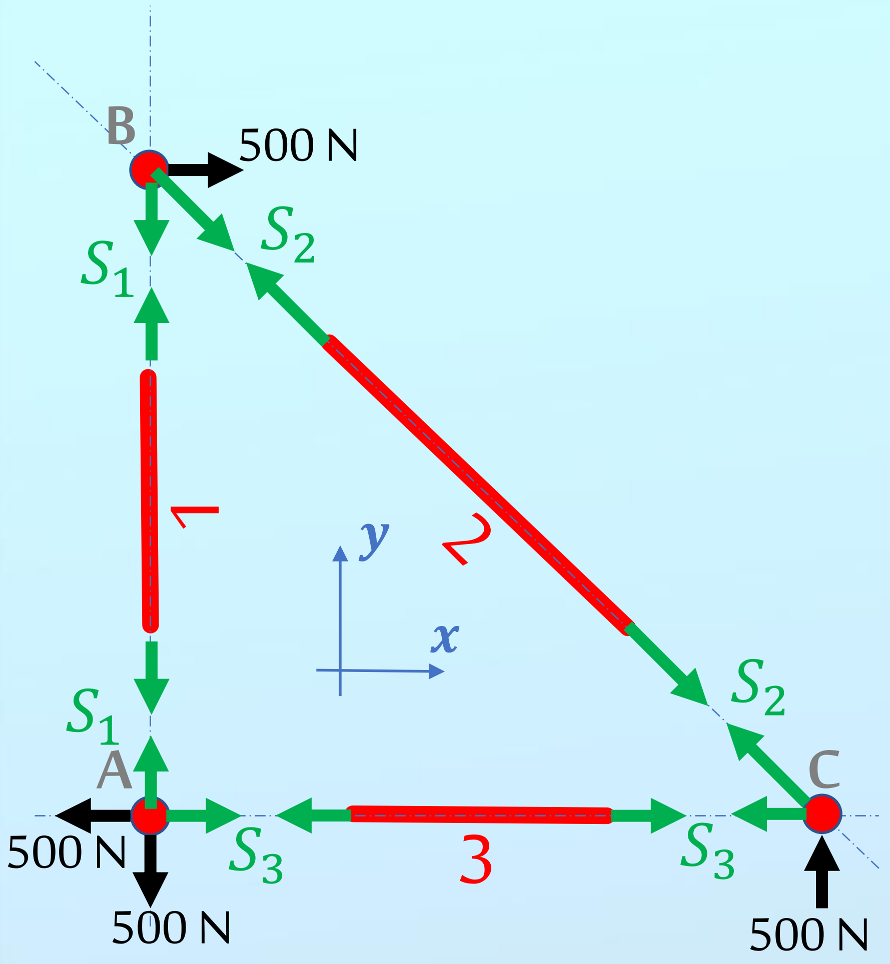
Determine the force in each member of the truss and indicate whether the members are in tension (T) or compression (C).

1) Reactions: F. B. D. of the Truss ردود الأفعال

$\rightarrow: -A_x + 500 = 0 \Rightarrow A_x = 500 \text{ N}$

$\circlearrowleft_{C}: -2(500) + 2A_y = 0 \Rightarrow A_y = 500 \text{ N}$

$\circlearrowleft_{A}: -2(500) + 2C_y = 0 \Rightarrow C_y = 500 \text{ N}$



Review & Illustrative Example 1.

Determine the force in each member of the truss and indicate whether the members are in tension (T) or compression (C).

2- Internal Forces: F. B. D. s.

F. B. D. of joint B:

$\rightarrow: S_2 \cos 45^\circ + 500 = 0 \quad \Rightarrow S_2 = -707 \text{ N}$

$\uparrow: -S_2 \sin 45^\circ - S_1 = 0 \quad \Rightarrow S_1 = -S_2 \sin 45^\circ = 500 \text{ N}$

F. B. D. of joint C:

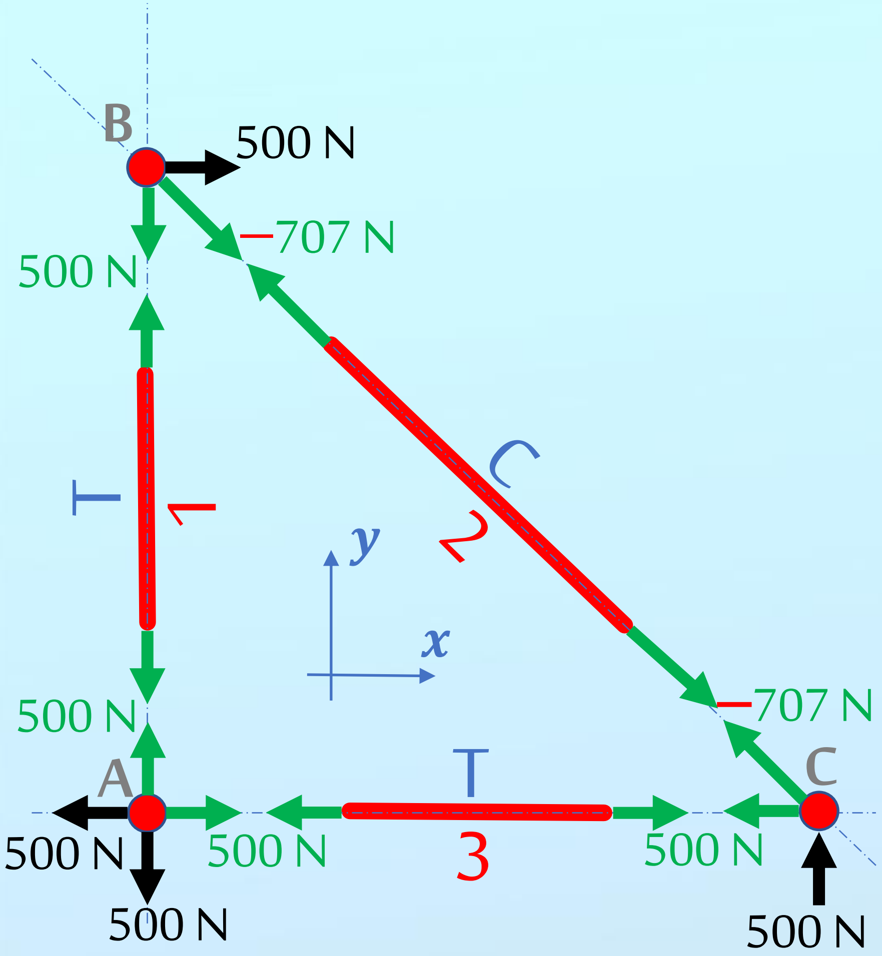
$\rightarrow: -S_2 \cos 45^\circ - S_3 = 0 \quad \Rightarrow S_3 = -S_2 \cos 45^\circ = +500 \text{ N}$

$\uparrow: check$

F. B. D. of joint A:

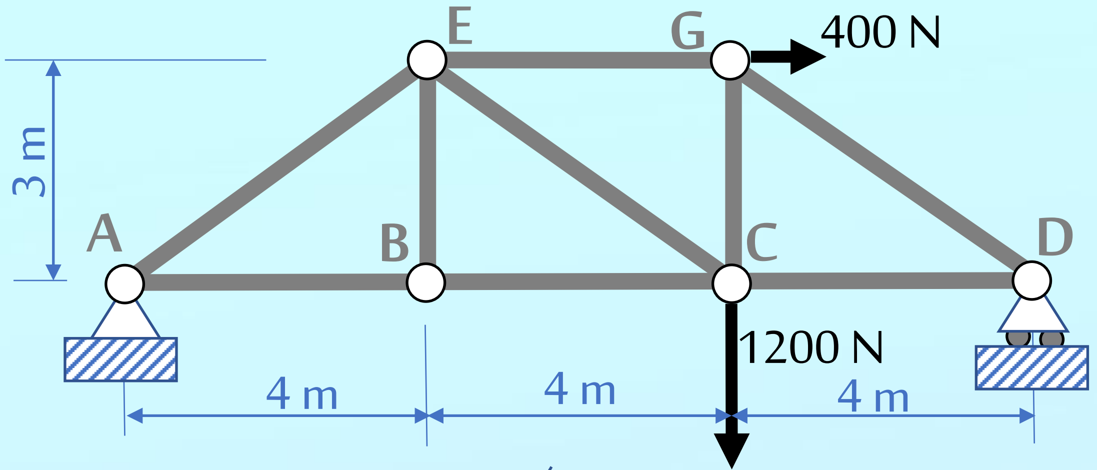
$\rightarrow: check$

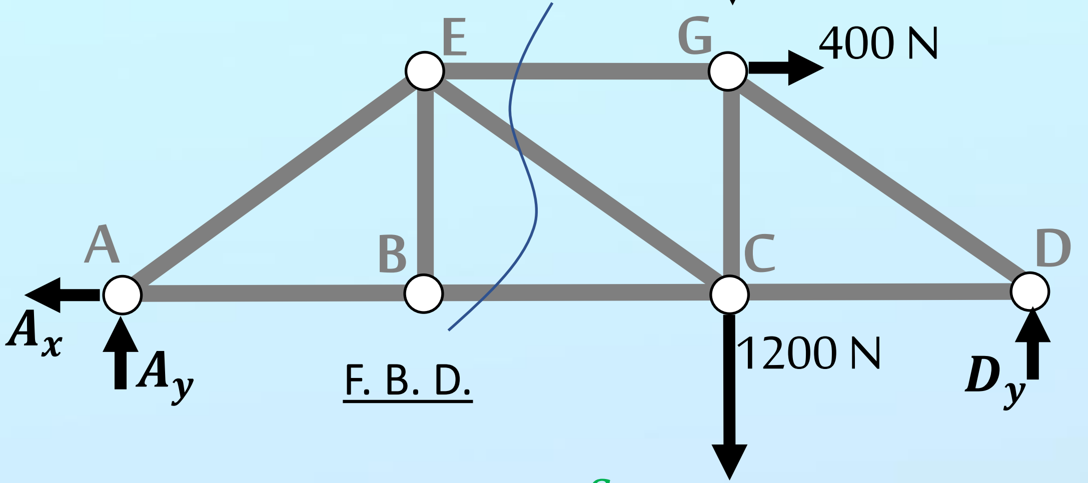
$\uparrow: check$



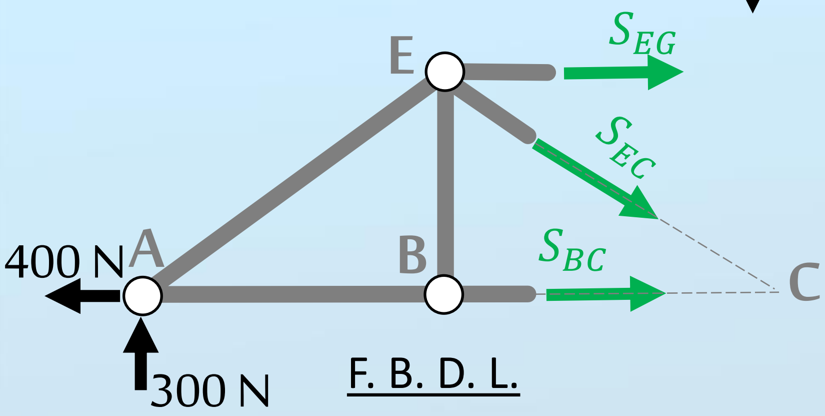
Review & Illustrative Example 1.

Determine the force in each member of the truss and indicate whether the members are in tension (T) or compression (C).





F. B. D.



F. B. D. L.

Review & Illustrative Example 2.

Determine the force in members EG, EC, and BC of the truss. Indicate whether the members are in tension or compression.

Solution:

1- Reactions: F. B. D. of the Truss

$\rightarrow: -A_x + 400 = 0 \Rightarrow A_x = 400$ N

$\circlearrowleft$ B: $-3(400) + 4(1200) - 12A_y = 0 \Rightarrow A_y = 300$ N

$\uparrow: +300 - 1200 + D_y = 0 \Rightarrow D_y = 900$ N

2- Section Internal Forces: F. B. D. L.

$\circlearrowleft$ E: $-3(400) - 4(300) + 3S_{BC} = 0 \Rightarrow S_{BC} = 800$ N (T)

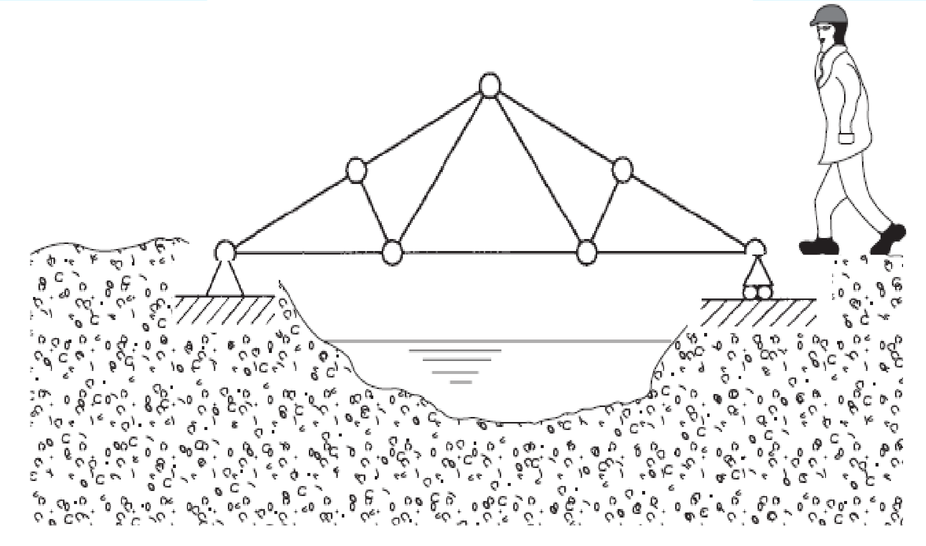
$\circlearrowleft$ C: $-8(300) - 3S_{EG} = 0 \quad \Rightarrow S_{EG} = -800$ N (C)

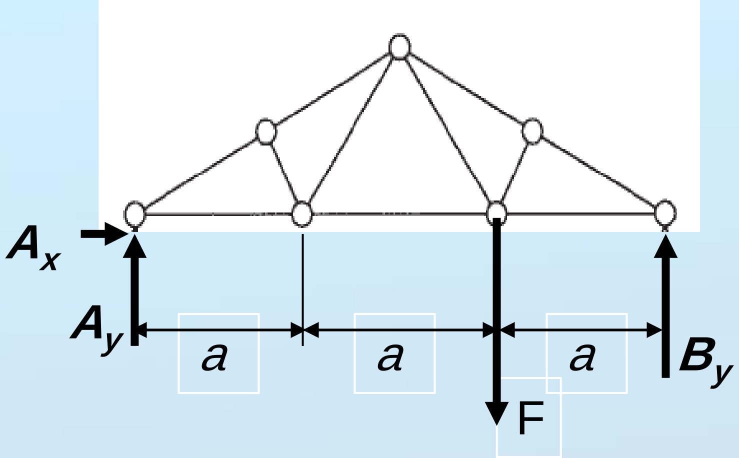
$\uparrow: +300 - S_{EC}\left(\frac{3}{5}\right) = 0 \quad \Rightarrow S_{EC} = 500$ N (T)

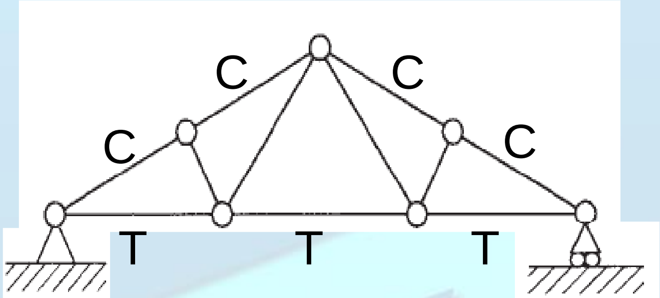
Beams, Frames, Arches

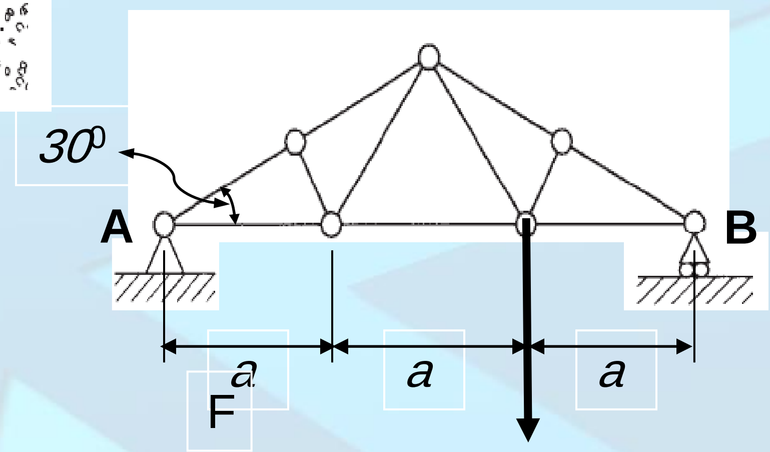
Beams are slender structural members that offer resistance to bending. They are among the most important elements in structural engineering.

الجيزان (ومفردها جائز) عناصر إنشائية هامة نحيلة تتلقى حمولات (أي قوى وعزوم) عرضية وتقاومها بالانعطاف

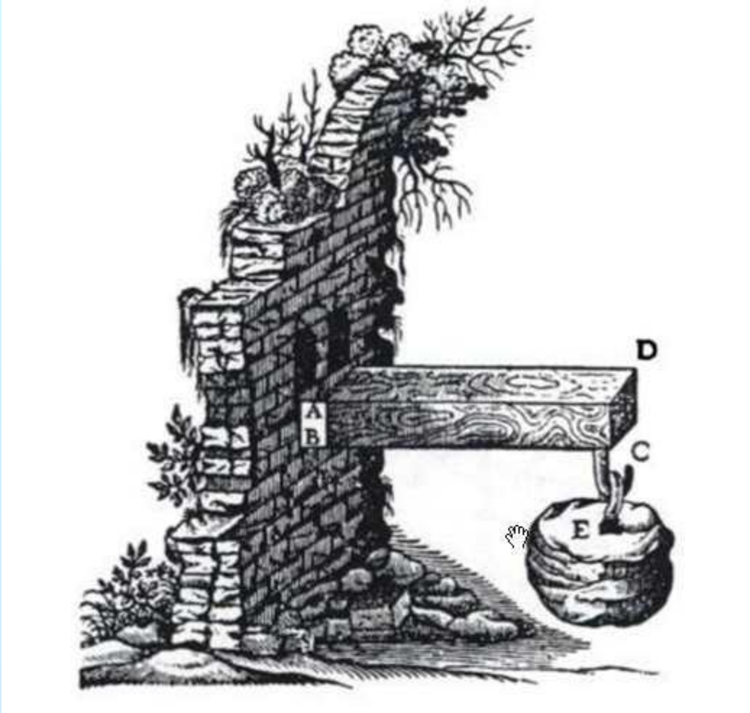








The basic function of a structure is to carry loads for which it has been designed & transmit forces from their points of applications to the supports. For example



Observing… the behavior of structures under the action of external loads, led to 2Qs (two questions) then to 2Ss+2Ss:

How strong is a str.? Strength notion & Stress concept.

How stiff is a str.? Stiffness notion & Strain concept.(*deformation concept*)

يهتم ميكانيك المواد بدراسة سلوك العناصر الإنشائية من خلال : المقاومة ومعيار قياسها الإجهادات، والصلابة ومعيار قياسها التشوهات.

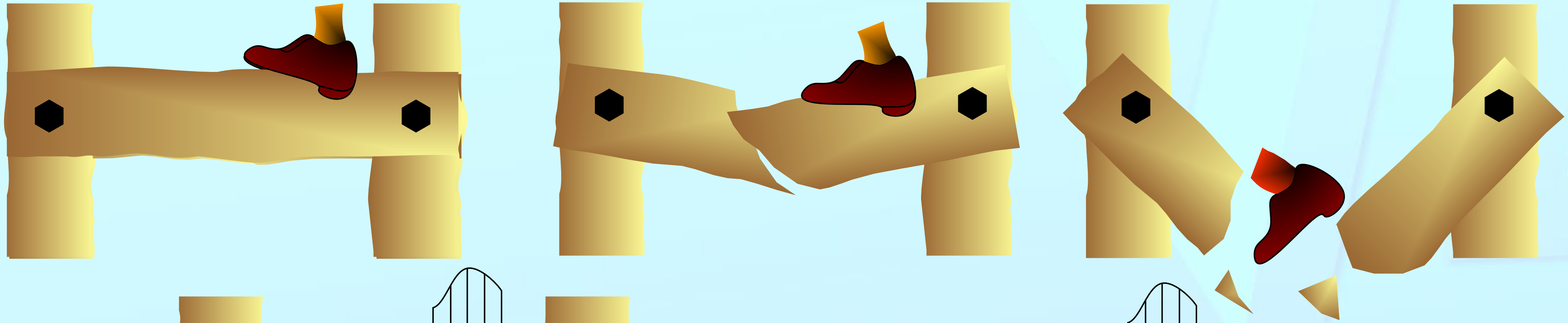
High strength is needed for safety.

But how to predetermine its safety degree?

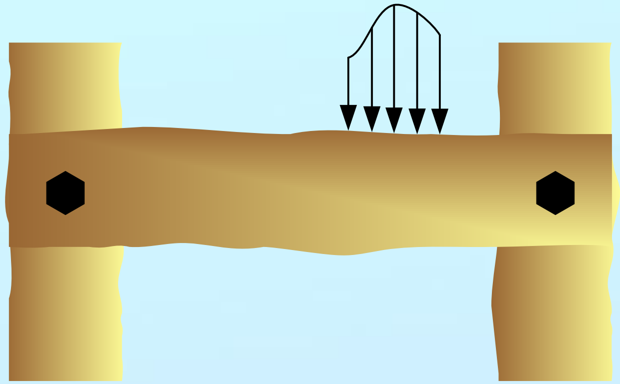
مقاومة العناصر الإنشائية (قدرتها على تلقي الحمولات ونقلها) ضرورية للأمان. فكيف نحدد درجة الأمان مسبقاً.

Balanced stiffness is needed.
But how to predetermine its degree?

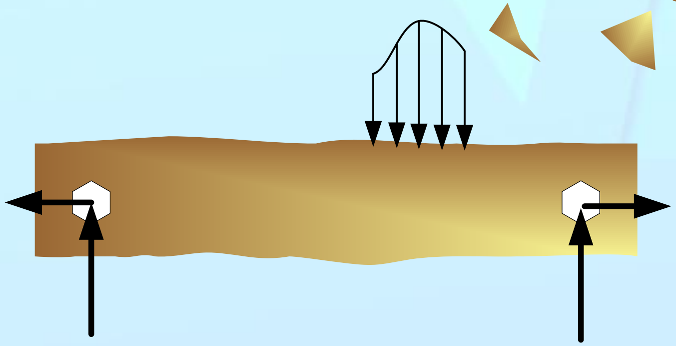
صلابة العناصر الإنشائية (قدرتها على الاحتفاظ بشكلها) ضرورية لأداء وظيفتها. فكيف نحدد الصلابة المطلوبة مسبقاً.



نمذجة الحمولات



Loads Modeling

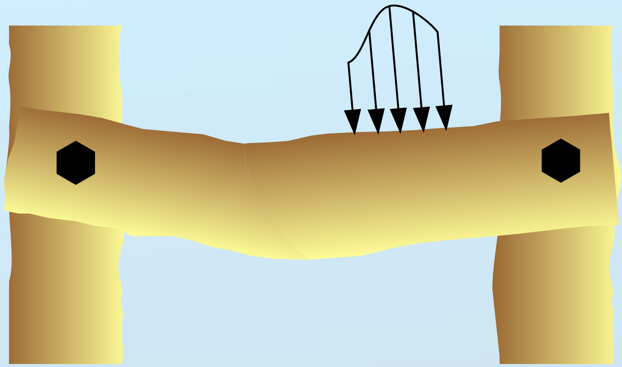


نمذجة الاستناد

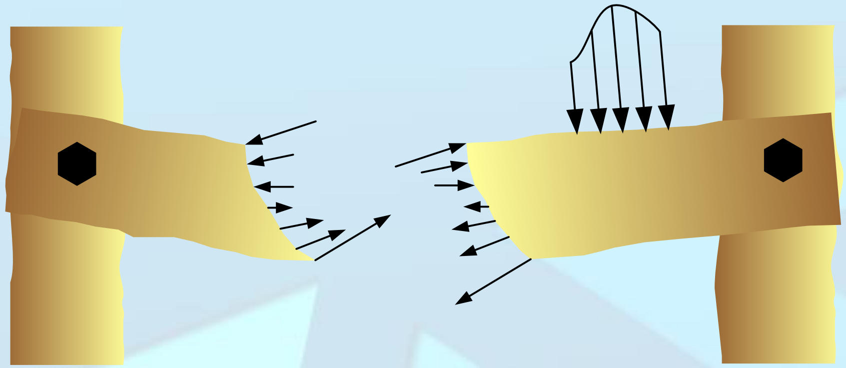
Supports Modeling

نمذجة

التشوهات



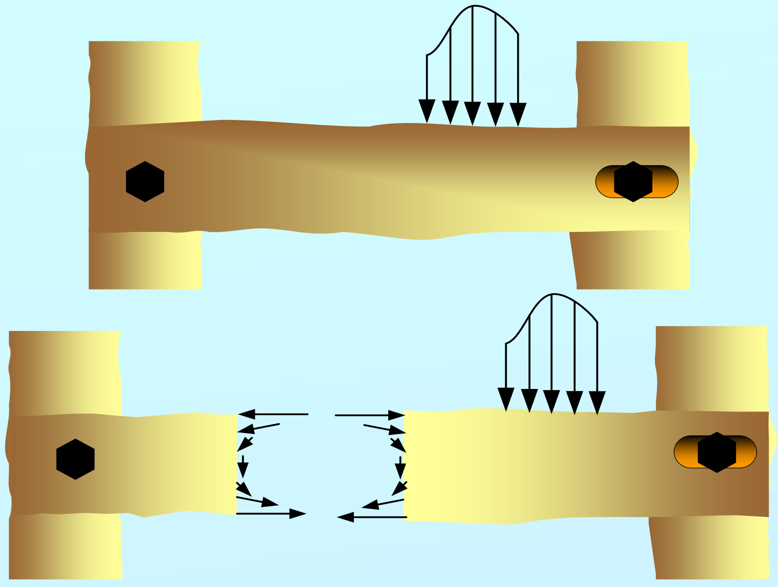
Strain (Deformation) modeling

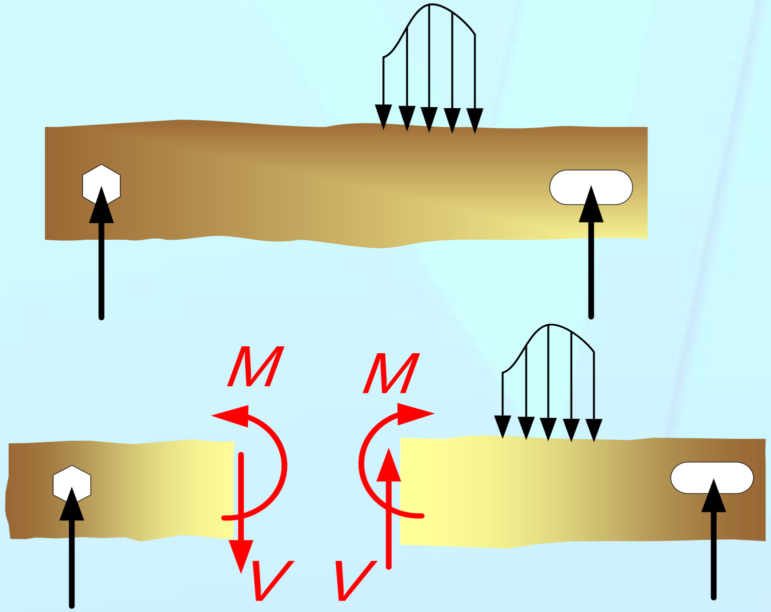


نمذجة

الاجهادات

Internal stress modeling





A slightly deformed structural member carries & transmits the external loads by developing an *internal force system*, that can be estimated without looking for the deformation, if the structure is determinate.

يتشوه عنصر إنشائي عندما يتلقى الحمولات فيولد جملة من القوى الداخلية تقوم بنقل الحمولات إلى مسانده. إذا كان هذا العنصر مقررا ستاتيكيا أمكننا تقدير جملة القوى الداخلية دون الاهتمام بالتشوهات.

This internal force system can be determined using two principles: global equilibrium & Partial (Cut) equilibrium

تحدد جملة القوى الداخلية في هذه الحالة باستخدام مبدأين هما: مبدأ التوازن الكلي ومبدأ التوازن الجزئي (أو مبدأ القطع)

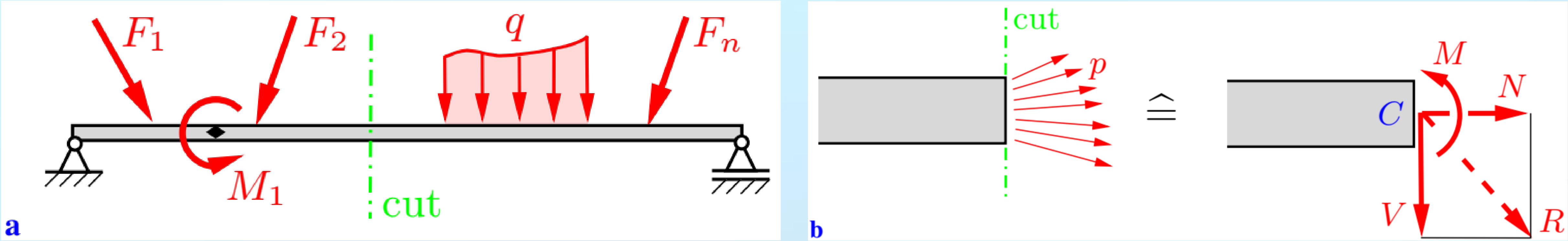
Knowledge of the internal forces is important in order :

1. to determine the load-bearing capacity of a beam,
2. to compute the properties (area,…) of the cross-section required to sustain a given load,
3. or to compute the deformation as we will see later.

1. قدرة التحمل
2. خصائص المقطع
3. التشوهات

المسائل المقررة والمستوية

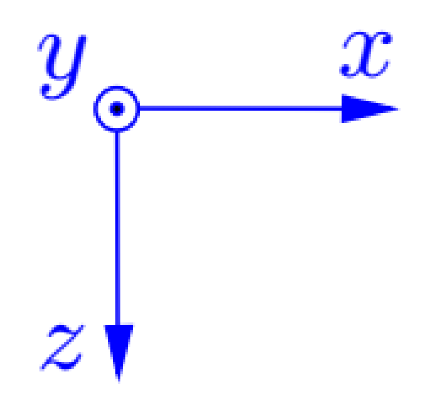
For simplicity, the discussion is limited to statically determinate plane problems,

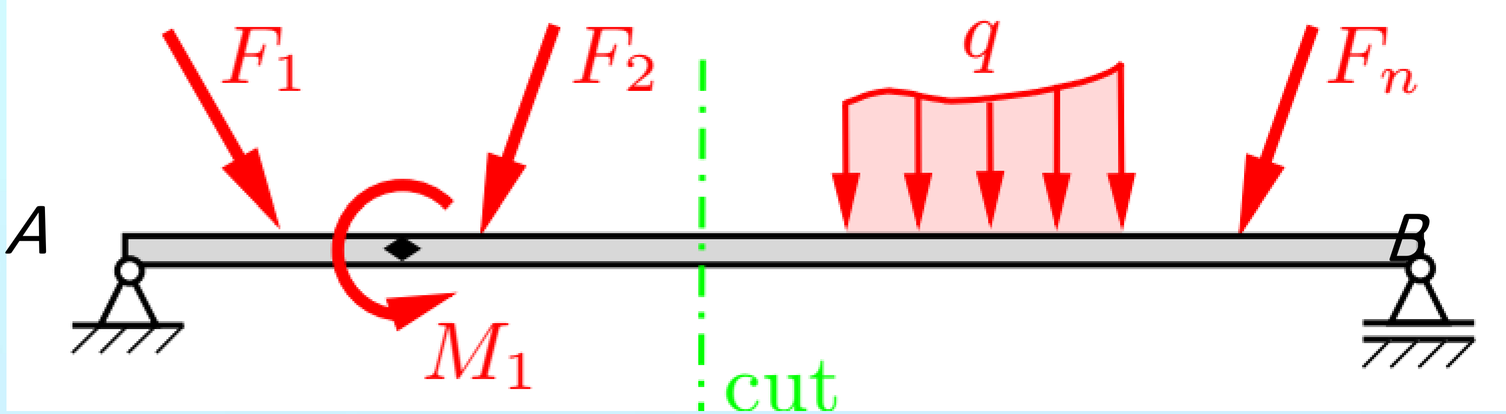


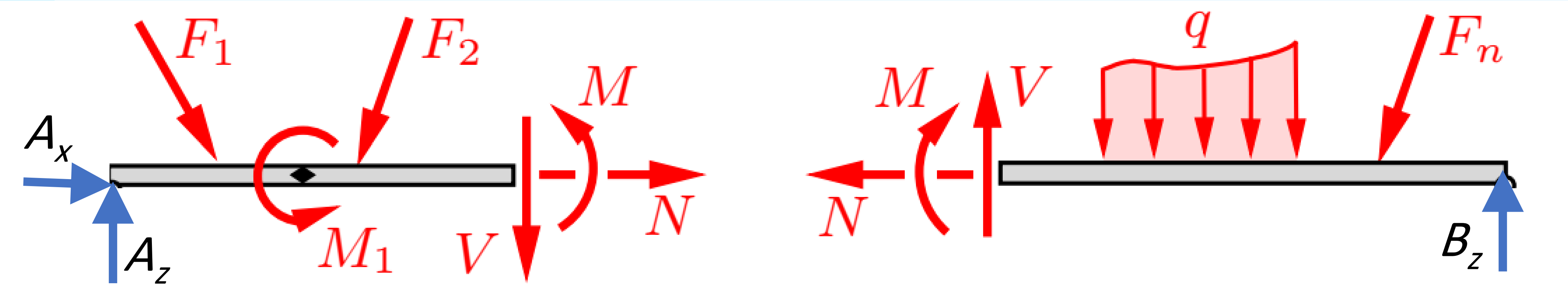
The quantities *N*, *V* & *M* are called the stress resultants (قوى المقطع أو القوى الداخلية أو محصلات الاجهادات). In particular

N is called the normal force (القوة الناظمية), *V* is called the shear force (قوة القص) and *M* is called the bending moment (عزم الانعطاف).

In order to determine the stress resultants (القوى الداخلية), the beam is divided by a cut into two segments (method of sections) (طريقة المقطع)



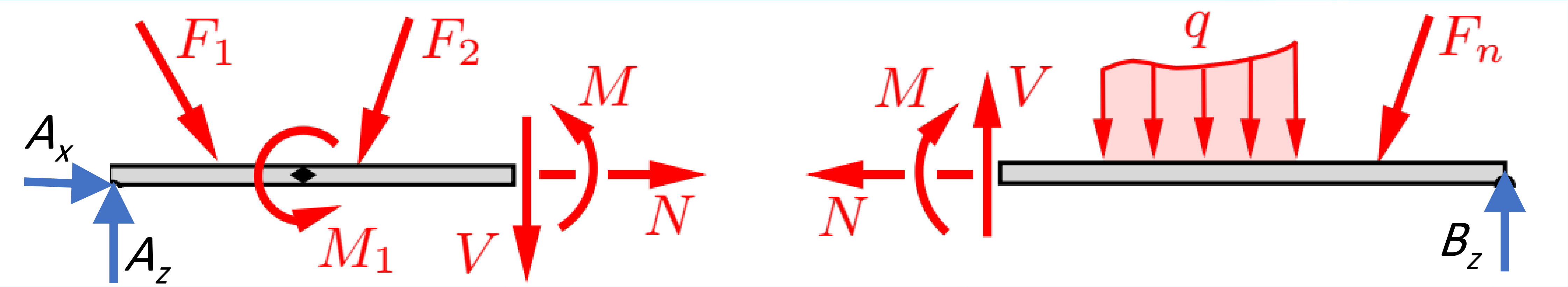




A F. B. D. of each segment includes all of the forces acting on it, i.e., the applied loads (forces and couples), the support reactions **& the stress resultants acting at the cut sections.** (مخطط جسم حر لكل جزء مع حمولاته وردود أفعال مسانده **وقوى المقطع**)

According to Newton’s third law (action = reaction) stress resultant act in opposite directions at the two faces of the segments of the beam.

وفق قانون نيوتن الثالث (رد الفعل يساوي ويعاكس الفعل) تكون القوى الداخلية على وجهي القطع متساوية ومتعاكسة.

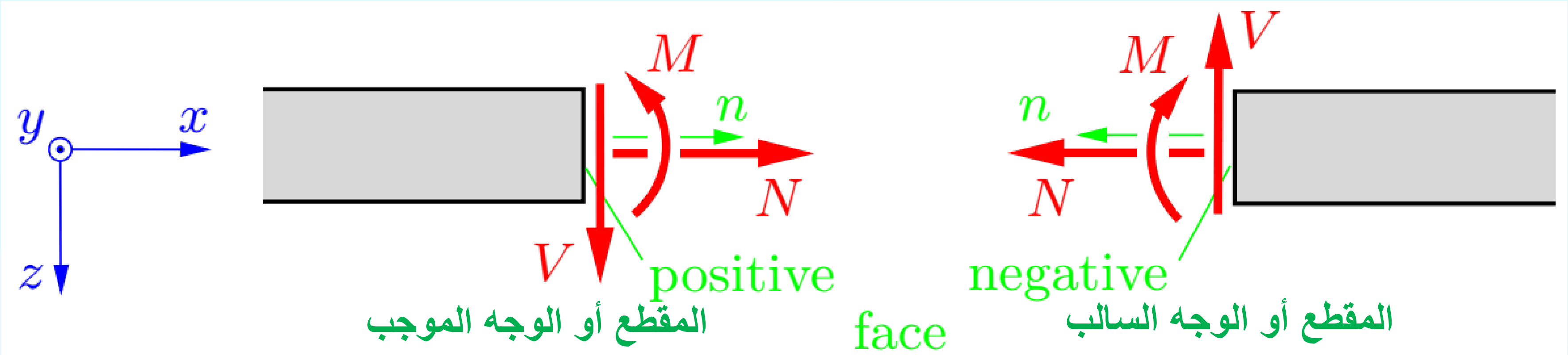


Since each part of the beam is in equilibrium, the 3 conditions of equilibrium for either part can be used to compute the three unknown stress resultants.

إن جزئي الجائز على طرفي القطع في حالة توازن، لذا فمن الممكن استخدام شروط التوازن الثلاثة لكل منهما من أجل حساب القوى الداخلية (قوى المقطع) المؤثرة على كل منهما.

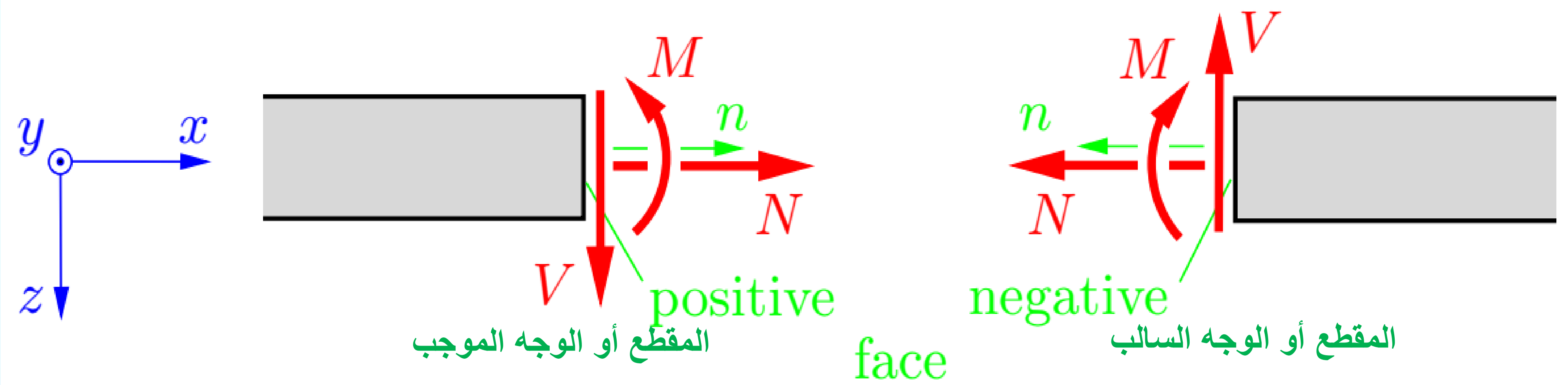
Before we can provide examples for the determination of the stress resultants, a sign convention must be introduced. Consider the two adjoining portions of the same beam shown in the next figure. The coordinate *x* coincides with the direction of the axis of the beam and points to the right; the coordinate *z* points downward.

جملة الاحداثيات والوجهين (المقطعين) الموجب والسالب



By cutting the beam, a left-hand face and a right-hand face are obtained (figure above). They are characterized by a normal vector ***n*** that points outward from the interior of the beam. If the vector ***n*** points in the positive (negative) direction of the *x-axis*, the corresponding face is called positive (negative).

The following sign convention is adopted: <u>اصطلاح</u> إشارة القوى الداخلية:



Positive stress resultants at a positive (negative) face point in the positive (negative) directions of the coordinates.

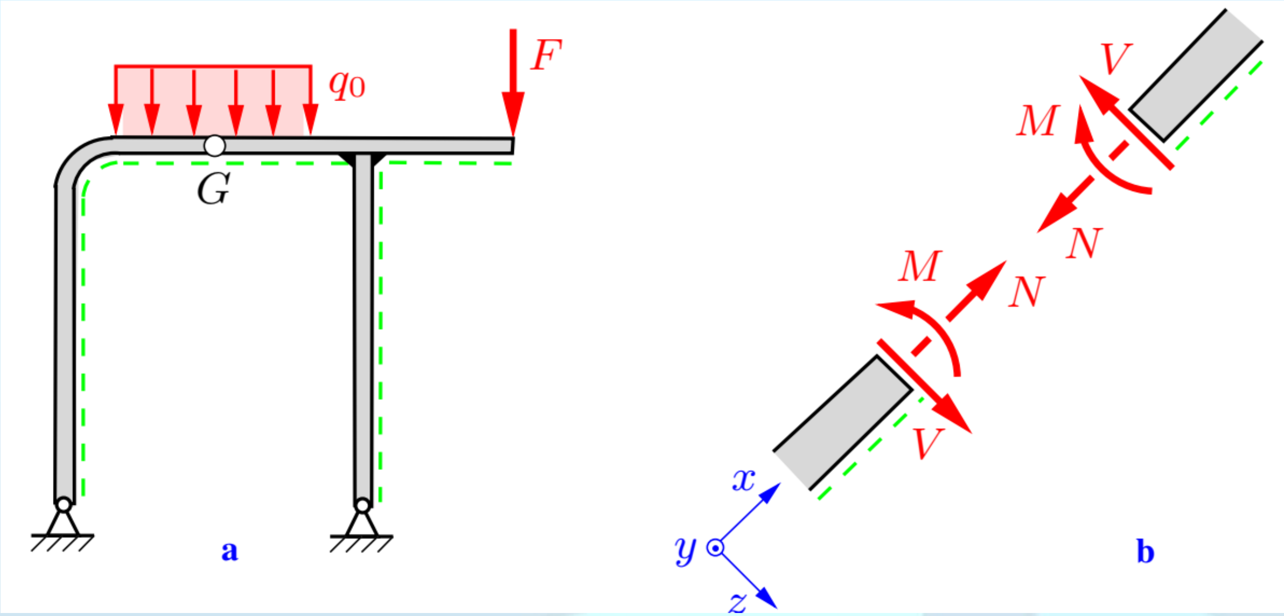
Here, the bending moment *M* has to be interpreted as a vector pointing in the direction of the *y-axis* (*positive direction according to the right-hand rule*)

The above figure shows the stress resultants with their positive directions. In the following examples, we shall strictly adhere to this sign convention.

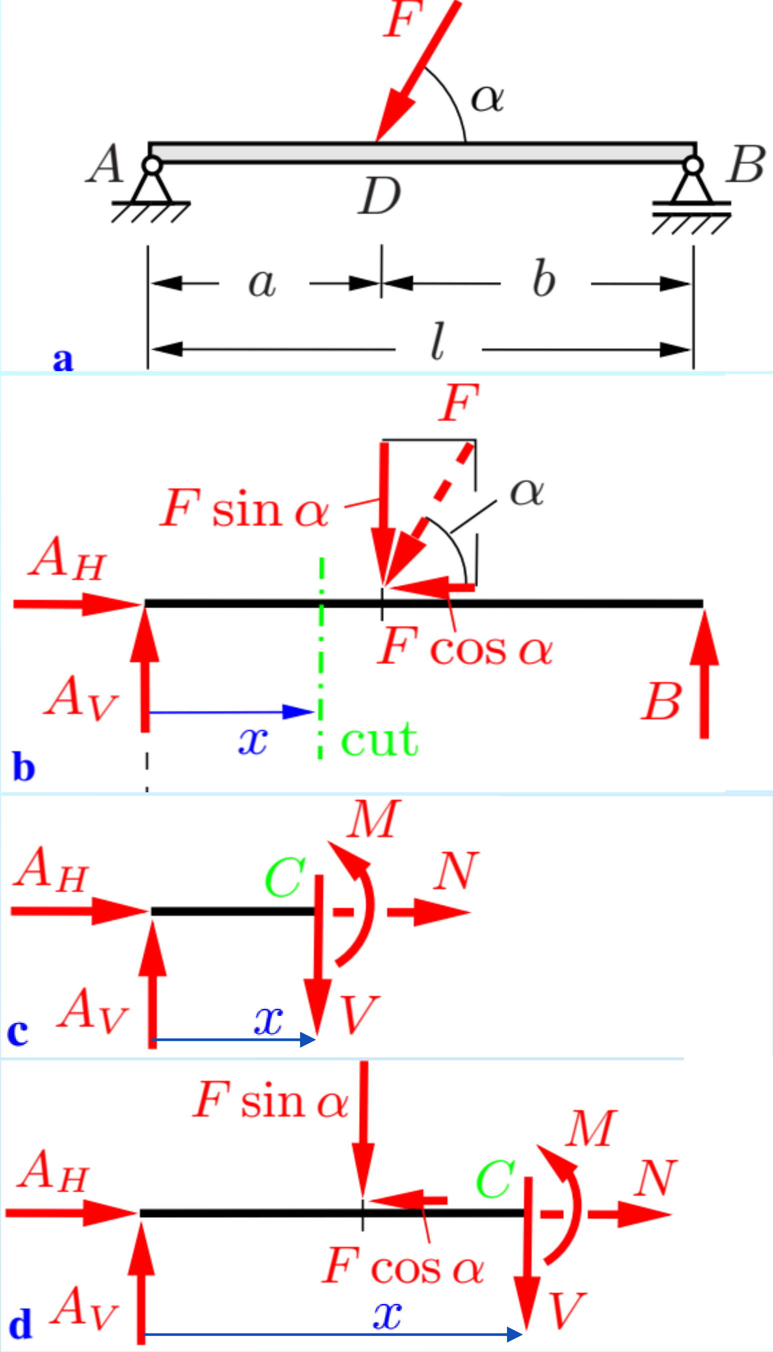
It should be noted, however, that different sign conventions exist.

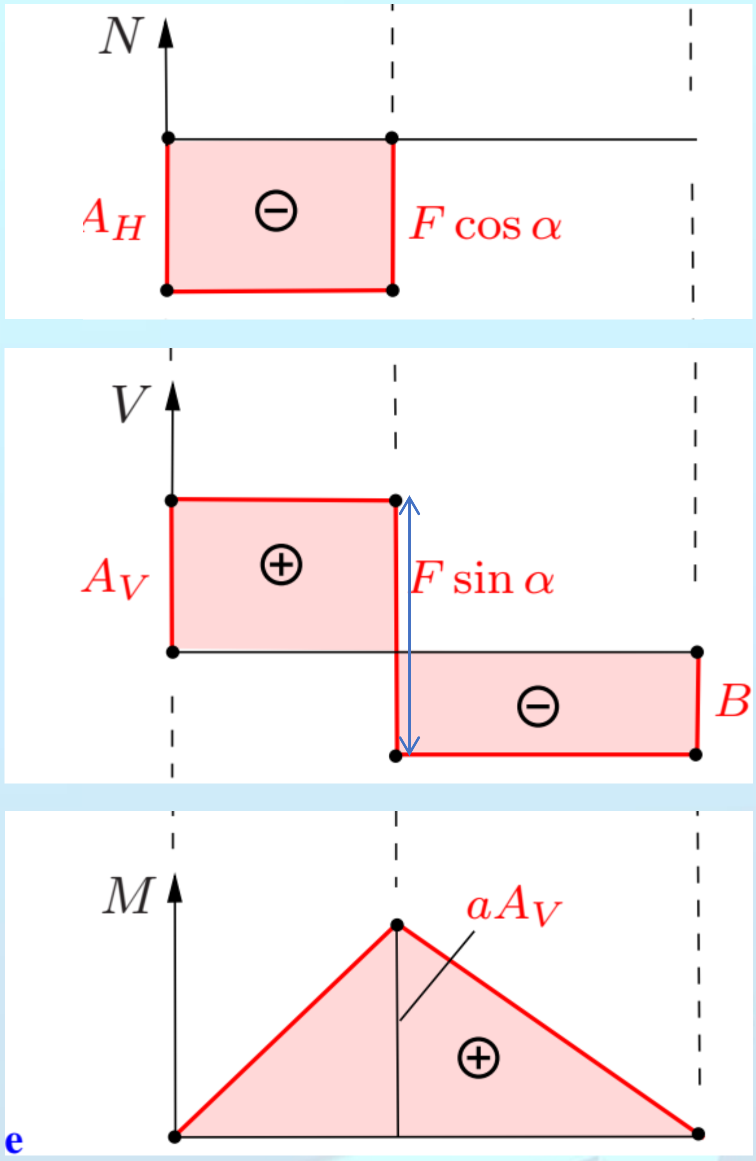
In the case of a horizontal beam, very often only the *x* coordinate is given. Then it is understood that the *z-axis* points downward.

The sign convention for frames and arches may be introduced by drawing a dashed line at one side of each part of the system. The side with the dashed line can then be interpreted as the "underneath side" of the respective part and the coordinate system can be chosen as the one for a beam: *x*-axis in the direction of the dashed line, *z-axis* toward the dashed line ("downward").



We will now determine the stress resultants for the simply supported beam shown in Fig.a.





Solution:

1. Reactions, Fig.b: $A_H = F\cos\alpha$,

$A_V = (b/l)F\sin\alpha,\ B = (a/l)F\sin\alpha$

2. Cut, Fig.c, $0 < x < a$:

$N = -A_H = -F\cos\alpha$

$V = A_V = (b/l)F\sin\alpha,$

$M = xA_V = [(b/l)F\sin\alpha]x.$

$x = 0 \Rightarrow M = 0,$

$x = a \Rightarrow M = aA_V$

3. Cut, Fig.d, $a < x < l$:

$N = A_H - F\cos\alpha = 0$

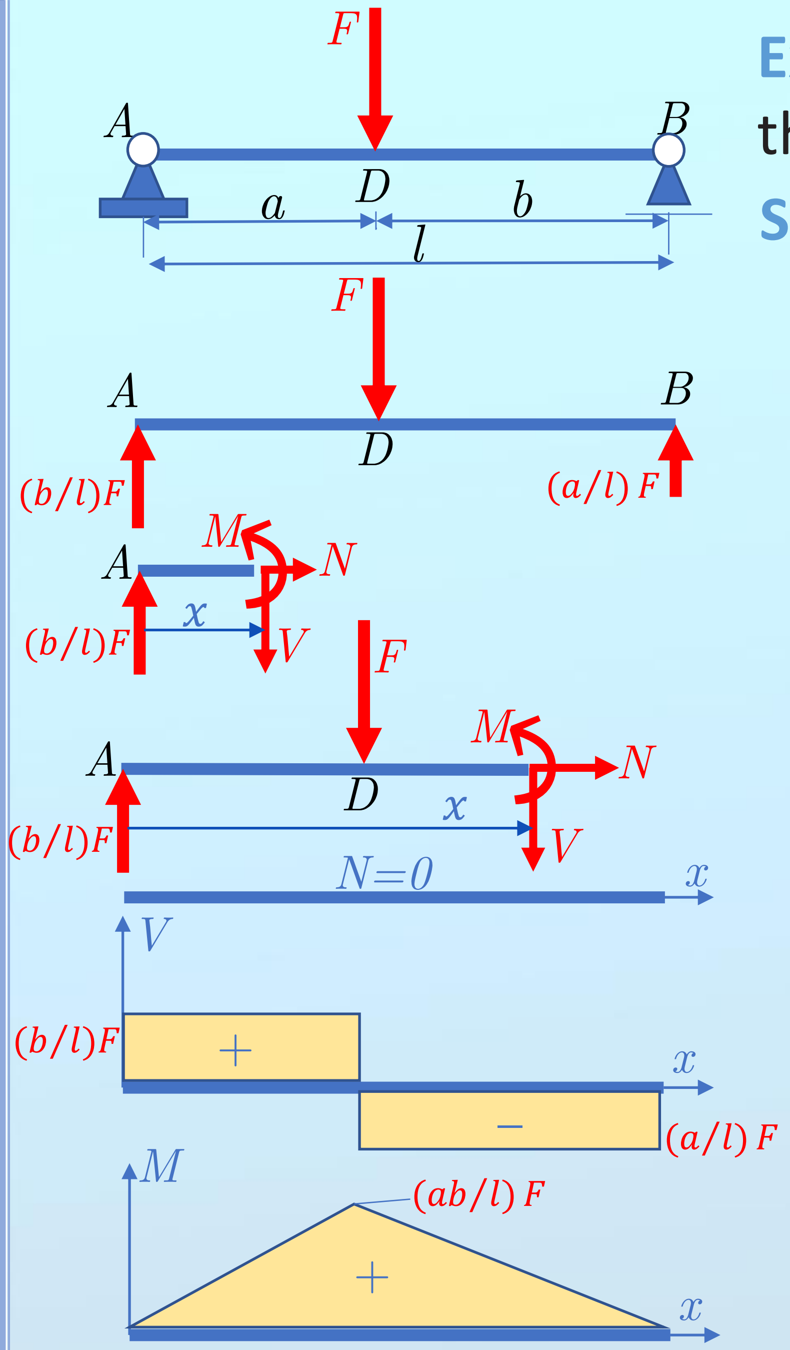
$V = A_V - F\sin\alpha = -B,$

$M = xA_V - (x - a)F\sin\alpha$

$x = a \Rightarrow M = aA_V$

$x = l \Rightarrow M = lA_V - bF\sin\alpha = 0$

Example 1. Draw the diagrams of the stress resultants for the beam shown in figure.



Solution:

0. Reactions: $\rightarrow$: $A_x = 0$

$\overset{\curvearrowleft}{B}$: $A_z = (b/l)F$, $\overset{\curvearrowleft}{A}$: $B_z = (a/l)$, $\uparrow$: $A_z + B_z = F$ (yes)

1. Cut: A…D, $0 < x < a$:

$\rightarrow: N = 0;$

$\uparrow: (b/l)F - V = 0 \Rightarrow V = (b/l)F;$

$\overset{\curvearrowleft}{x}: M - x(b/l)F = 0 \Rightarrow M = x(b/l)F$

$x = 0: M = 0; x = a: M = (ab/l)F$

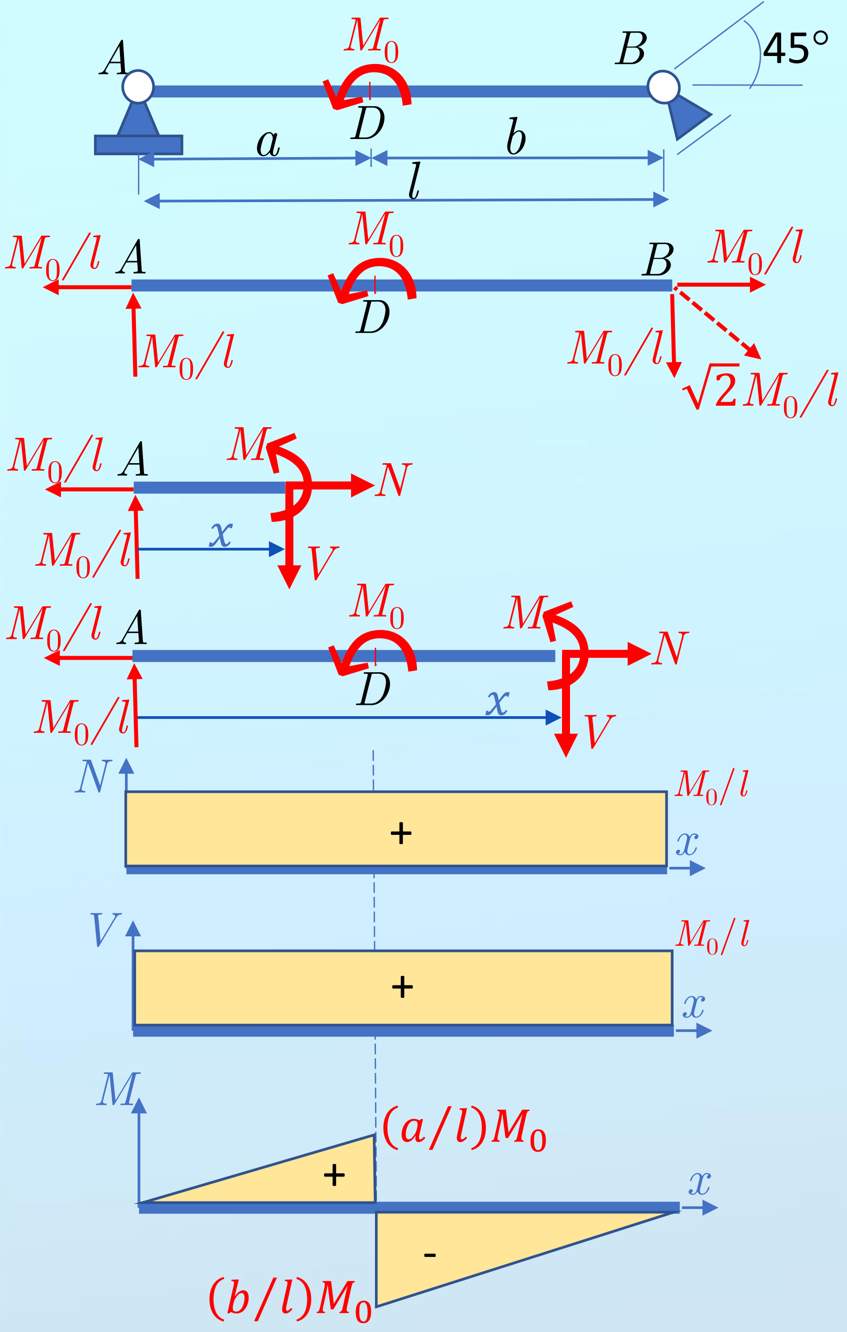
2. Cut: D…B, $a < x < l$:

$\rightarrow: N = 0;$

$\uparrow: (b/l)F - F - V = 0 \Rightarrow V = (b/l)F - F = [(b-l)/l]F = -(a/l)F;$

$\overset{\curvearrowleft}{x}: M + (x-a)F - x(b/l)F = 0 \Rightarrow M = (l-x)(a/l)F;$

$x = a: M = (ab/l)F; x = l: M = 0$



Example 2. Draw the diagrams of the stress resultants for the beam shown in figure.

Solution:

0. Reactions:

$\overset{\curvearrowleft}{B}: -lA_z + M_0 = 0 \Rightarrow A_z = M_0/l;$

$\overset{\curvearrowleft}{A}: +lB_z + M_0 = 0 \Rightarrow B_z = -M_0/l\,(\downarrow) \Rightarrow B_x = -M_0/l\,(\rightarrow)$

$\rightarrow: -A_x + B_x = 0 \Rightarrow A_x = M_0/l(\leftarrow)$

1. Cut: A…D, $0 < x < a$:

$\rightarrow: N = M_0/l;$

$\uparrow: M_0/l - V = 0 \Rightarrow V = M_0/l$

$\overset{\curvearrowleft}{X}: M - x(M_0/l) = 0 \Rightarrow M = (x/l)M_0,$

$x = 0: M = 0; x = a: M = (a/l)M_0.$

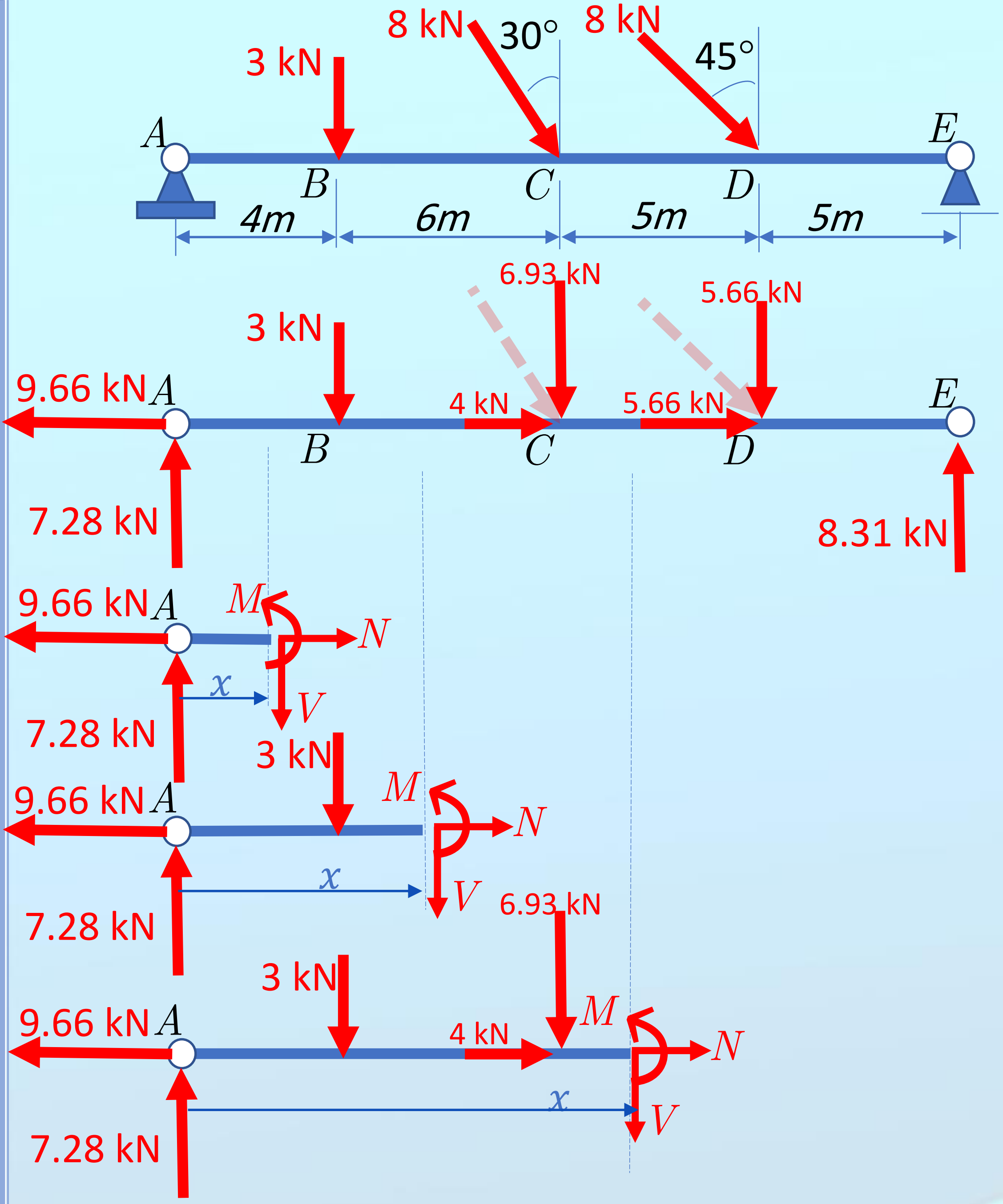
2. Cut: D…B, $a < x < l$:

$\rightarrow: N = M_0/l;$

$\uparrow: M_0/l - V = 0 \Rightarrow V = M_0/l$

$\overset{\curvearrowleft}{X}: M + M_0 - x(M_0/l) = 0 \Rightarrow M = -M_0 + (x/l)M_0 = (\frac{x-l}{l})M_0;$

$x = a: M = -(b/l)M_0; x = l: M = 0.$



Example 3. Draw the diagrams of the stress resultants for the beam shown in figure.

Solution:

0. Reactions:

$\rightarrow: -A_x + 4 + 5.66 = 0 \Rightarrow A_x = 9.66 \text{ kN } (\leftarrow)$

$\overset{\curvearrowleft}{E}: +5(5.66) + 10(6.93) + 16(3) - 20A_z = 0 \Rightarrow \ A_z = 7.28 \ \text{kN}$

$\overset{\curvearrowleft}{A}: -4(3) - 10(6.93) - 15(3) + 20E_z = 0 \Rightarrow \ E_z = 8.31 \ \text{kN}$

1. Cut: A…B, $0 < x < 4\text{m}$:

$\rightarrow: N = 9.66 \text{ kN}; \qquad \uparrow: V = 7.28 \text{ kN}$

$\overset{\curvearrowleft}{X}: \ M - x(7.28) = 0 \Rightarrow M = 7.28x,$

$x = 0: M = 0; x = 4: M = 29.1 \text{ kNm}.$

2. Cut: B…C, $4 < x < 10\text{m}$:

$\rightarrow: N = 9.66 \text{ kN}; \qquad \uparrow: V = 7.28 - 3 = 4.28 \text{ kN}$

$\overset{\curvearrowleft}{X}: \ M - x(7.28) + (x-4)(3) = 0 \Rightarrow M = 4.28x + 12,$

$x = 4: M = 29.1 \text{ kNm}; x = 10: M = 54.8 \text{ kNm}.$

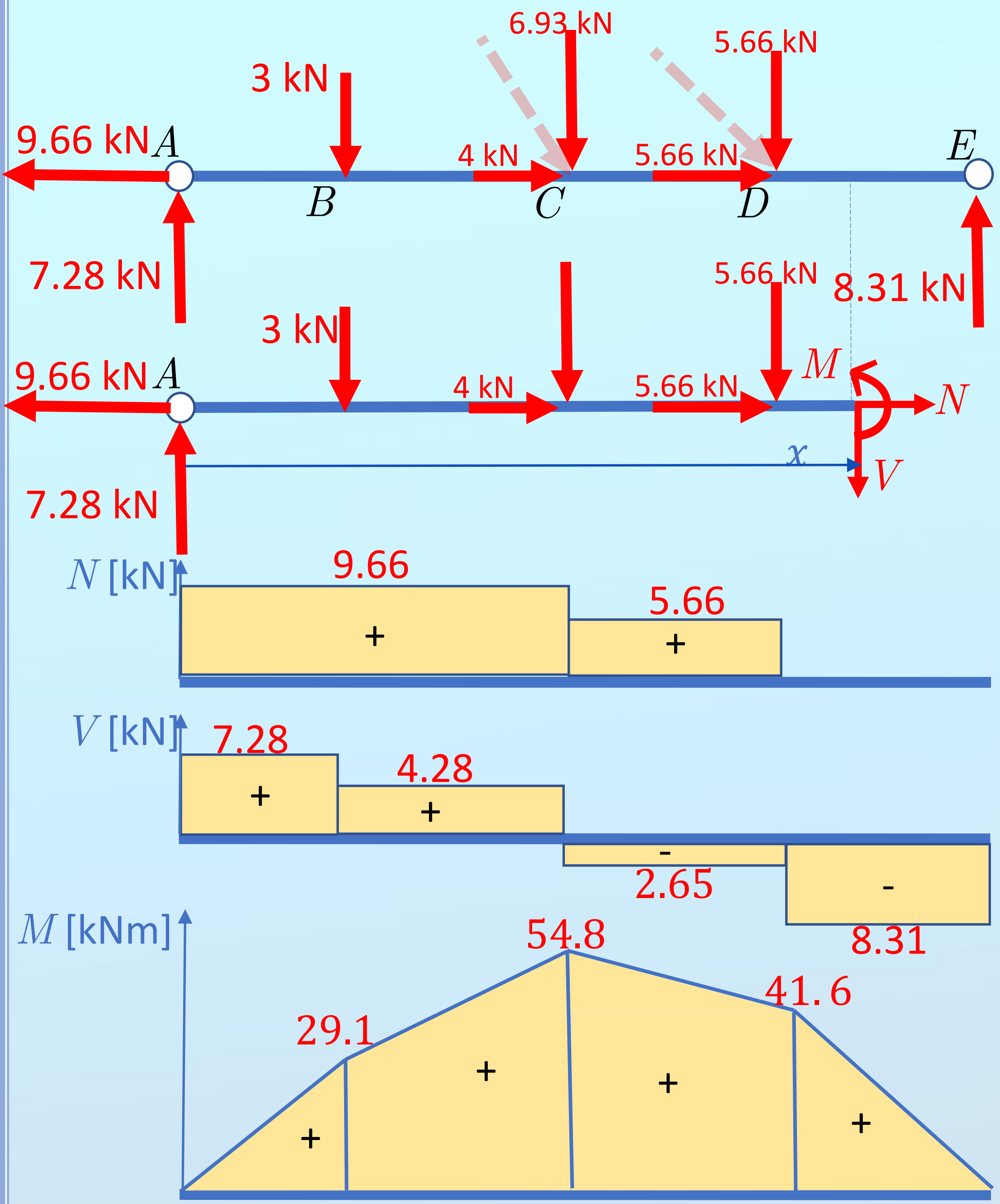
3. Cut: C…D, $10 < x < 15\text{m}$:

$\rightarrow: N = 5.66 \text{ kN}; \qquad \uparrow: V = 7.28 - 3 - 6.93 = -2.65\text{kN}$

$\overset{\curvearrowleft}{X}: \ M - x(7.28) + (x-4)(3) + (x-10)(6.93) = 0$

$\Rightarrow M = -2.65x + 81.3,$

$x = 10: M = 54.8 \text{ kNm}; x = 15: M = 41.6 \text{ kNm}.$



4. Cut: D…E, $15 < x < 20\text{m}$:

$\rightarrow: N = 0; \uparrow: V = 7.28 - 3 - 6.93 - 5.66 = -8.31\text{kN}$

$\overset{\curvearrowleft}{X}: M - x(7.28) + (x-4)(3) + (x-10)(6.93) + (x - 15)(5.66) = 0 \Rightarrow M = -8.31x + 166.2,$

$x = 15: M = 41.6\text{kNm}; x = 20: M = 0.$