Lecture #3

Moments, Couples, and Force Couple Systems

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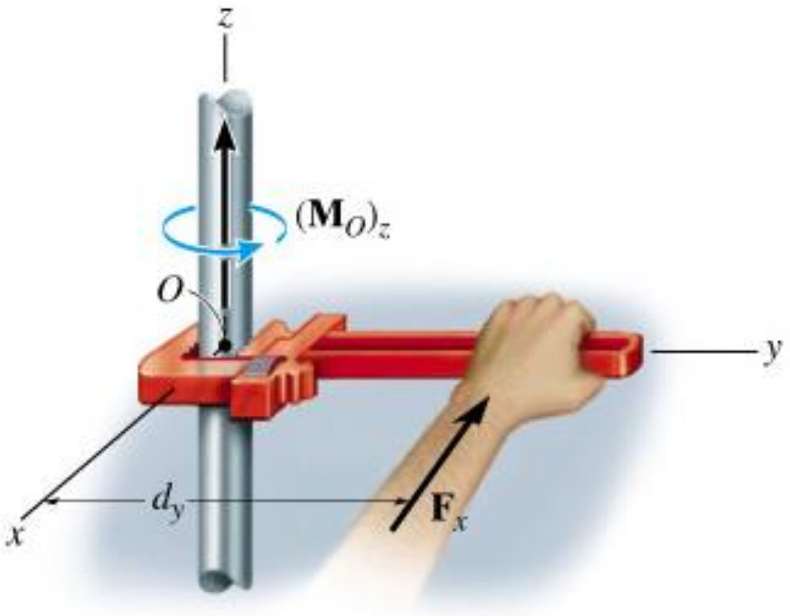
Introduction to Moments

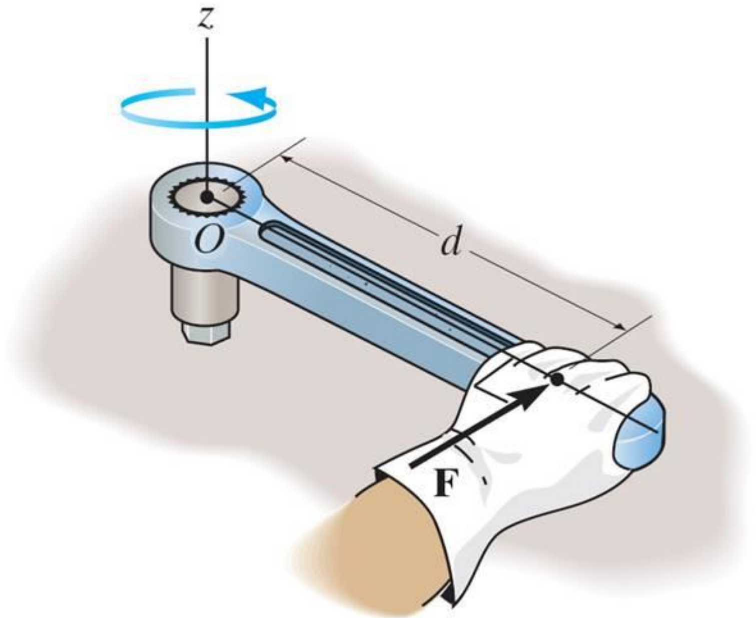
The tendency of a force to rotate a rigid body about any defined axis is called the Moment of the force about the axis



(a)

MOMENT OF A FORCE – SCALAR FORMULATION

$M = F \cdot d$



The **moment**, M, of a force about a point provides a measure of the tendency for rotation (sometimes called a torque)

$M = F \cdot d$

Moment caused by a Force

- The Moment of Force (F) about an axis through Point (A) or for short, the Moment of F about A, is the product of the magnitude of the force and the perpendicular distance between Point (A) and the line of action of Force (F)
  - $\boxed{M_A F = F \cdot d}$

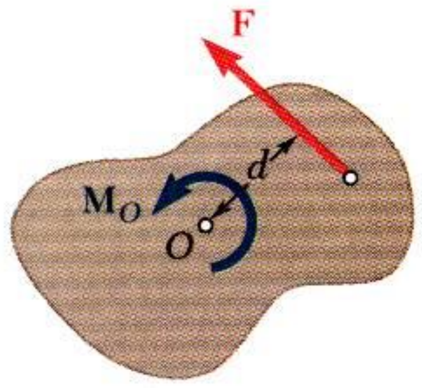
Units of a Moment

- The units of a Moment are:
  - *N·m* in the SI system
  - *lbs.ft* or *lbs.in* in the US Customary system

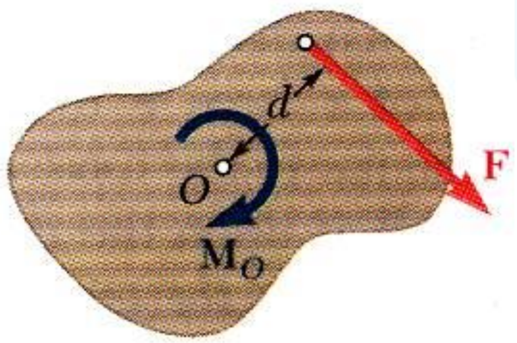
Properties of a Moment

- Moments not only have a magnitude, they also have a sense to them.

- The sense of a moment is clockwise or counter-clockwise depending on which way it will tend to make the object rotate



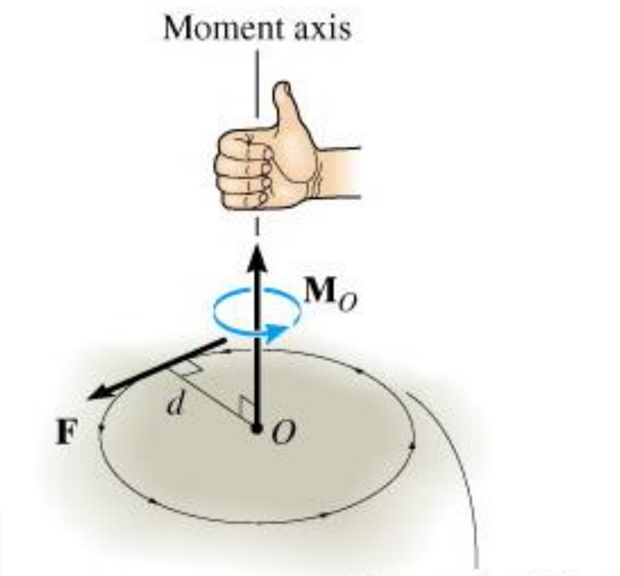
(a) $M_O = + Fd$



(b) $M_O = - Fd$

Properties of a Moment

- **The sense of a Moment is defined by the direction it is acting on the Axis and can be found using Right Hand Rule**



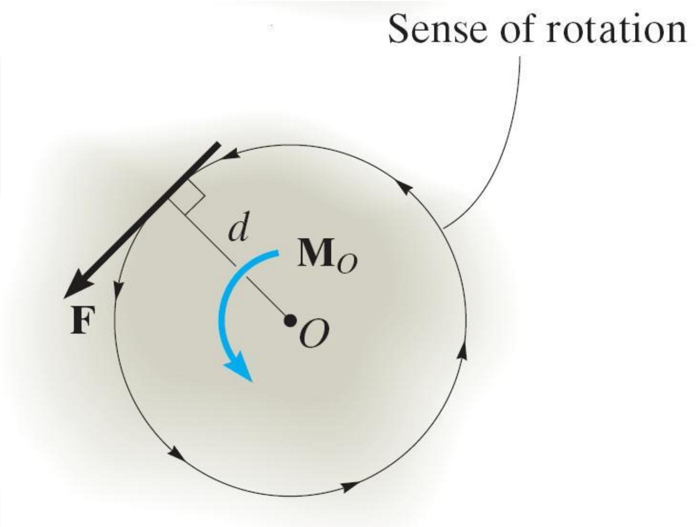
(a)

Varignon’s Theorem

- The moment of a Force about any axis is equal to the sum of the moments of its components about that axis
- This means that resolving or replacing forces with their resultant force will not affect the moment on the object being analyzed

MOMENT OF A FORCE - SCALAR FORMULATION

In the 2-D case, the magnitude of the moment is $M_o = F\,d$



As shown, d is the perpendicular distance from point O to the line of action of the force.

In 2-D, the direction of M_O is either clockwise or counter-clockwise, depending on the tendency for rotation

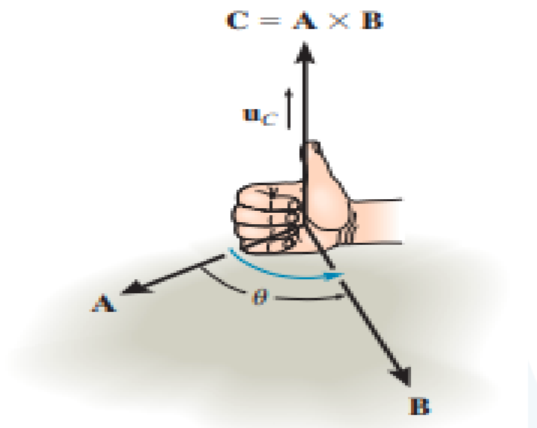
Cross Products

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

Magnitude : $\boxed{C = A.B \sin\theta}$

Direction :Vector C has a direction that is perpendicular to the plane containing A and B, such that C is specified by the right –hand rule , curling the fingers of the right hand from vector A (cross) to vector B , the thumb points in the direction of C. $\boxed{\boldsymbol{C} = (A.B \sin\theta\,)\boldsymbol{U_C}}$

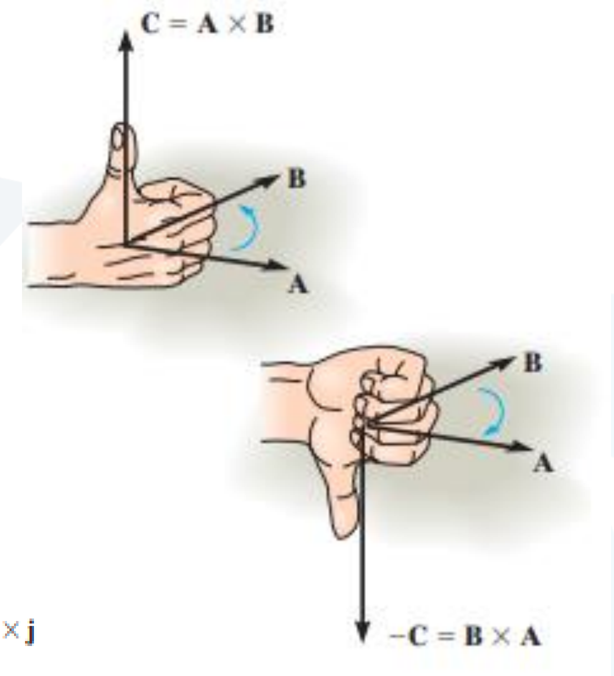
Where the scalar $A.B \sin\theta$ defines the magnitude of C and the unit vector $\boldsymbol{U_C}$ defines the direction of C



$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

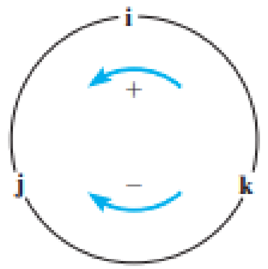
$$a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$

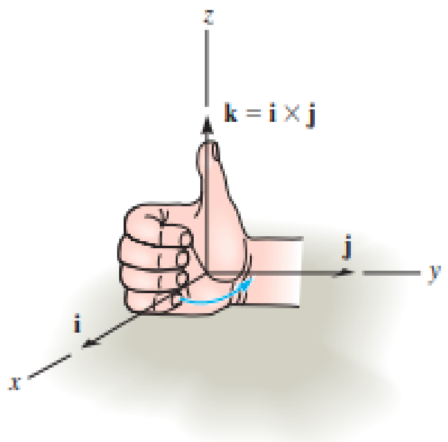
$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$



Cartesian Vector Formulation.

$$\begin{array}{lll}
\mathbf{i} \times \mathbf{j} = \mathbf{k} & \mathbf{i} \times \mathbf{k} = -\mathbf{j} & \mathbf{i} \times \mathbf{i} = 0 \\
\mathbf{j} \times \mathbf{k} = \mathbf{i} & \mathbf{j} \times \mathbf{i} = -\mathbf{k} & \mathbf{j} \times \mathbf{j} = 0 \\
\mathbf{k} \times \mathbf{i} = \mathbf{j} & \mathbf{k} \times \mathbf{j} = -\mathbf{i} & \mathbf{k} \times \mathbf{k} = 0
\end{array}$$





$$\begin{aligned}\mathbf{A}\times\mathbf{B} &= (A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k})\times(B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k})\\ &= A_xB_x(\mathbf{i}\times\mathbf{i})+A_xB_y(\mathbf{i}\times\mathbf{j})+A_xB_z(\mathbf{i}\times\mathbf{k})\\ &\quad + A_yB_x(\mathbf{j}\times\mathbf{i})+A_yB_y(\mathbf{j}\times\mathbf{j})+A_yB_z(\mathbf{j}\times\mathbf{k})\\ &\quad + A_zB_x(\mathbf{k}\times\mathbf{i})+A_zB_y(\mathbf{k}\times\mathbf{j})+A_zB_z(\mathbf{k}\times\mathbf{k})\end{aligned}$$

$$\mathbf{A}\times\mathbf{B}=(A_yB_z-A_zB_y)\mathbf{i}-(A_xB_z-A_zB_x)\mathbf{j}+(A_xB_y-A_yB_x)\mathbf{k}$$

$$\boxed{\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}}$$

$$\begin{vmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{vmatrix}$$

For element **i**: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

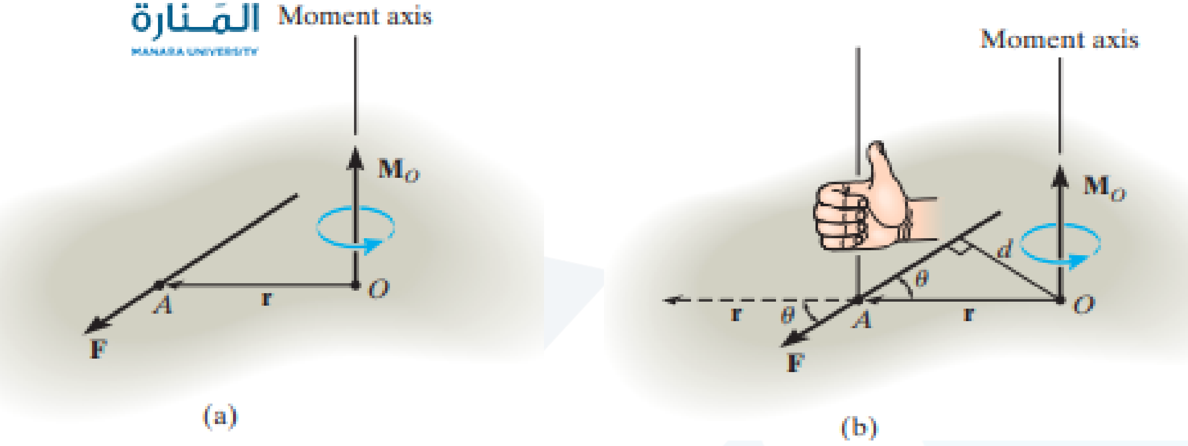
Remember the negative sign

For element **j**: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

For element **k**: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

Moment of a Force—Vector Formulation

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$



Magnitude: $M_O = rF \sin\theta = F(r \sin\theta) = Fd$

Direction:

The direction and sense of MO are determined by the right-hand rule as it applies to the cross product. Thus, sliding r to the dashed position and curling the right-hand fingers from r toward F, "r cross F," the thumb is directed upward or perpendicular to the plane containing r and F and this is in the same direction as MO.

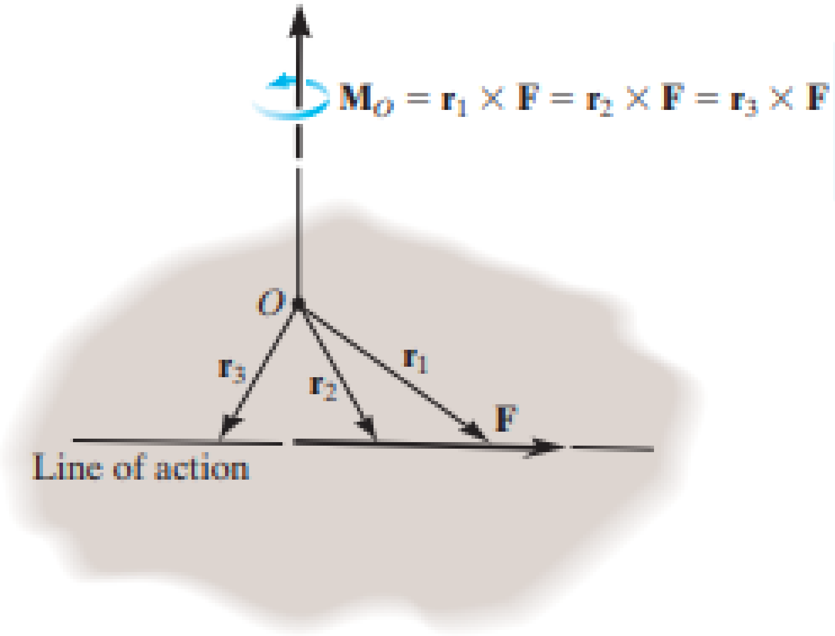
Principle of Transmissibility

we can use any position vector r measured from point O to any point on the line of action of the force F

Since F can be applied at any point along its line of action and still create this same moment about point O

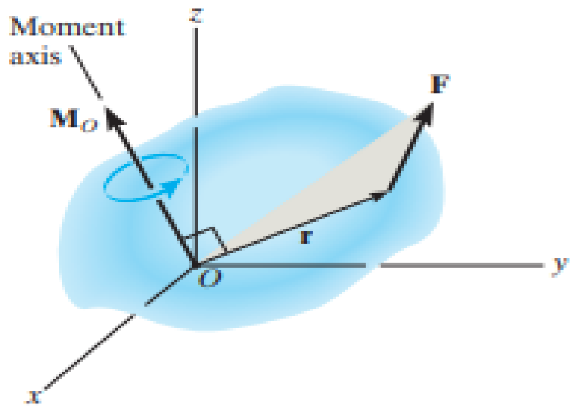
F can be considered a **sliding vector**. This property is called the **principle of transmissibility** of a force.

$$\mathbf{M}_O = \mathbf{r}_1 \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} = \mathbf{r}_3 \times \mathbf{F}$$



Cartesian Vector Formulation.

If we establish x, y, z coordinate axes, then the position vector r and force F can be expressed as Cartesian vectors,

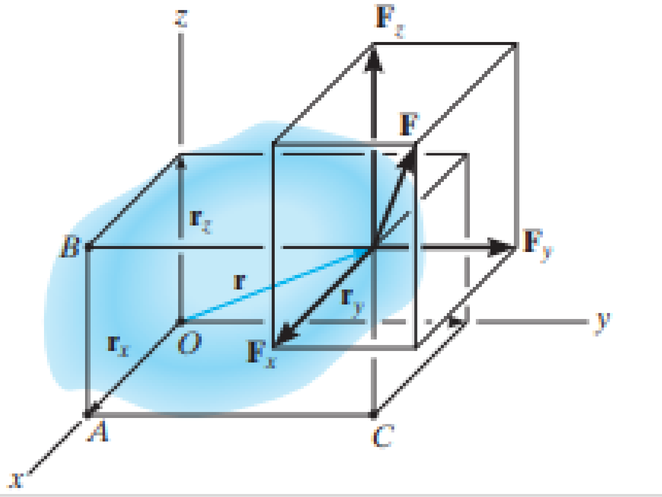


(a)

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

-rx, ry, rz represent the x, y, z components of the position vector drawn from point O to any point on the line of action of the force

- Fx, Fy, Fz represent the x, y, z components of the force vector

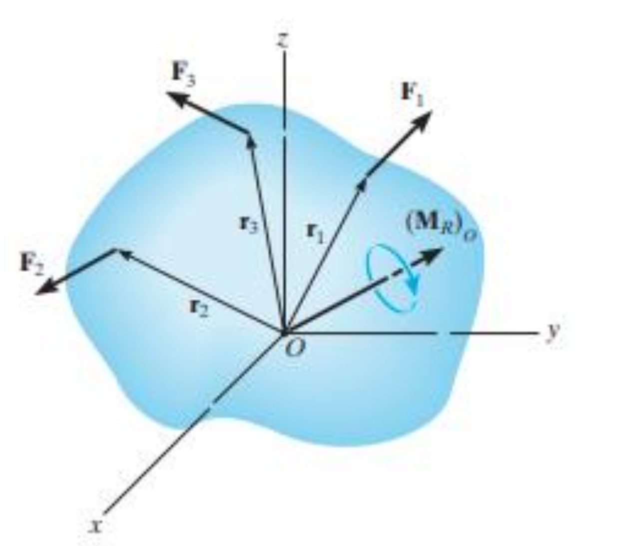


$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k}$$

Resultant Moment of a System of Forces.

If a body is acted upon by a system of forces, the resultant moment of the forces about point O can be determined by vector addition of the moment of each force. This resultant can be written symbolically as:

$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$



Example 1: Determine the moment produced by the force F in about point O. Express the result as a Cartesian vector.

As shown in Fig b, either $\mathbf{r}_A$ or $\mathbf{r}_B$ can be used to determine the moment about point O. These position

$$\mathbf{r}_A = \{12\mathbf{k}\}\text{ m and } \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\text{ m}$$

Force **F** expressed as a Cartesian vector is

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\text{ kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\text{ m}}{\sqrt{(4\text{ m})^2 + (12\text{ m})^2 + (-12\text{ m})^2}}\right]$$

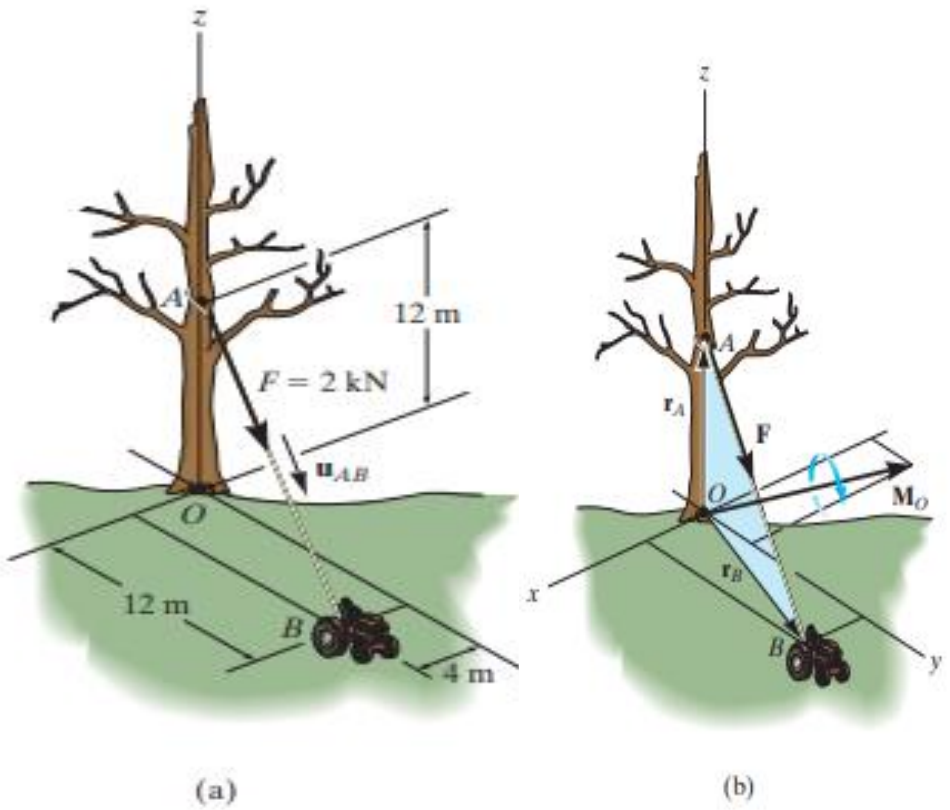
$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\text{ kN}$$

Thus

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\text{ kN}\cdot\text{m}$$



$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

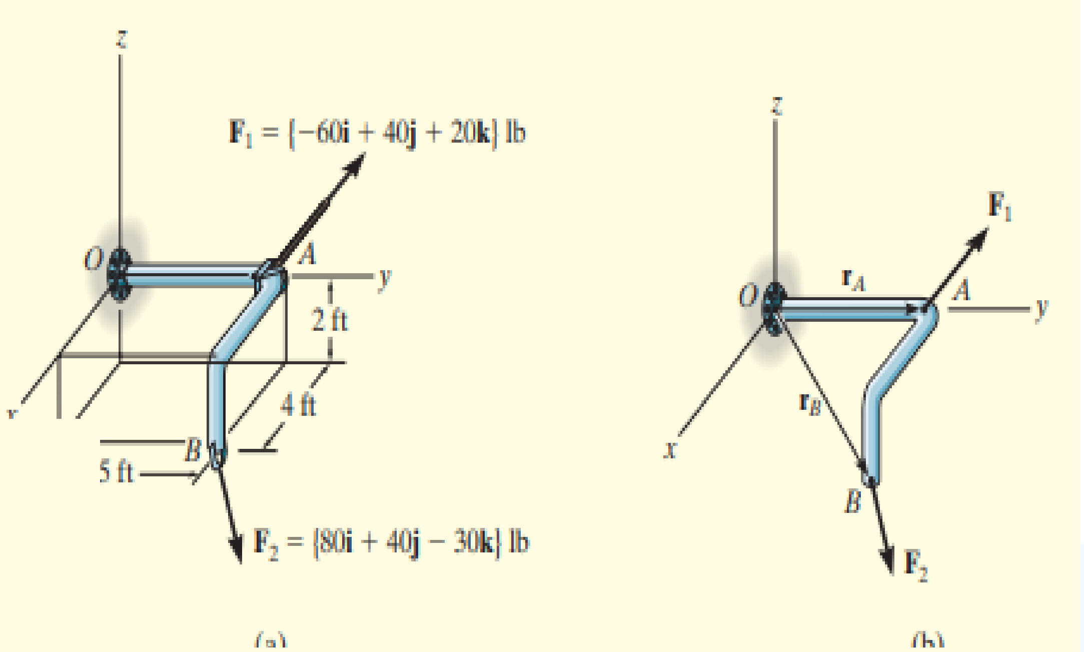
$$= [12(-1.376) - 0(1.376)]\mathbf{i} - [4(-1.376) - 0(0.4588)]\mathbf{j} + [4(1.376) - 12(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN} \cdot \text{m} \qquad \textit{Ans.}$$

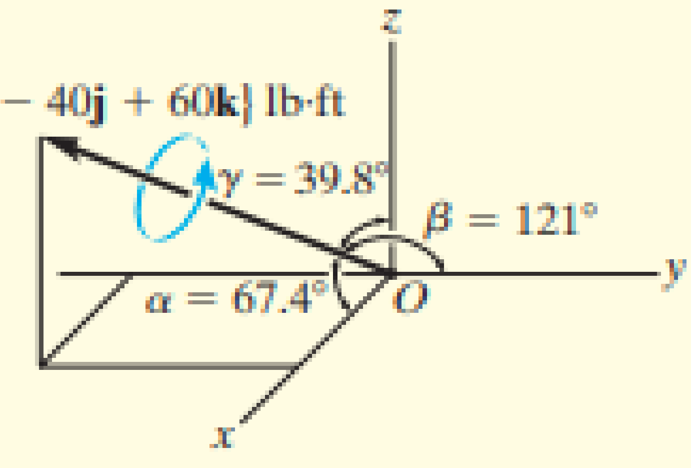
Example 2 :Two forces act on the rod shown in Fig. a. Determine the resultant moment they create about the flange at O. Express the result as a Cartesian vector.

Solution :Position vectors are directed from point O to each force as shown in Fig. b. These vectors are

$$\mathbf{r}_A = \{5\mathbf{j}\}\ \text{ft}$$
$$\mathbf{r}_B = \{4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$



$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$
$$= \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$
$$= [5(20) - 0(40)]\mathbf{i} - [0]\mathbf{j} + [0(40) - (5)(-60)]\mathbf{k}$$
$$+ [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k}$$
$$= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\}\ \text{lb}\cdot\text{ft} \qquad \textit{Ans.}$$



(c)

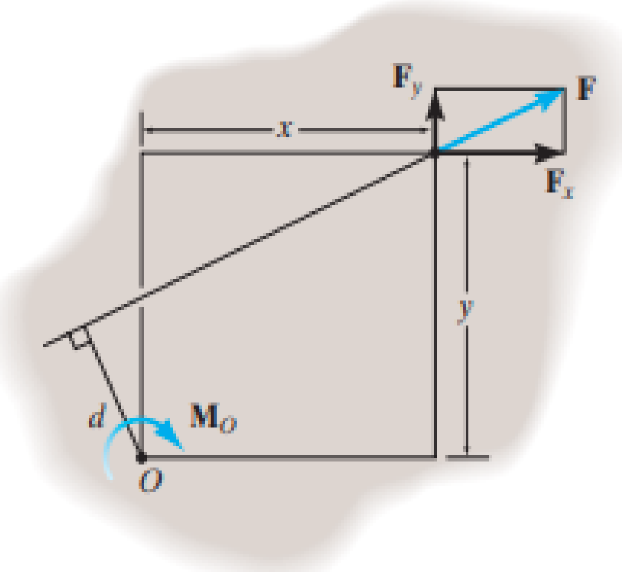
Principle of moment : (Varignon's theorem)

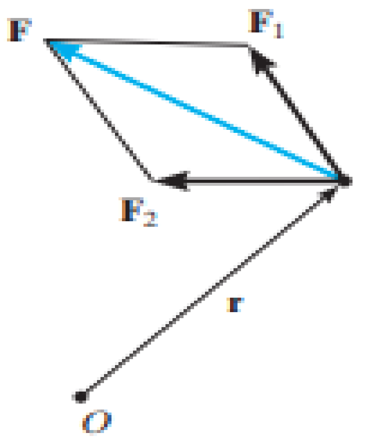
the moment of a force about a point is equal to the sum of the moments of the components of the force about the point

$$M_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

$$M_O = F_x y - F_y x$$

$$M_O = Fd.$$





Important Points

- The moment of a force creates the tendency of a body to turn about an axis passing through a specific point O.
- Using the right-hand rule, the sense of rotation is indicated by the curl of the fingers, and the thumb is directed along the moment axis, or line of action of the moment.
- The magnitude of the moment is determined from MO = F.d, where d is called the **moment arm**, which represents the perpendicular or shortest distance from point O to the line of action of the force.
- In three dimensions the vector cross product is used to determine the moment, i.e., MO = r x F. Remember that **r** is directed from point O to any point on the line of action of F.

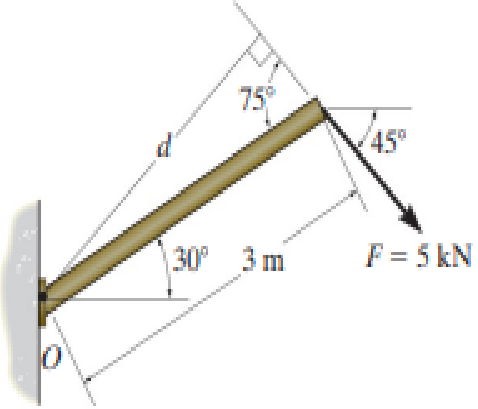
Example 3:Determine the moment of the force in Fig. a about point O

SOLUTION I :The moment arm d in Fig. a can be found from trigonometry.

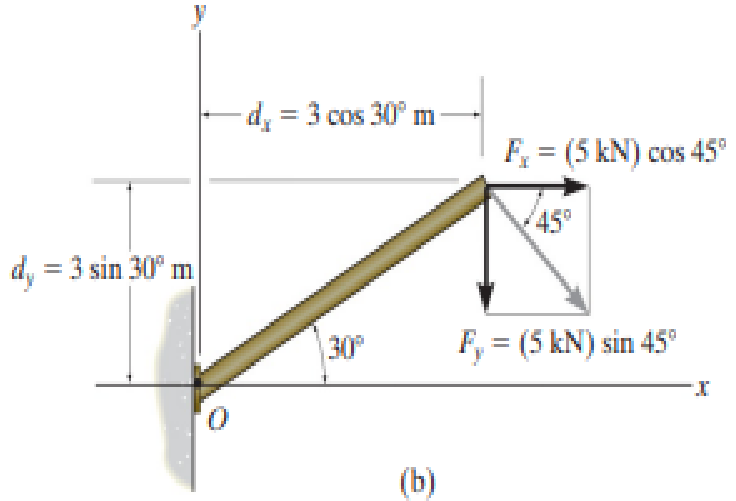
d = (3 m) sin 75 = 2.898 m Thus,

M_O = F.d = (5 kN)(2.898 m) = 14.5 kN

. Since the force tends to rotate or orbit clockwise about point O, the moment is directed into the page



(a)

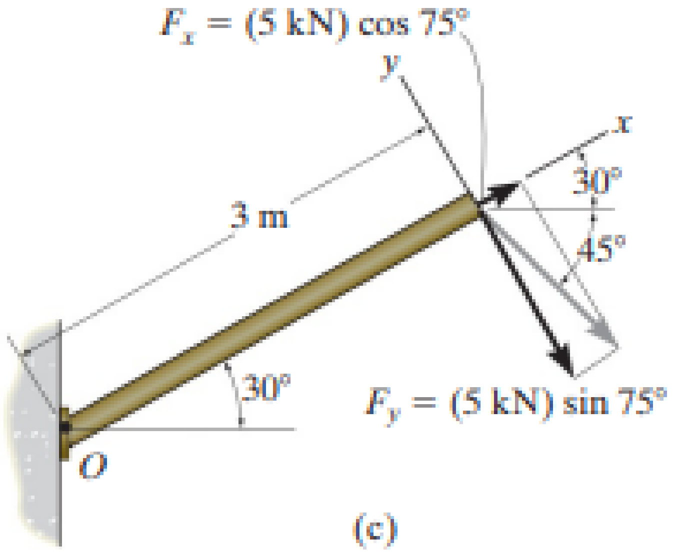


(b)

SOLUTION II :The x and y components of the force are indicated in Fig. b. Considering counterclockwise moments as positive, and applying the principle of moments, we have:

$$\circlearrowleft + M_O = -F_x d_y - F_y d_x$$
$$= -(5\cos 45°\text{ kN})(3\sin 30°\text{ m}) - (5\sin 45°\text{ kN})(3\cos 30°\text{ m})$$
$$= -14.5\text{ kN}\cdot\text{m} = 14.5\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

SOLUTION III : The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. c. Here Fx produces no moment about point O since its line of action passes through this point. Therefore,



$$\circlearrowleft + M_O = -F_y d_x$$

$$= -(5 \sin 75° \text{ kN})(3 \text{ m})$$

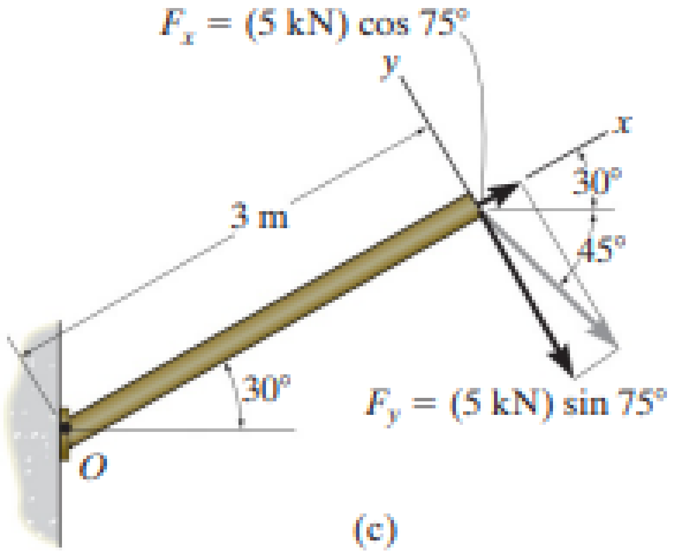
$$= -14.5 \text{ kN}\cdot\text{m} = 14.5 \text{ kN}\cdot\text{m} \circlearrowright$$

SOLUTION III :The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. c. Here Fx produces no moment about point O since its line of action passes through this point. Therefore,

$$\circlearrowleft + M_O = -F_y d_x$$

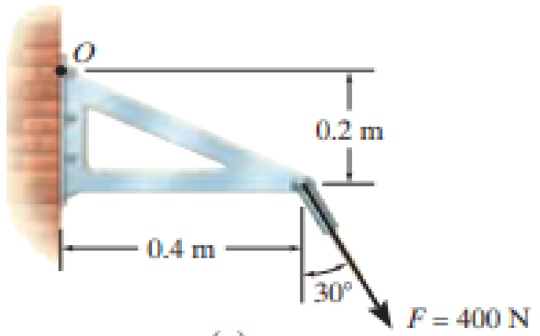
$$= -(5 \sin 75^\circ \text{ kN})(3 \text{ m})$$

$$= -14.5 \text{ kN}\cdot\text{m} = 14.5 \text{ kN}\cdot\text{m} \circlearrowright$$



(c)

Example 4: Force F acts at the end of the angle bracket in Fig. a. Determine the moment of the force about point O.

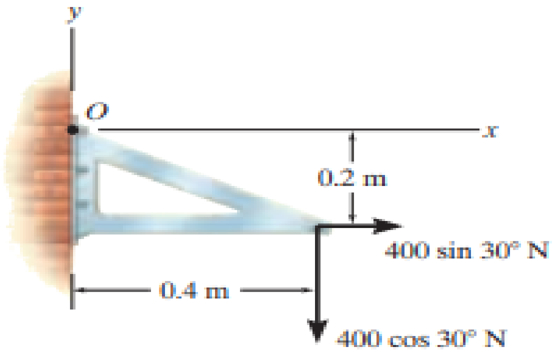


(a)

SOLUTION I :(Scalar Analysis) The force is resolved into its x and y components, Fig. b, then

$$\circlearrowleft + M_O = 400 \sin 30^\circ \text{ N}(0.2 \text{ m}) - 400 \cos 30^\circ \text{ N}(0.4 \text{ m})$$
$$= -98.6 \text{ N}\cdot\text{m} = 98.6 \text{ N}\cdot\text{m} \circlearrowright$$

$$\mathbf{M}_O = \{-98.6\mathbf{k}\} \text{ N}\cdot\text{m}$$

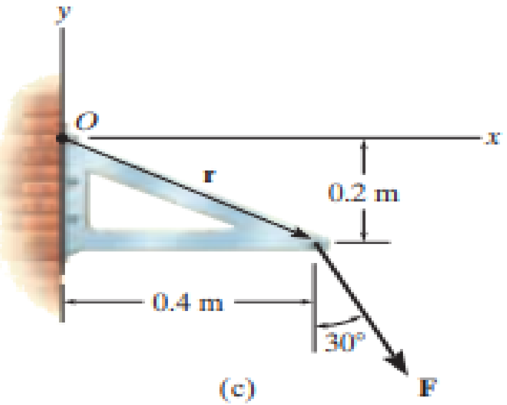


(b)

SOLUTION II (Vector Analysis) Using a Cartesian vector approach, the force and position vectors, Fig. c, are

$$\mathbf{r} = \{0.4\mathbf{i} - 0.2\mathbf{j}\} \text{ m}$$
$$\mathbf{F} = \{400 \sin 30^\circ \mathbf{i} - 400 \cos 30^\circ \mathbf{j}\} \text{ N}$$
$$= \{200.0\mathbf{i} - 346.4\mathbf{j}\} \text{ N}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & -0.2 & 0 \\ 200.0 & -346.4 & 0 \end{vmatrix}$$
$$= 0\mathbf{i} - 0\mathbf{j} + [0.4(-346.4) - (-0.2)(200.0)]\mathbf{k}$$
$$= \{-98.6\mathbf{k}\} \text{ N}\cdot\text{m}$$



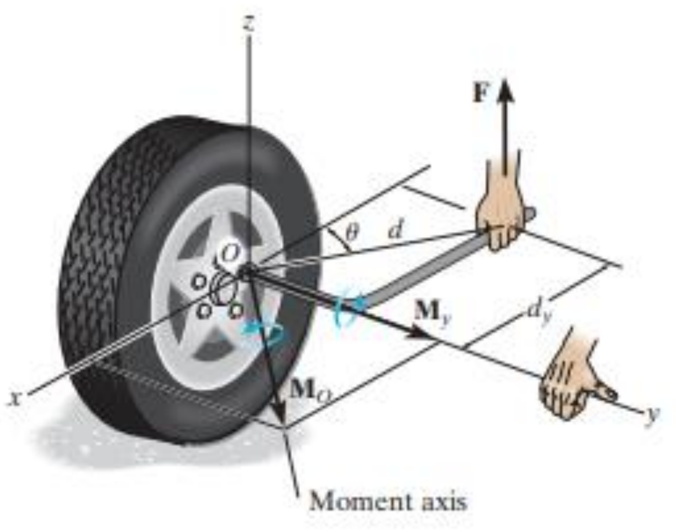
(c)

Moment of force about a specified axis

Scalar Analysis. To use a scalar analysis in the case of the lug nut in Fig. a, the moment arm, or perpendicular distance from the axis to the line of action of the force, is $d_y = d.\cos\theta$. Thus, the moment of F about the y axis is:

$M_y = F.d_y = F(d\cos\theta)$. According to the right-hand rule, M_y is directed along the positive y axis as shown in the figure. In general, for any axis **a**, the moment is:

$$\boxed{M_a = Fd_a}$$

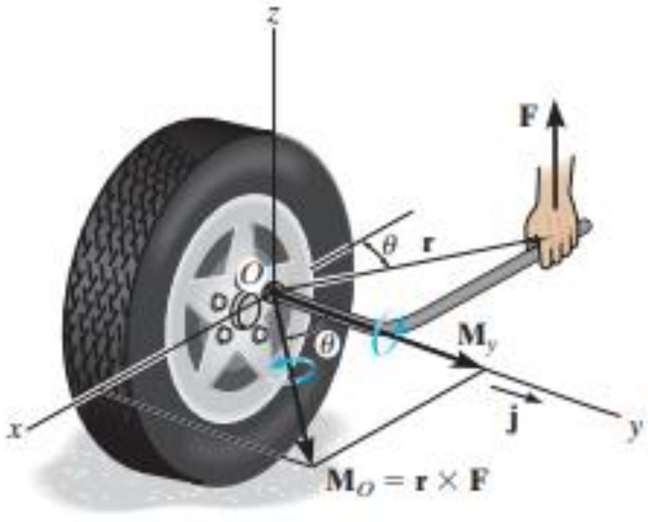


(a)

Vector Analysis.

we must first determine the moment of the force about any point O on the y axis, $M_O = r \times F$. The component M_y along the y axis is the projection of M_O

using the dot product : $M_y = j \cdot M_O = j \cdot (r \times F)$



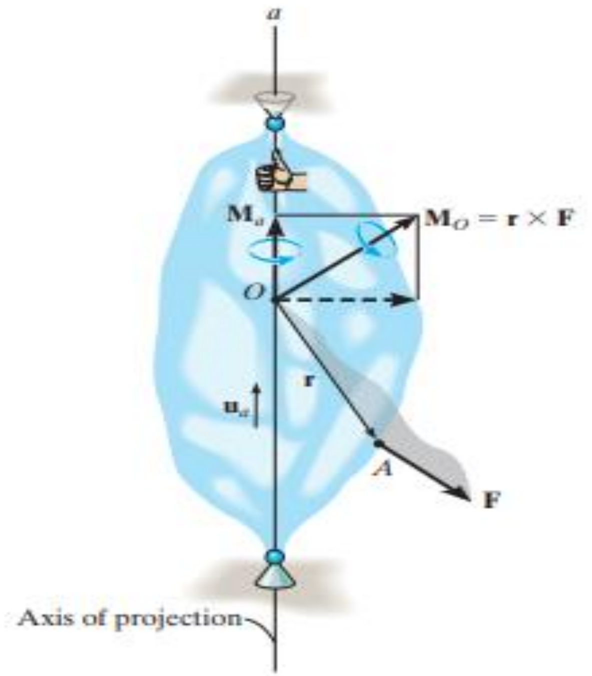
(b)

We can generalize this approach by letting u_a be the unit vector that specifies the direction of the a axis. Then the moment of F about a point O on the axis is $M_O = r \times F$, and the projection of this moment onto the a axis is $M_a = u_a \cdot (r \times F)$. This combination is referred to as the scalar triple product

$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



where uax , uay , uaz represent the x, y, z components of the unit vector defining the direction of the a axis rx, ry, rz represent the x, y, z components of the position vector extended from any point O on the a axis to any point A on the line of action of the force Fx, Fy, Fz represent the x, y, z components of the force vector

Provided Ma is determined, we can then express Ma as a Cartesian vector, namely

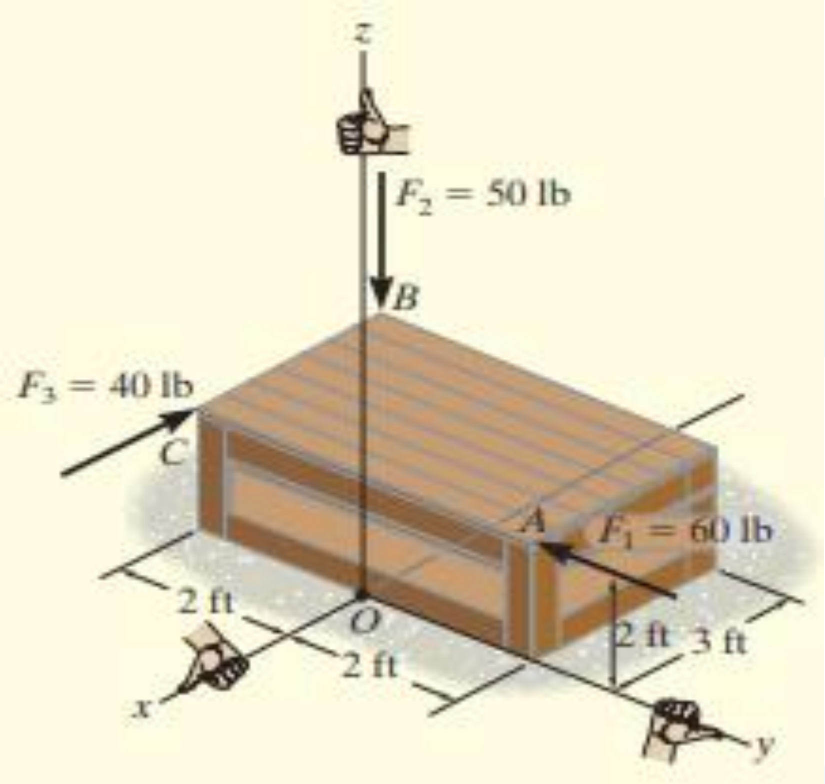
$$\mathbf{M}_a = M_a\mathbf{u}_a$$

Important Points

- The moment of a force about a specified axis can be determined provided the perpendicular distance da from the force line of action to the axis can be determined. Ma = F .da.
- If vector analysis is used, Ma = ua . (r x F), where ua defines the direction of the axis .and r is extended from any point on the axis to any point on the line of action of the force.
- If Ma is calculated as a negative scalar, then the sense of direction of Ma is opposite to ua.
- The moment Ma expressed as a Cartesian vector is determined from Ma = Ma.ua

Example 5:Determine the resultant moment of the three forces in Fig. about the x axis, the y axis, and the z axis

SOLUTION A force that is parallel to a coordinate axis or has a line of action that passes through the axis does not produce any moment or tendency for turning about that axis. Therefore, defining the positive direction of the moment of a force according to the right-hand rule, as shown in the figure, we have



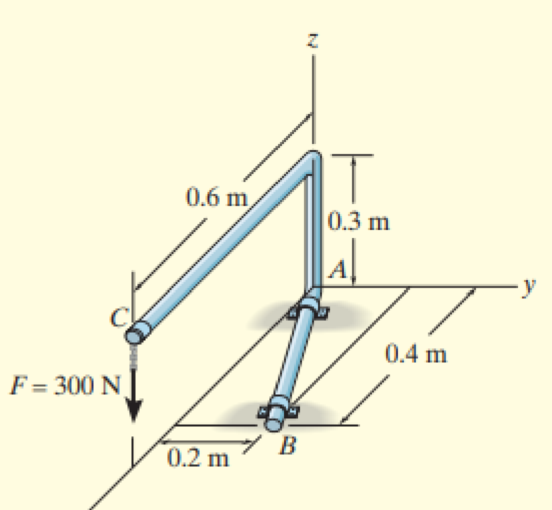
$$M_x = (60\text{ lb})(2\text{ ft}) + (50\text{ lb})(2\text{ ft}) + 0 = 220\text{ lb}\cdot\text{ft}$$

$$M_y = 0 - (50\text{ lb})(3\text{ ft}) - (40\text{ lb})(2\text{ ft}) = -230\text{ lb}\cdot\text{ft}$$

$$M_z = 0 + 0 - (40\text{ lb})(2\text{ ft}) = -80\text{ lb}\cdot\text{ft}$$

Example 6 : Determine the moment M_{AB} produced by the force F in Fig. a, which tends to rotate the rod about the AB axis

SOLUTION: A vector analysis using $M_{AB} = \mathbf{u}_B \cdot (\mathbf{r} \times \mathbf{F})$ will be considered for the solution rather than trying to find the moment arm or perpendicular distance from the line of action of F to the AB axis. Each of the terms in the equation will now be identified



(a)

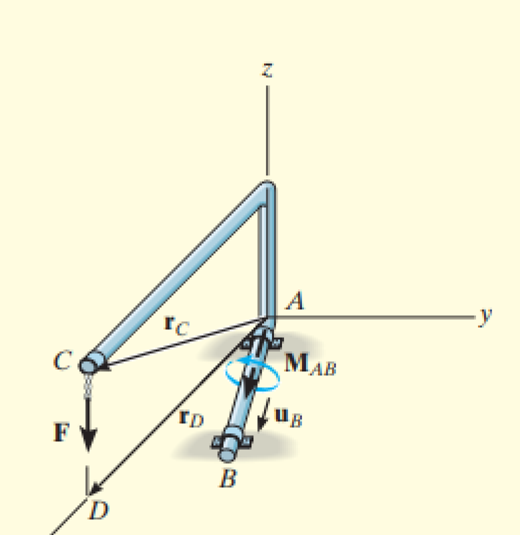
Unit vector $\mathbf{u}_B$ defines the direction of the AB axis of the rod

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{\{0.4\mathbf{i} + 0.2\mathbf{j}\}\ \text{m}}{\sqrt{(0.4\ \text{m})^2 + (0.2\ \text{m})^2}} = 0.8944\mathbf{i} + 0.4472\mathbf{j}$$

Vector r is directed from any point on the AB axis to any point on the line of action of the force. For example, position vectors $\mathbf{r}_C$ and $\mathbf{r}_D$ are suitable, Fig. b.(Although not shown, $\mathbf{r}_{BC}$ or $\mathbf{r}_{BD}$ can also be used.) For simplicity, we choose $\mathbf{r}_D$, where

$$\mathbf{r}_D = \{0.6\mathbf{i}\}\ \text{m}$$

$$\mathbf{F} = \{-300\mathbf{k}\}\ \text{N}$$



(b)

$$M_{AB} = \mathbf{u}_B \cdot (\mathbf{r}_D \times \mathbf{F}) = \begin{vmatrix} 0.8944 & 0.4472 & 0 \\ 0.6 & 0 & 0 \\ 0 & 0 & -300 \end{vmatrix}$$

$$= 0.8944[0(-300) - 0(0)] - 0.4472[0.6(-300) - 0(0)] + 0[0.6(0) - 0(0)]$$

$$= 80.50 \text{ N} \cdot \text{m}$$

This positive result indicates that the sense of $\mathbf{M}_{AB}$ is in the same direction as $\mathbf{u}_B$.

Expressing $\mathbf{M}_{AB}$ in Fig. as a Cartesian vector yields

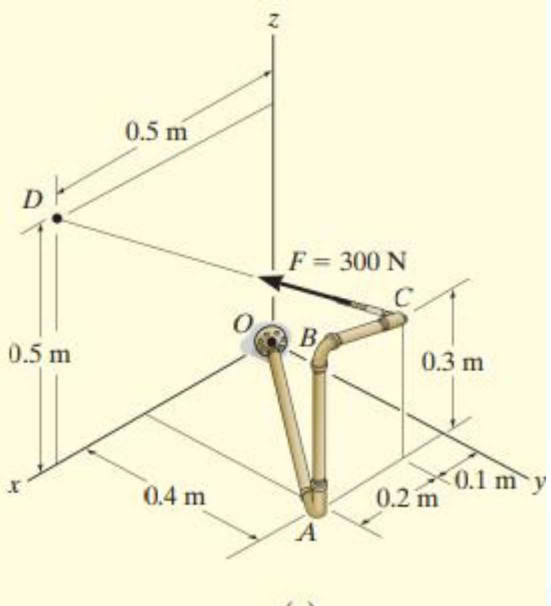
$$\mathbf{M}_{AB} = M_{AB}\mathbf{u}_B = (80.50 \text{ N} \cdot \text{m})(0.8944\mathbf{i} + 0.4472\mathbf{j})$$

$$= \{72.0\mathbf{i} + 36.0\mathbf{j}\} \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$

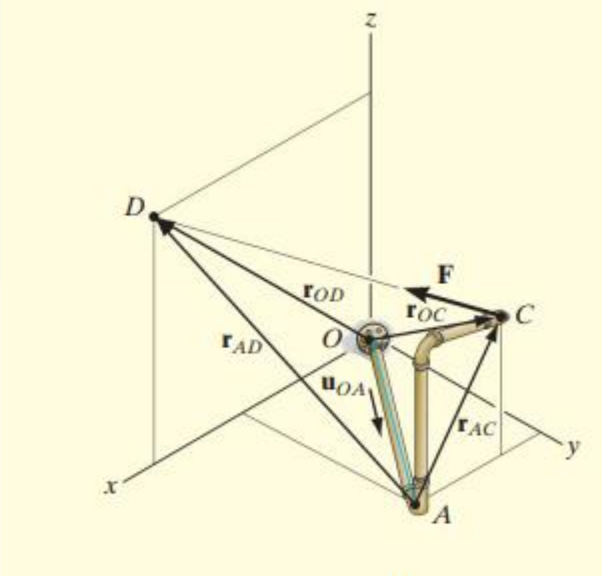
Example 6 :Determine the magnitude of the moment of force **F** about segment **OA** of the pipe assembly.

SOLUTION :The moment of F about the OA axis is determined from MOA = **u**OA . (r x F), where **r** is a position vector extending from any point on the OA axis to any point on the line of action of F. As indicated in Fig. b, either **r**OD, **r**OC, **r**AD, or **r**AC can be used; however, **r**OD will be considered since it will simplify the calculation. The unit vector **u**OA, which specifies the direction of the OA axis, is :

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{\{0.3\mathbf{i} + 0.4\mathbf{j}\}\ \text{m}}{\sqrt{(0.3\ \text{m})^2 + (0.4\ \text{m})^2}} = 0.6\mathbf{i} + 0.8\mathbf{j}$$



(a)



(b)

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{\{0.3\mathbf{i} + 0.4\mathbf{j}\}\text{ m}}{\sqrt{(0.3\text{ m})^2 + (0.4\text{ m})^2}} = 0.6\mathbf{i} + 0.8\mathbf{j}$$

and the position vector $\mathbf{r}_{OD}$ is

$$\mathbf{r}_{OD} = \{0.5\mathbf{i} + 0.5\mathbf{k}\}\text{ m}$$

The force **F** expressed as a Cartesian vector is

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right)$$

$$= (300\text{ N})\left[\frac{\{0.4\mathbf{i} - 0.4\mathbf{j} + 0.2\mathbf{k}\}\text{ m}}{\sqrt{(0.4\text{ m})^2 + (-0.4\text{ m})^2 + (0.2\text{ m})^2}}\right]$$

$$= \{200\mathbf{i} - 200\mathbf{j} + 100\mathbf{k}\}\text{ N}$$

Therefore,

$$M_{OA} = \mathbf{u}_{OA} \cdot (\mathbf{r}_{OD} \times \mathbf{F})$$

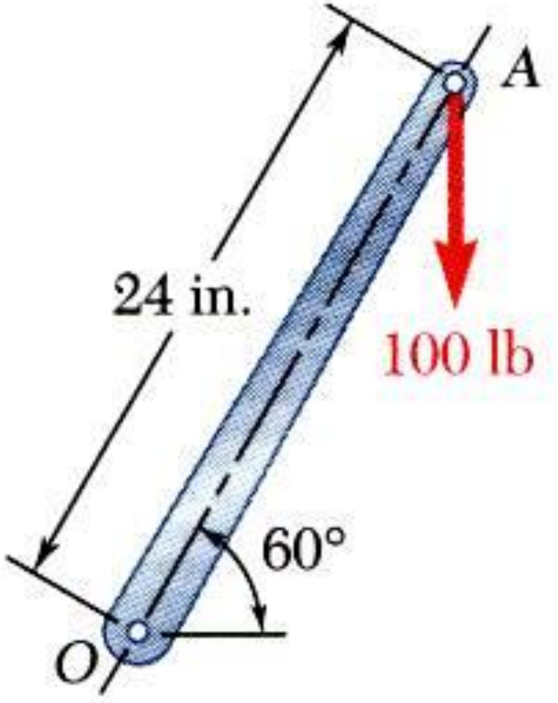
$$= \begin{vmatrix} 0.6 & 0.8 & 0 \\ 0.5 & 0 & 0.5 \\ 200 & -200 & 100 \end{vmatrix}$$

$$= 0.6[0(100) - (0.5)(-200)] - 0.8[0.5(100) - (0.5)(200)] + 0$$

$$= 100\text{ N}\cdot\text{m}$$ *Ans.*

Example 7

- A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at *O*.
- Determine:
  a) Moment about *O*,
  b) Horizontal force at *A* which creates the same moment,
  c) Smallest force at A which produces the same moment,
  d) Location for a 240-lb vertical force to produce the same moment,
  e) Whether any of the forces from b, c, and d is equivalent to the original force.



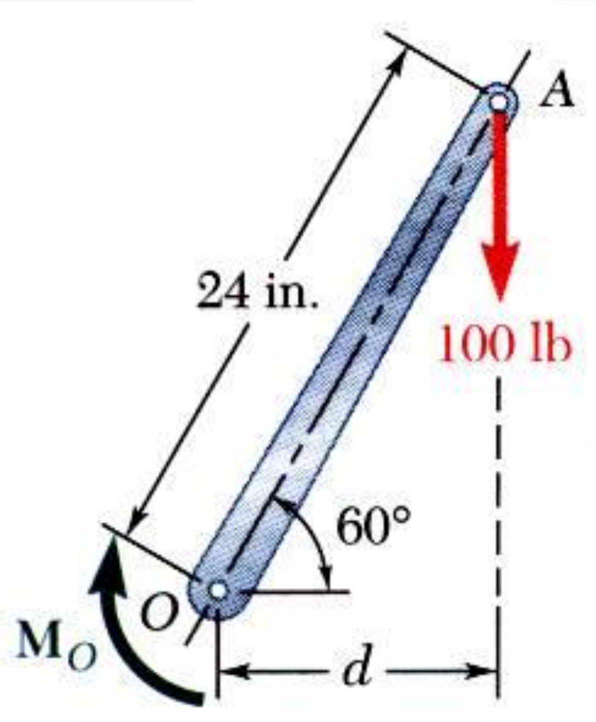
a) Moment about O is equal to the product of the force and the perpendicular distance between the line of action of the force and O. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper

$$M_O = Fd$$

$$d = (24\,\text{in.})\cos 60° = 12\,\text{in.}$$

$$M_O = (100\,\text{lb})(12\,\text{in.})$$

$$\boxed{M_O = 1200\,\text{lb}\cdot\text{in}}$$



b) **Horizontal force at *A* that produces the same moment,**

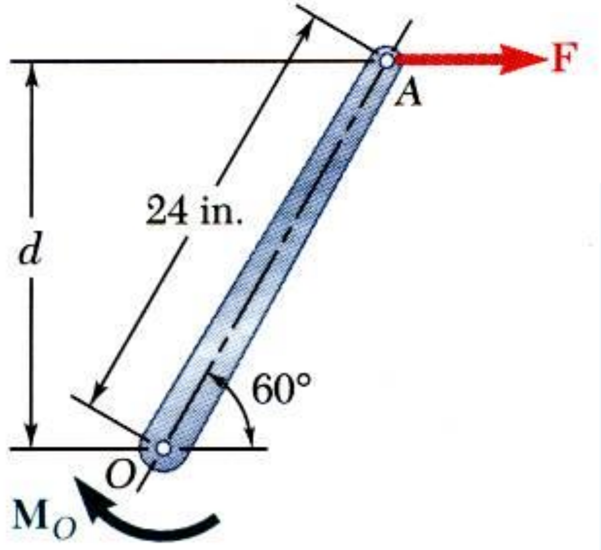
$$d = (24\text{ in.})\sin 60° = 20.8\text{ in.}$$

$$M_O = Fd$$

$$1200\text{ lb}\cdot\text{in.} = F(20.8\text{ in.})$$

$$F = \frac{1200\text{ lb}\cdot\text{in.}}{20.8\text{ in.}}$$

$$\boxed{F = 57.7\text{ lb}}$$



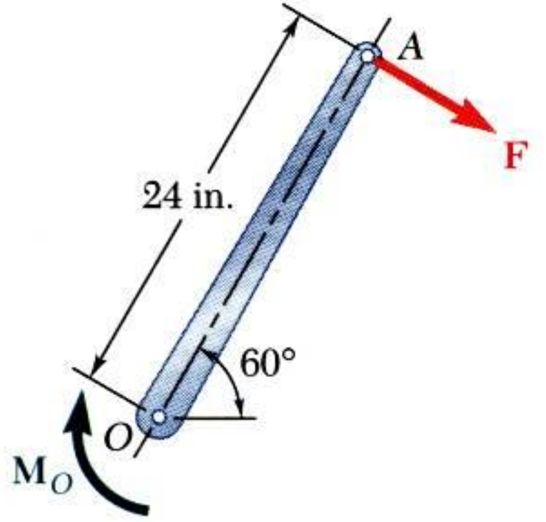
c) The smallest force at *A* to produce the same moment occurs when the perpendicular distance is a maximum or when *F* is perpendicular to *OA*

$$M_O = Fd$$

$$1200\,\text{lb}\cdot\text{in.} = F(24\text{ in.})$$

$$F = \frac{1200\,\text{lb}\cdot\text{in.}}{24\text{ in.}}$$

$$\boxed{F = 50\,\text{lb}}$$



d) To determine the point of application of a 240 lb force to produce the same moment,

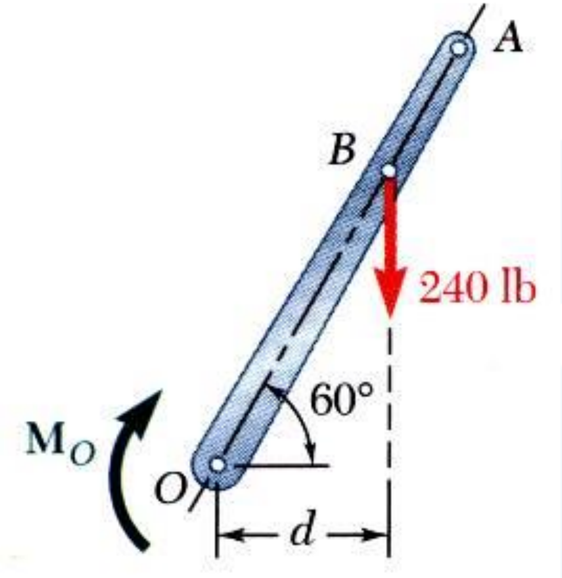
$$M_O = Fd$$

$$1200\,\text{lb}\cdot\text{in.} = (240\,\text{lb})d$$

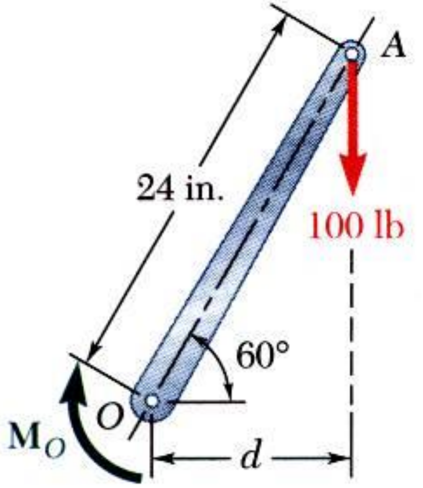
$$d = \frac{1200\,\text{lb}\cdot\text{in.}}{240\,\text{lb}} = 5\,\text{in.}$$

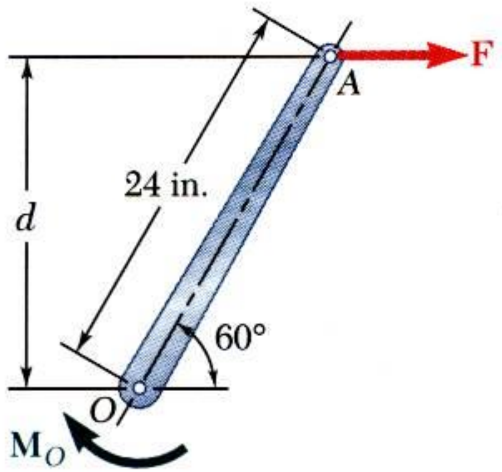
$$OB\cos 60^\circ = 5\,\text{in.}$$

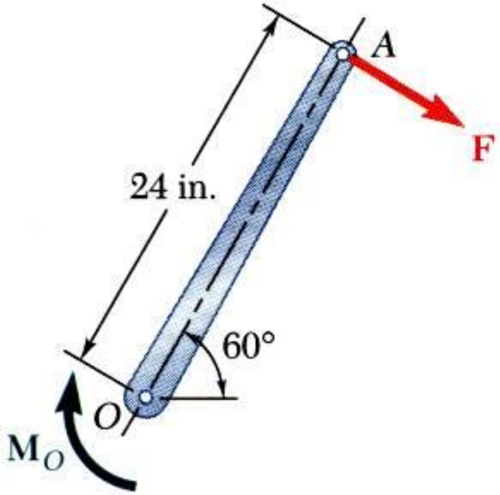
$$\boxed{OB = 10\,\text{in.}}$$

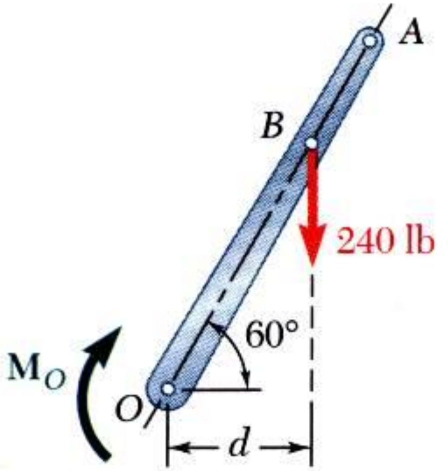


e) Although each of the forces in parts b), c), and d) produces the same moment as the 100 lb force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 lb force



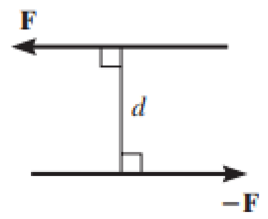






Force Couples

- A Couple is defined as two Forces having the same magnitude, parallel lines of action, and opposite sense
- In this situation, the sum of the forces in each direction is zero, so a couple does not affect the sum of forces equations
- A force couple will however tend to rotate the body.

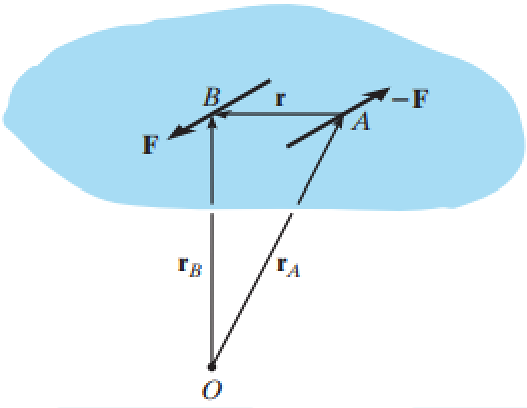


$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

However $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

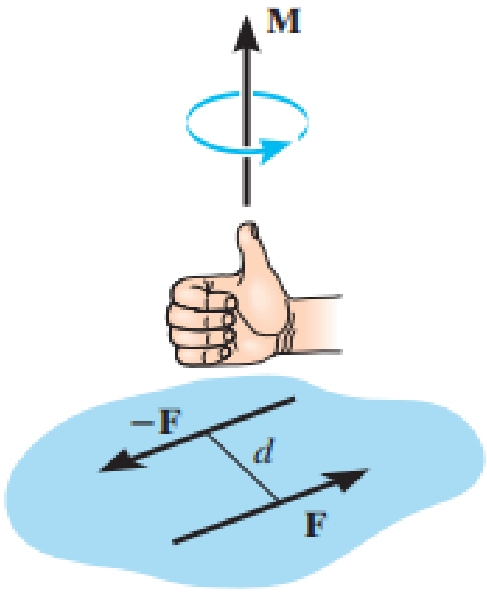
This result indicates that a couple moment is a free vector, i.e., it can act at any point since M depends only upon the position vector r directed between the forces and not the position vectors rA and rB, directed from the arbitrary point O to the forces.



Scalar Formulation. The moment of a couple, M,, is defined as having a magnitude of

$$M = Fd$$

where **F** is the magnitude of one of the forces and **d** is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right-hand rule, where the thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces. In all cases, M will act perpendicular to the plane containing these forces



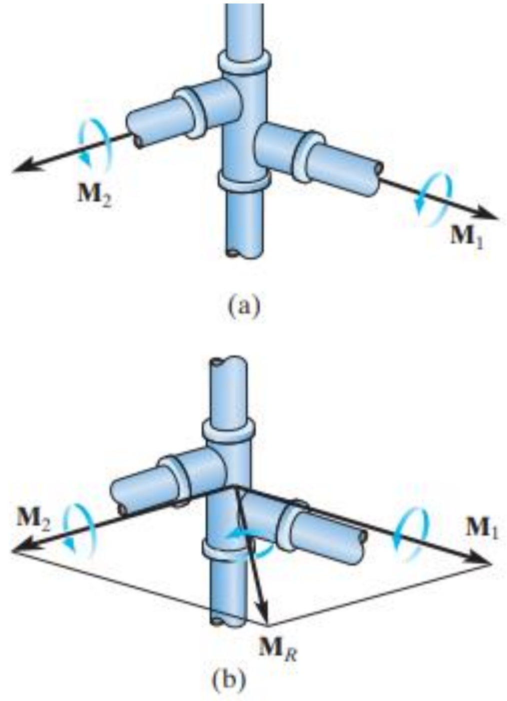
Vector Formulation. The moment of a couple can also be expressed by the vector cross product using

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Equivalent Couples. If two couples produce a moment with the same magnitude and direction, then these two couples are equivalent since

Resultant Couple Moment. Since couple moments are vectors, their resultant can be determined by vector addition

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$



If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as:

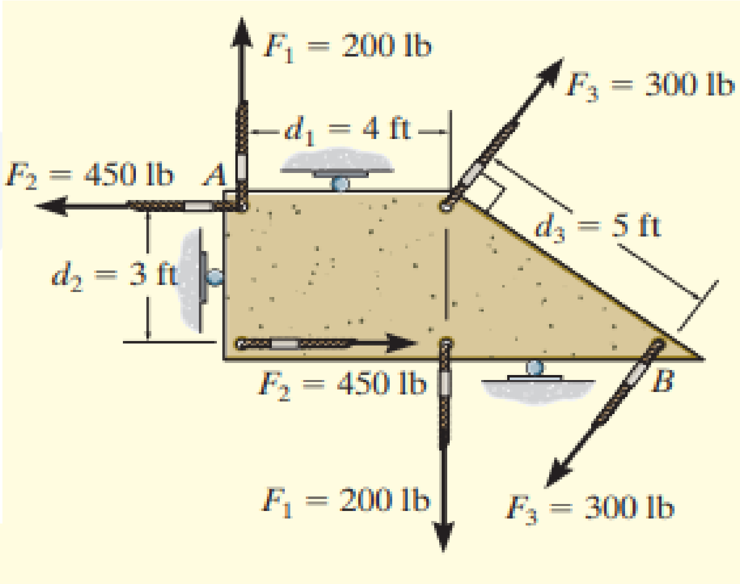
$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$

Important Points

- A couple moment is produced by two noncollinear forces that are equal in magnitude but opposite in direction. Its effect is to produce pure rotation, or tendency for rotation in a specified direction.
- A couple moment is a free vector, and as a result it causes the same rotational effect on a body regardless of where the couple moment is applied to the body.
- The moment of the two couple forces can be determined about any point. For convenience, this point is often chosen on the line of action of one of the forces in order to eliminate the moment of this force about the point.
- In three dimensions the couple moment is often determined using the vector formulation, $M = r \times F$, where r is directed from any point on the line of action of one of the forces to any point on the line of action of the other force F.
- A resultant couple moment is simply the vector sum of all the couple moments of the system

Example9 :**Determine the resultant couple moment of the three couples acting on the plate**

SOLUTION: As shown the perpendicular distances between each pair of couple forces are d1 = 4 ft, d2 = 3 ft, and d3 = 5 ft. Considering counterclockwise couple moments as positive, we have



$$\circlearrowleft + M_R = \Sigma M;\ M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= -(200\ \text{lb})(4\ \text{ft}) + (450\ \text{lb})(3\ \text{ft}) - (300\ \text{lb})(5\ \text{ft})$$

$$= -950\ \text{lb}\cdot\text{ft} = 950\ \text{lb}\cdot\text{ft} \circlearrowright \qquad \textit{Ans.}$$

Example 10 : Determine the magnitude and direction of the couple moment acting on the gear

Solution :The easiest solution requires resolving each force into its components as shown in Fig. b. The couple moment can be determined by summing the moments of these force components about any point, for example, the center O of the gear or point A. If we consider counterclockwise moments as positive, we have

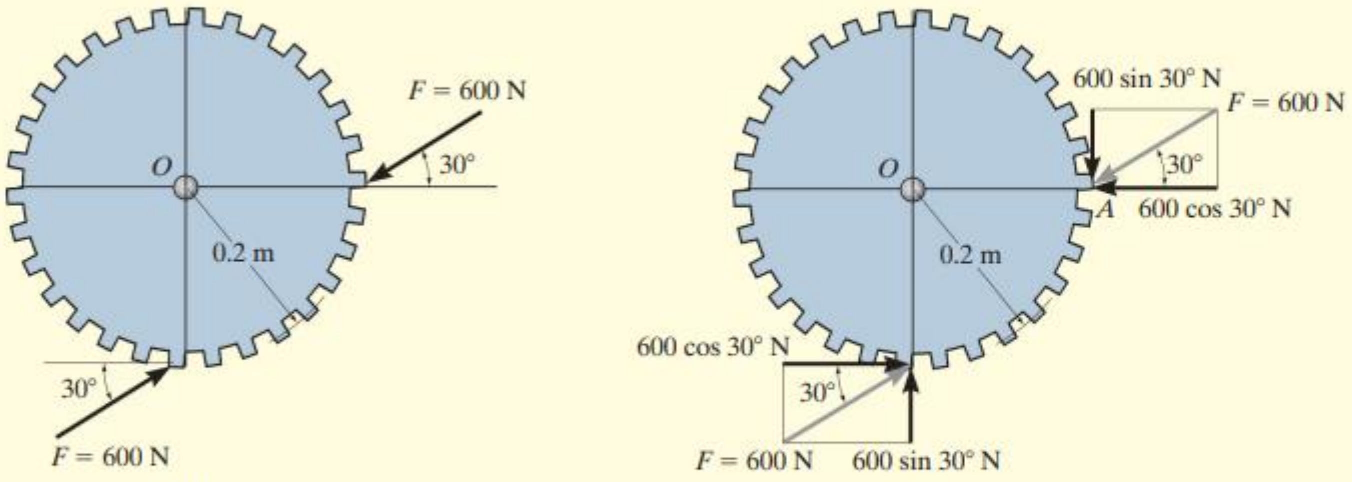
$$\circlearrowleft + M = \Sigma M_O;\quad M = (600 \cos 30^\circ\ \text{N})(0.2\ \text{m}) - (600 \sin 30^\circ\ \text{N})(0.2\ \text{m})$$

$$= 43.9\ \text{N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

or

$$\circlearrowleft + M = \Sigma M_A;\quad M = (600 \cos 30^\circ\ \text{N})(0.2\ \text{m}) - (600 \sin 30^\circ\ \text{N})(0.2\ \text{m})$$

$$= 43.9\ \text{N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$



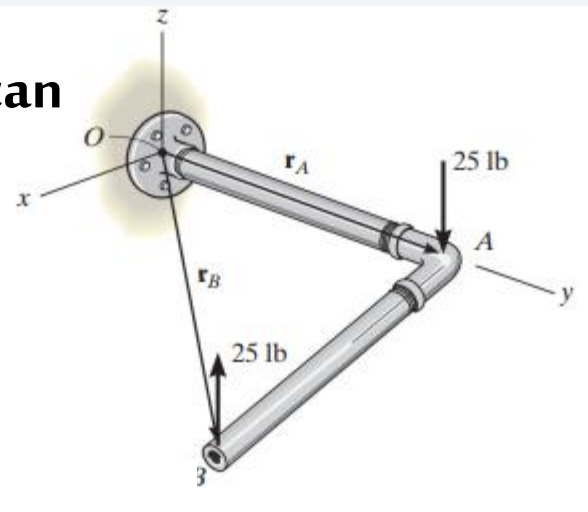
(a) (b)

Example11 :Determine the couple moment acting on the pipe shown in Fig. Segment AB is directed 30° below the x–y plane

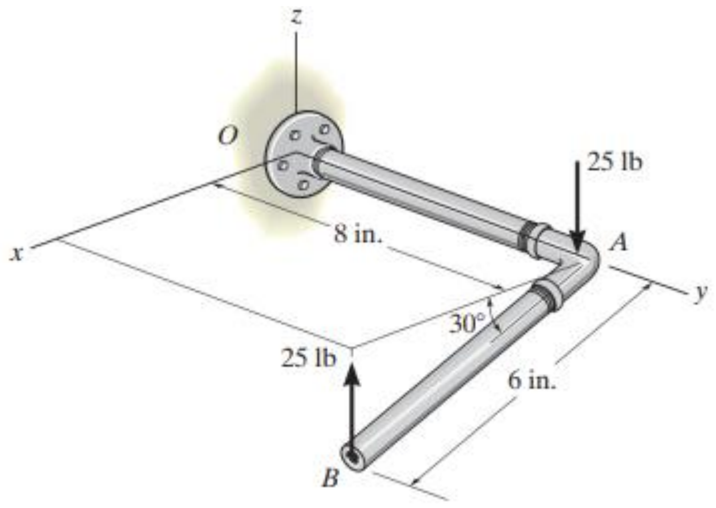
Solution I (vector analysis)

The moment of the two couple forces can be found about any point. If point O is considered,

Fig. b, we have



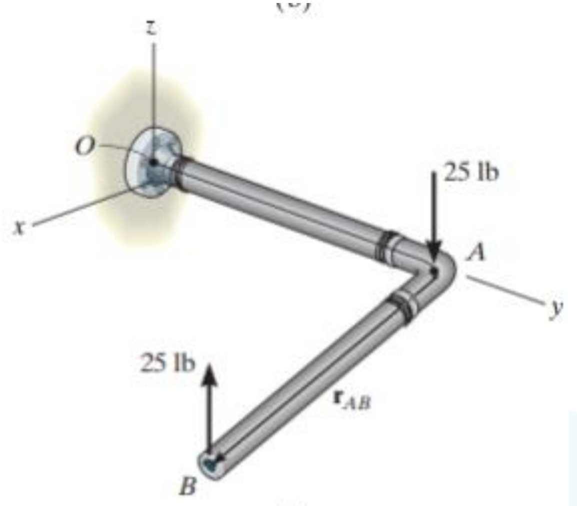
(b)



(a)

$$\mathbf{M} = \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k})$$
$$= (8\mathbf{j}) \times (-25\mathbf{k}) + (6\cos 30^\circ\mathbf{i} + 8\mathbf{j} - 6\sin 30^\circ\mathbf{k}) \times (25\mathbf{k})$$
$$= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i}$$
$$= \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.} \qquad \textit{Ans.}$$

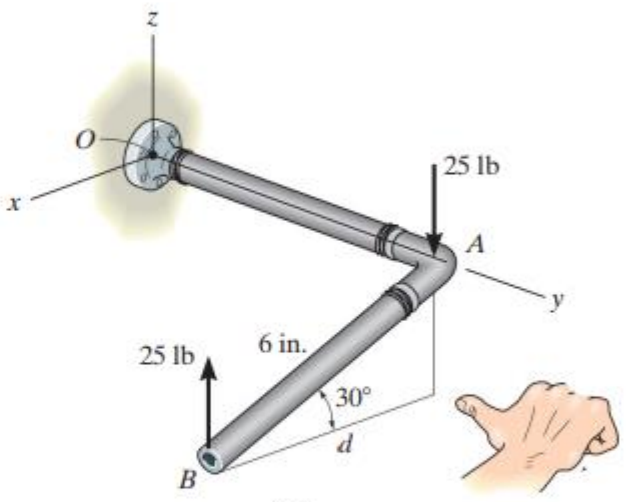
It is easier to take moments of the couple forces about a point lying on the line of action of one of the forces, e.g., point A, Fig.c. In this case the moment of the force at A is zero, so that



(c)

$$\mathbf{M} = \mathbf{r}_{AB} \times (25\mathbf{k})$$
$$= (6 \cos 30^\circ \mathbf{i} - 6 \sin 30^\circ \mathbf{k}) \times (25\mathbf{k})$$
$$= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.}$$

Solution II (scalar analysis) Although this problem is shown in three dimensions, the geometry is simple enough to use the scalar equation M = F.d. The perpendicular distance between the lines of action of the couple forces is d = 6 cos 30 = 5.196 in., Fig. d. Hence, taking moments of the forces about either point A or point B yields

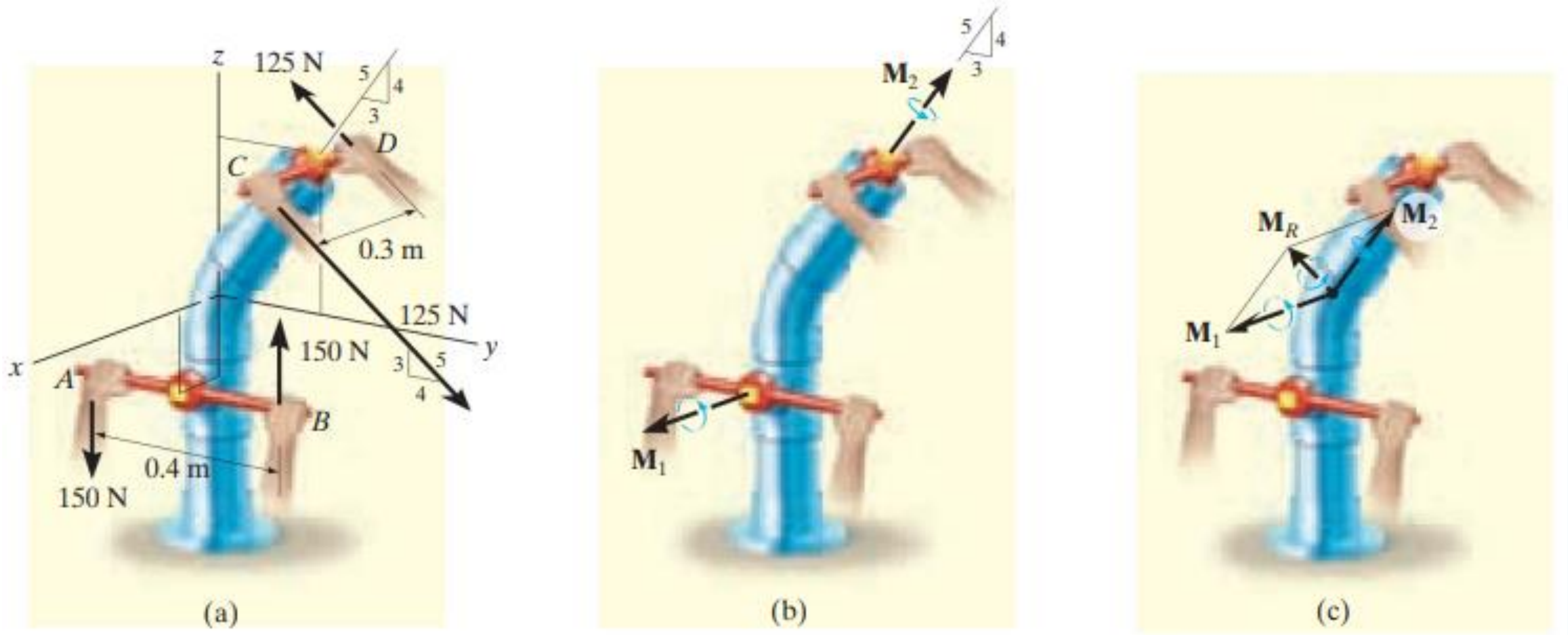


(d)

$$M = Fd = 25\text{ lb}(5.196\text{ in.}) = 129.9\text{ lb}\cdot\text{in.}$$

Applying the right-hand rule, M acts in the -j direction. Thus, M = {-130j} lb . in

Example 12: Replace the two couples acting on the pipe column. by a resultant couple moment



Solution (vector analysis) The couple moment $M1$, developed by the forces at A and B, can easily be determined from a scalar formulation.

$M1 = F.d = 150\ N0.4\ m(= 60\ N\ .\ m\ )$

By the right-hand rule, $M1$ acts in the $+i$ direction, Fig. b. Hence, $M1 = \{60i\}\ N\ .\ m$

Vector analysis will be used to determine $M2$, caused by forces at C and D. If moments are calculated about point D, Fig. a, $M2 = r_{DC} \times F_C$, then

$$\mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C = (0.3\mathbf{i}) \times \left[125\left(\tfrac{4}{5}\right)\mathbf{j} - 125\left(\tfrac{3}{5}\right)\mathbf{k}\right]$$

$$= (0.3\mathbf{i}) \times [100\mathbf{j} - 75\mathbf{k}] = 30(\mathbf{i} \times \mathbf{j}) - 22.5(\mathbf{i} \times \mathbf{k})$$

$$= \{22.5\mathbf{j} + 30\mathbf{k}\}\ \text{N} \cdot \text{m}$$

Since $M1$ and $M2$ are free vectors, they may be moved to some arbitrary point and added vectorially, Fig. c. The resultant couple moment becomes

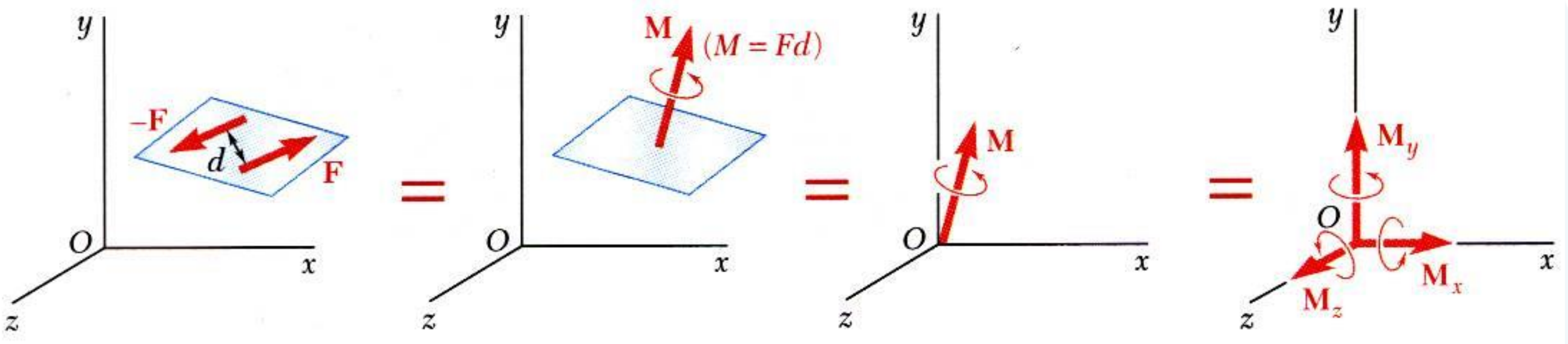
$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = \{60\mathbf{i} + 22.5\mathbf{j} + 30\mathbf{k}\}\ \text{N} \cdot \text{m}$$

Couples are Free Vectors

- The point of action of a Couple does not matter
- The plane that the Couple is acting in does not matter
- All that matters is the orientation of the plane the Couple is acting in
- Therefore, a Force Couple is said to be a Free Vector and can be applied at any point on the body it is acting

Vector Addition of Couples

- By applying Varignon's Theorem to the Forces in the Couple, it can be proven that couples can be added and resolved as Vectors



Force Couple Systems

- As a result of this it can be stated that any force (F) acting on a rigid body may be moved to any given point on the rigid body as long as a moment equal to moment of (F) about the axis is added to the rigid body.

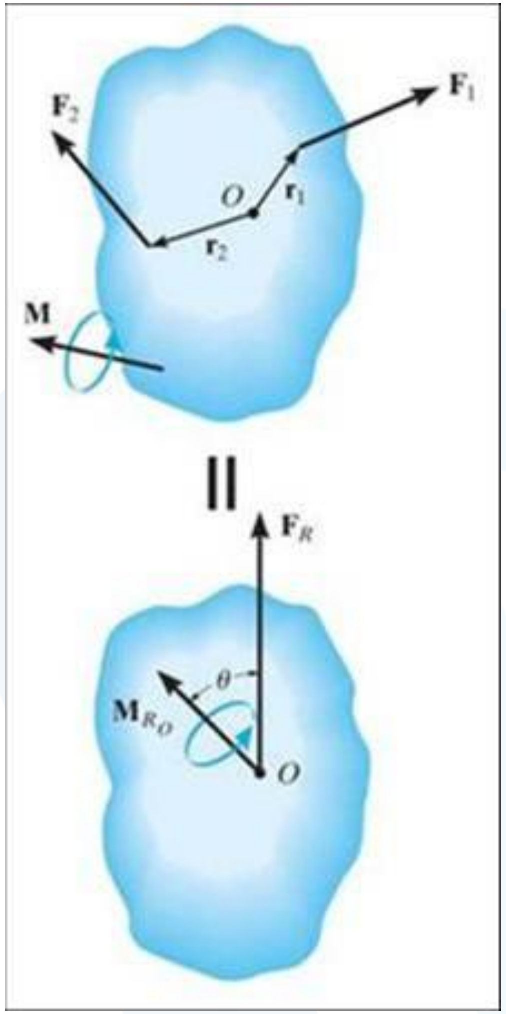
Resolution of a System of Forces in 3D

- Each Force can be Resolved into a Force and Moment at the point of interest using the method just discussed.
- The Resultant Force can then be found by Vector Addition.
- The Resultant Moment must also be found using Vector Addition.

SIMPLIFICATION OF FORCE AND COUPLE SYSTEM

When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect.

The two force and couple systems are called equivalent systems since they have the same external effect on the body.



نقل تأثير القوة إلى نقطة تقع على حاملها

(a) (b) (c)

نقل تأثير القوة إلى نقطة لا تقع على حاملها

(a) (b) (c)

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM

When several forces and couple moments act on a body,

you can move each force and its associated couple moment

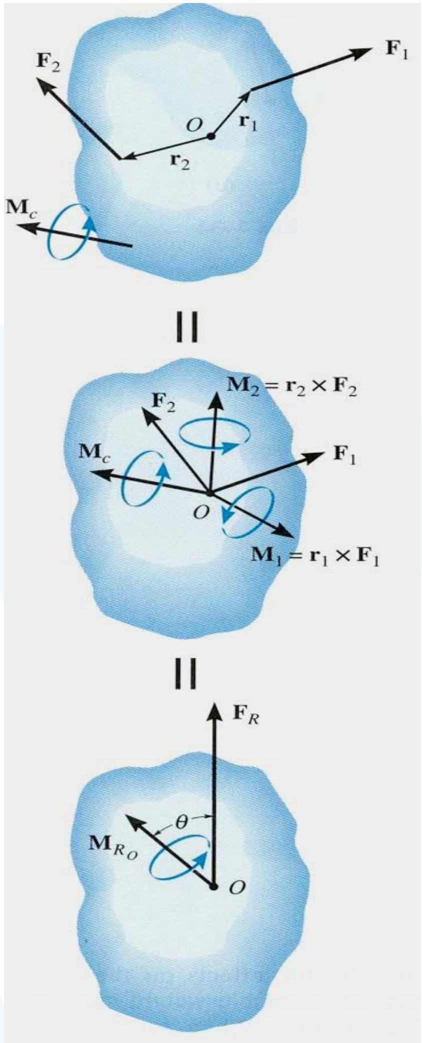
to a common point O.

Now you can add all the forces

and couple moments together and

find one resultant force-couple moment pair.

$$F_R = \Sigma F$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$



SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (continued)

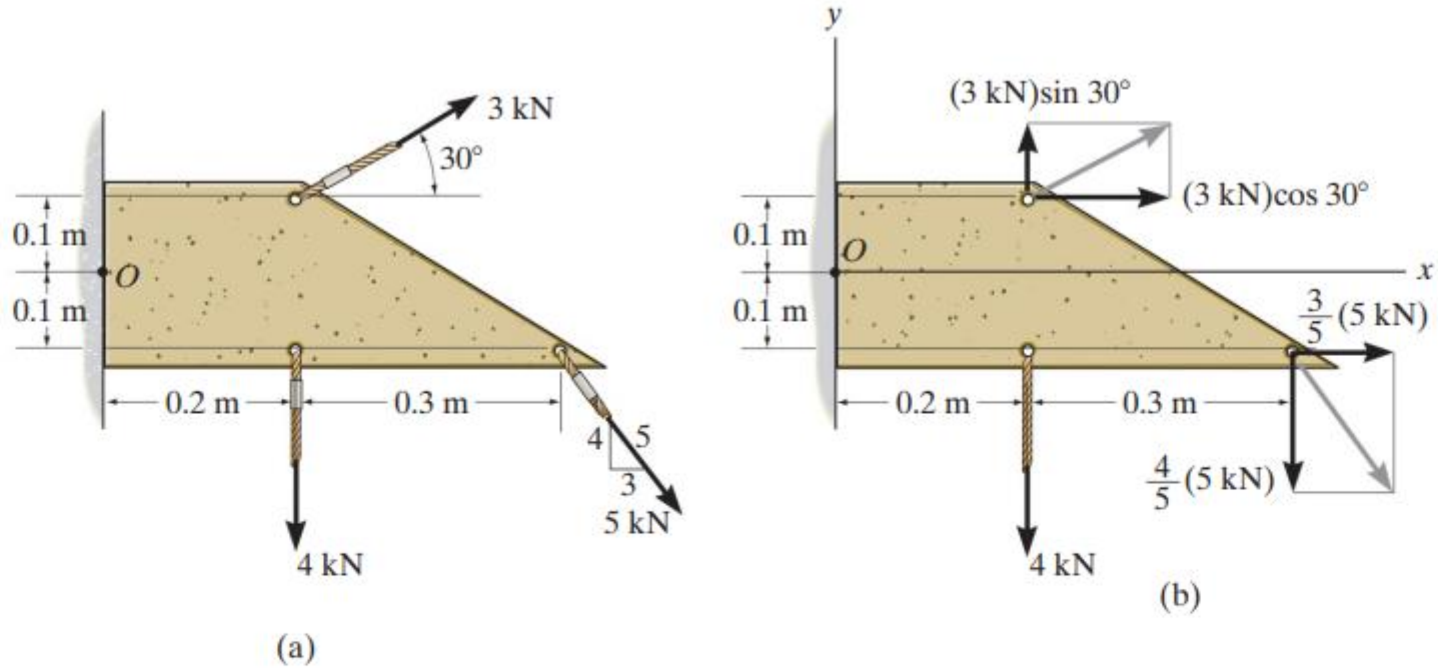
If the force system lies in the x-y plane (a 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

Important Points

- Force is a sliding vector, since it will create the same external effects on a body when it is applied at any point P along its line of action. This is called the principle of transmissibility.
- A couple moment is a free vector since it will create the same external effects on a body when it is applied at any point P on the body.
- When a force is moved to another point P that is not on its line of action, it will create the same external effects on the body if a couple moment is also applied to the body. The couple moment is determined by taking the moment of the force about point P.

Example 13 : Replace the force and couple system shown in Fig. a by an equivalent resultant force and couple moment acting at point O.



SOLUTION

Force Summation. The 3 kN and 5 kN forces are resolved into their x and y components as shown in Fig. 4–37b. We have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3\text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5\text{ kN}) = 5.598\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3\text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5\text{ kN}) - 4\text{ kN} = -6.50\text{ kN} = 6.50\text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37c, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598\text{ kN})^2 + (6.50\text{ kN})^2} = 8.58\text{ kN} \qquad \textit{Ans.}$$

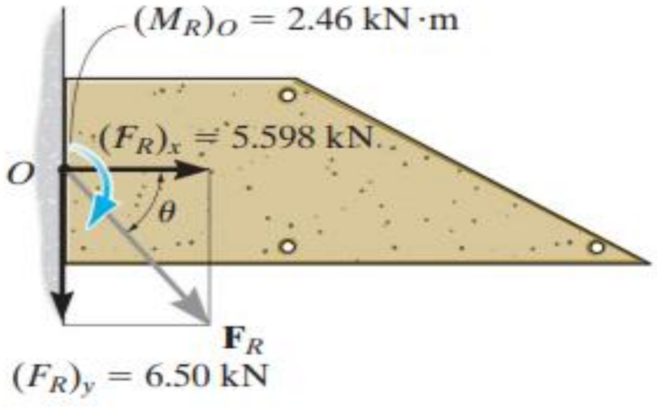
Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50\text{ kN}}{5.598\text{ kN}}\right) = 49.3^\circ \qquad \textit{Ans.}$$

Moment Summation. The moments of 3 kN and 5 kN about point O will be determined using their x and y components. Referring to Fig. 4–37b, we have

$$\zeta + (M_R)_O = \Sigma M_O;$$

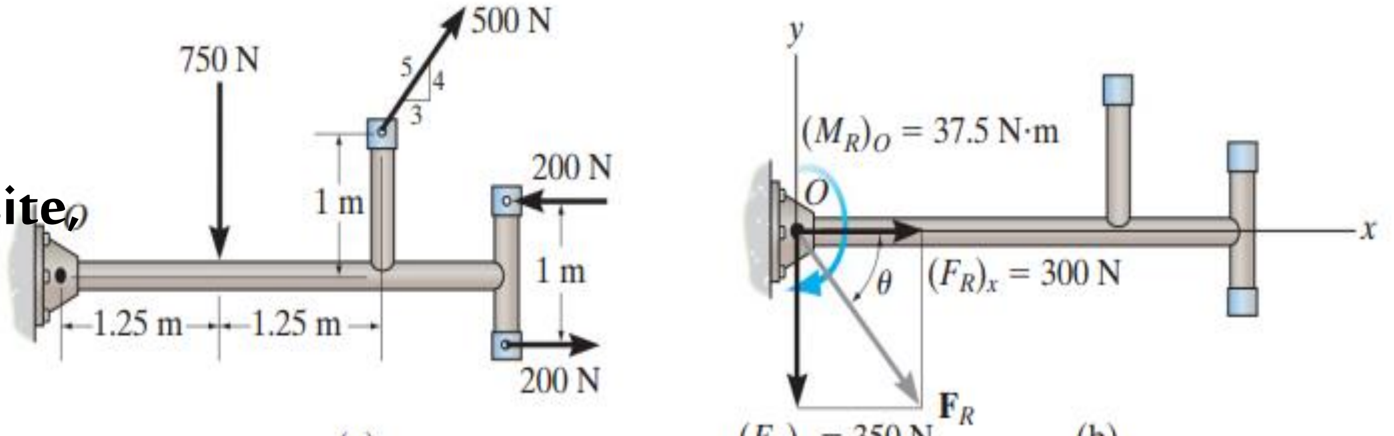
$$(M_R)_O = (3\text{ kN})\sin 30^\circ(0.2\text{ m}) - (3\text{ kN})\cos 30^\circ(0.1\text{ m}) + \left(\tfrac{3}{5}\right)(5\text{ kN})(0.1\text{ m})$$
$$- \left(\tfrac{4}{5}\right)(5\text{ kN})(0.5\text{ m}) - (4\text{ kN})(0.2\text{ m})$$
$$= -2.46\text{ kN}\cdot\text{m} = 2.46\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$



(c)

Example13 : Replace the force and couple system acting on the member in Fig. a by an equivalent resultant force and couple moment acting at point O

SOLUTION Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its x and y components, thus,



(a) (b)

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x;\quad (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;\quad (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N} \qquad \textit{Ans.}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4^\circ \qquad \textit{Ans.}$$

Moment Summation. Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig. 4–38*a*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

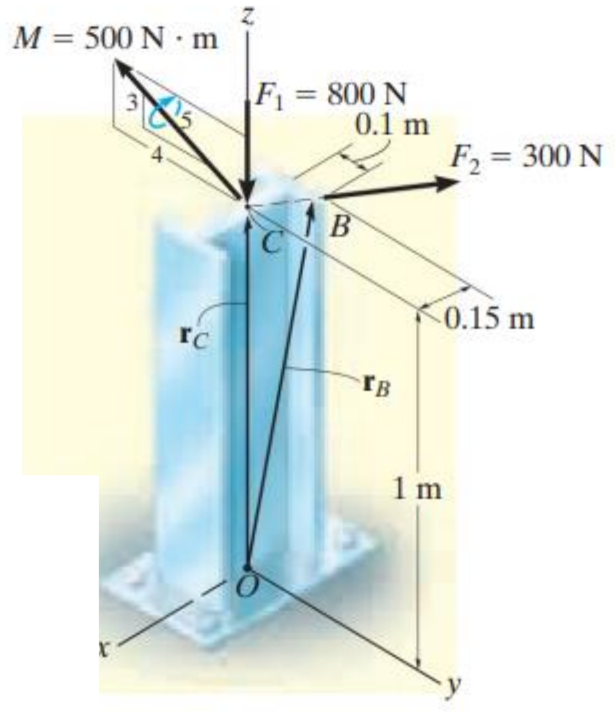
$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

Example 14 : The structural member is subjected to a couple moment M and forces F1 and F2 in Fig. 4–39a. Replace this system by an equivalent resultant force and couple moment acting at its base, point O.

SOLUTION (VECTOR ANALY SIS) The three-dimensional aspects of the problem can be simplified by using a Cartesian vector analysis. Expressing the forces and couple moment as Cartesian vectors, we have



(a)

$$\mathbf{F}_1 = \{-800\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = (300 \text{ N})\mathbf{u}_{CB}$$

$$= (300 \text{ N})\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right)$$

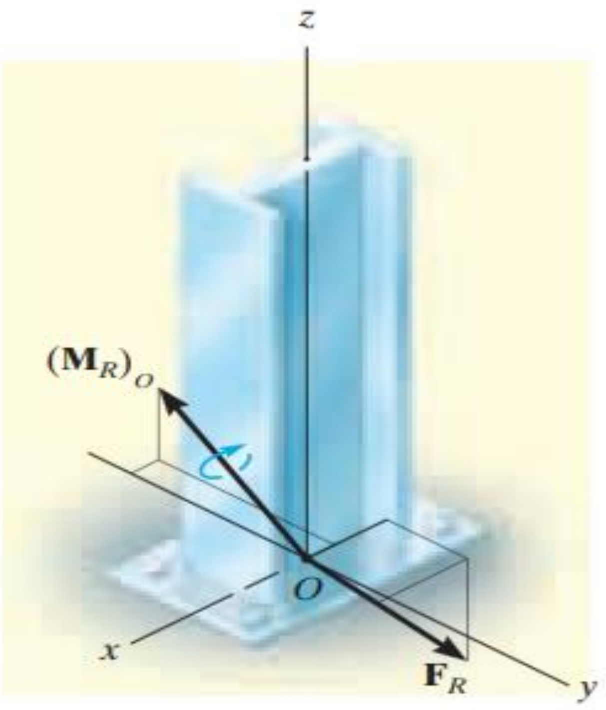
$$= 300 \text{ N}\left[\frac{\{-0.15\mathbf{i} + 0.1\mathbf{j}\} \text{ m}}{\sqrt{(-0.15 \text{ m})^2 + (0.1 \text{ m})^2}}\right] = \{-249.6\mathbf{i} + 166.4\mathbf{j}\} \text{ N}$$

$$\mathbf{M} = -500\left(\tfrac{4}{5}\right)\mathbf{j} + 500\left(\tfrac{3}{5}\right)\mathbf{k} = \{-400\mathbf{j} + 300\mathbf{k}\} \text{ N} \cdot \text{m}$$

Force Summation.

$$\mathbf{F}_R = \Sigma \mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = -800\mathbf{k} - 249.6\mathbf{i} + 166.4\mathbf{j}$$

$$= \{-250\mathbf{i} + 166\mathbf{j} - 800\mathbf{k}\}\ \text{N}$$



(b)

Moment Summation.

$$(\mathbf{M}_R)_o = \Sigma \mathbf{M} + \Sigma \mathbf{M}_O$$

$$(\mathbf{M}_R)_o = \mathbf{M} + \mathbf{r}_C \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

$$(\mathbf{M}_R)_o = (-400\mathbf{j} + 300\mathbf{k}) + (1\mathbf{k}) \times (-800\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.1 & 1 \\ -249.6 & 166.4 & 0 \end{vmatrix}$$

$$= (-400\mathbf{j} + 300\mathbf{k}) + (0) + (-166.4\mathbf{i} - 249.6\mathbf{j})$$

$$= \{-166\mathbf{i} - 650\mathbf{j} + 300\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

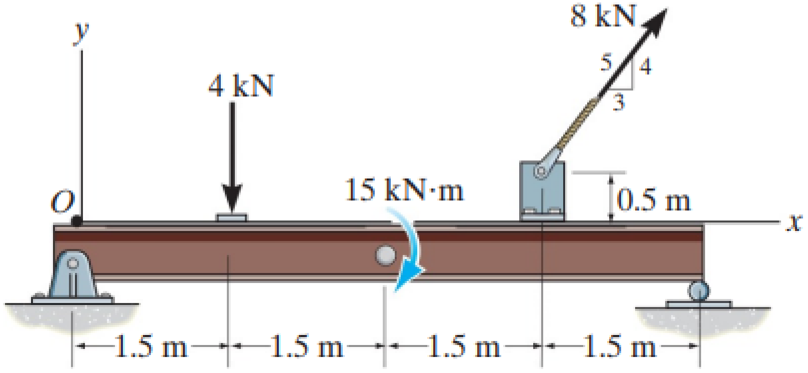
Further Simplification of a Force and Couple System

Procedure for Analysis The technique used to reduce a coplanar or parallel force system to a single resultant force follows a similar procedure outlined in the previous section.

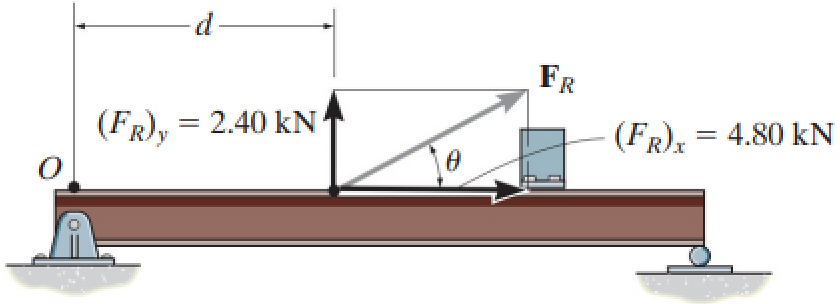
- **Establish the x, y, z, axes and locate the resultant force FR an arbitrary distance away from the origin of the coordinates. Force Summation.**
- **The resultant force is equal to the sum of all the forces in the system.**
- **For a coplanar force system, resolve each force into its x and y components. Positive components are directed along the positive x and y axes, and negative components are directed along the negative x and y axes. Moment Summation.**
- **The moment of the resultant force about point O is equal to the sum of all the couple moments in the system plus the moments of all the forces in the system about O.**
- **This moment condition is used to find the location of the resultant force from point O.**

$$(M_R)_O = F_R d = \Sigma M_O \text{ or } d = \Sigma M_O / F_R.$$

Example 15 :Replace the force and couple moment system acting on the beam in Fig. a by an equivalent resultant force, and find where its line of action intersects the beam, measured from point O.



(a)



(b)

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SOLUTION

Force Summation. Summing the force components,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ$$

Moment Summation. We must equate the moment of FR about point O in Fig. 4–44b to the sum of the moments of the force and couple moment system about point O in Fig. a. Since the line of action of (FR)x acts through point O, only (FR)y produces a moment about this point. Thus

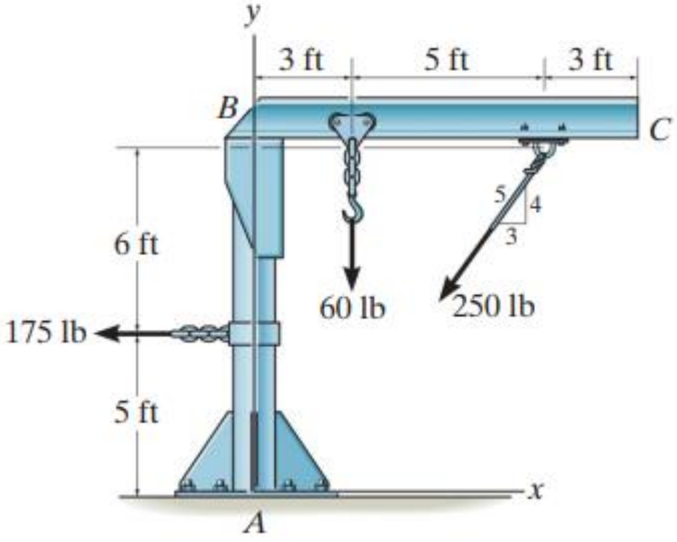
Moment Summation. We must equate the moment of $\mathbf{F}_R$ about point O in Fig. 4–44*b* to the sum of the moments of the force and couple moment system about point O in Fig. 4–44*a*. Since the line of action of $(\mathbf{F}_R)_x$ acts through point O, *only* $(\mathbf{F}_R)_y$ *produces a moment* about this point. Thus,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m} \qquad \textit{Ans.}$$

Example16 :The jib crane shown in Fig. a is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column AB and boom BC.



(a)

SOLUTION

Force Summation. Resolving the 250-lb force into x and y components and summing the force components yields

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = -250 \text{ lb}\left(\tfrac{3}{5}\right) - 175 \text{ lb} = -325 \text{ lb} = 325 \text{ lb} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -250 \text{ lb}\left(\tfrac{4}{5}\right) - 60 \text{ lb} = -260 \text{ lb} = 260 \text{ lb}\downarrow$$

As shown by the vector addition in Fig. b,

$$F_R = \sqrt{(325 \text{ lb})^2 + (260 \text{ lb})^2} = 416 \text{ lb} \qquad \textit{Ans.}$$

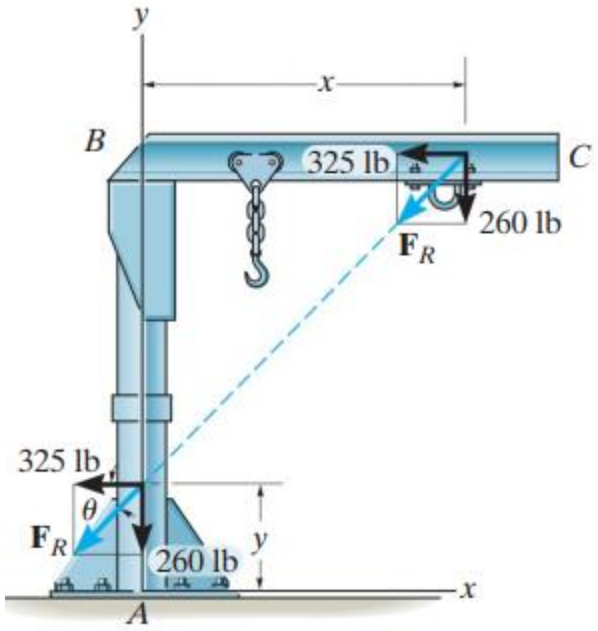
$$\theta = \tan^{-1}\left(\frac{260 \text{ lb}}{325 \text{ lb}}\right) = 38.7^\circ \qquad \textit{Ans.}$$

Moment Summation. Moments will be summed about point A. Assuming the line of action of $\mathbf{F}_R$ *intersects* AB at a distance y from A, Fig. 4–45b, we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(y) + 260\text{ lb }(0)$$

$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

$$y = 2.29\text{ ft} \qquad \textit{Ans.}$$



(b)

By the principle of transmissibility, $\mathbf{F}_R$ can be placed at a distance x where it intersects BC, Fig. b. In this case we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(11\text{ ft}) - 260\text{ lb }(x)$$

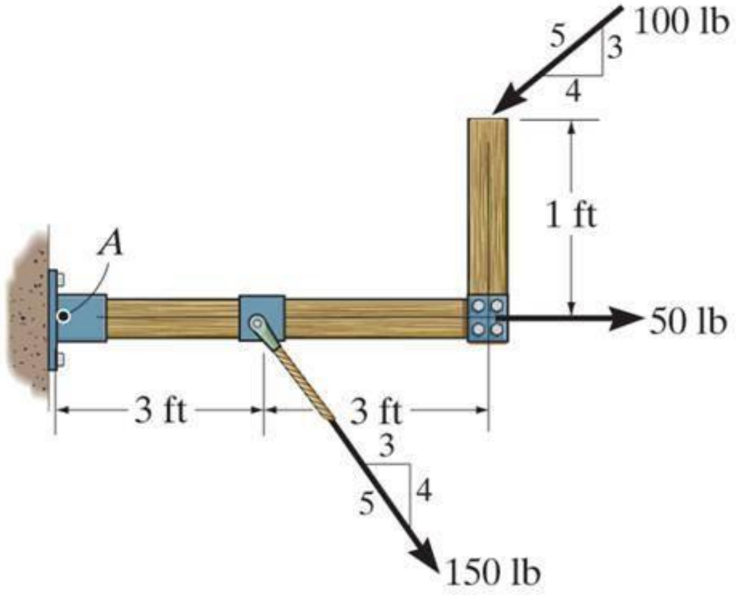
$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

$$x = 10.9\text{ ft} \qquad \textit{Ans.}$$

EXAMPLE 17

Given: A 2-D force system with geometry as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A.



Plan: 1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force component to A.

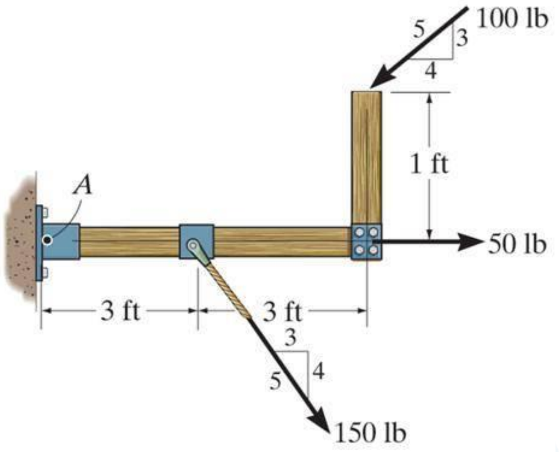
3) Shift F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE (continued)

$+\rightarrow \Sigma F_{Rx} = 150\ (3/5) + 50 - 100\ (4/5)$
$= 60 \text{ lb}$

$+\downarrow \Sigma F_{Ry} = 150\ (4/5) + 100\ (3/5)$
$= 180 \text{ lb}$

$+\quad M_{RA} = 100\ (4/5)\ 1 - 100\ (3/5)\ 6$
$- 150(4/5)\ 3 = - 640 \text{ lb}\cdot\text{ft}$



$F_R = (60^2 + 180^2)^{1/2} = 190 \text{ lb}$

$\theta = \tan^{-1}(180/60) = 71.6°$

The equivalent single force F_R can be located at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 640 / 180 = 3.56 \text{ ft.}$