Exercises 2: Matrices

CEDC102 : Linear Algebra and Matrix Theory

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Find, if possible, (a) $\boldsymbol{A}+\boldsymbol{B}$, (b) $\boldsymbol{A}-\boldsymbol{B}$, (c) $2\boldsymbol{A}$, (d) $2\boldsymbol{A}-\boldsymbol{B}$, and (e) $\boldsymbol{B}+1/2\boldsymbol{A}$

① $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix},\quad B=\begin{bmatrix}-3 & -2\\ 4 & 2\end{bmatrix}$

$(a)\ A+B=\begin{bmatrix}-2 & 0\\ 6 & 3\end{bmatrix}\qquad (b)\ A-B=\begin{bmatrix}4 & 4\\ -2 & -1\end{bmatrix}\qquad (c)\ 2A=\begin{bmatrix}2 & 4\\ 4 & 2\end{bmatrix}$

$(d)\ 2A-B=\begin{bmatrix}5 & 6\\ 0 & 0\end{bmatrix}\qquad (e)\ B+\frac{1}{2}A=\begin{bmatrix}-5/2 & -1\\ 5 & 5/2\end{bmatrix}$

② $A=\begin{bmatrix}2 & 1 & 1\\ -1 & -1 & 4\end{bmatrix},\quad B=\begin{bmatrix}2 & -3 & 4\\ -3 & 1 & -2\end{bmatrix}$

$$(a)\ A+B=\begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix} \qquad (b)\ A-B=\begin{bmatrix} 0 & 4 & -3 \\ 2 & -2 & 6 \end{bmatrix} \qquad (c)\ 2A=\begin{bmatrix} 4 & 2 & 2 \\ -2 & -2 & 8 \end{bmatrix}$$

$$(d)\ 2A-B=\begin{bmatrix} 2 & 5 & -2 \\ 1 & -3 & 10 \end{bmatrix} \qquad (e)\ B+\frac{1}{2}A=\begin{bmatrix} 3 & -5/2 & 9/2 \\ -7/2 & 1/2 & 0 \end{bmatrix}$$

3 $A=\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix}, \quad B=\begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix}$

$$(c)\ 2A=\begin{bmatrix} 12 & 0 & 6 \\ -2 & -8 & 0 \end{bmatrix}$$

All other operations are not defined (A and B are different sizes)

Find, if possible, (a) AB, (b) BA

① $A=\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix},\quad B=\begin{bmatrix}2 & -1\\ -1 & 8\end{bmatrix}$

$$(a)\ AB=\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix}\begin{bmatrix}2 & -1\\ -1 & 8\end{bmatrix}=\begin{bmatrix}1(2)+2(-1) & 1(-1)+2(8)\\ 4(2)+2(-1) & 4(-1)+2(8)\end{bmatrix}=\begin{bmatrix}0 & 15\\ 6 & 12\end{bmatrix}$$

$$(b)\ BA=\begin{bmatrix}2 & -1\\ -1 & 8\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix}=\begin{bmatrix}2(1)+(-1)(4) & 2(2)+(-1)(2)\\ -1(1)+8(4) & -1(2)+8(2)\end{bmatrix}=\begin{bmatrix}-2 & 2\\ 31 & 14\end{bmatrix}$$

② $A=\begin{bmatrix}2 & 1\\ -3 & 4\\ 1 & 6\end{bmatrix},\quad B=\begin{bmatrix}0 & -1 & 0\\ 4 & 0 & 2\\ 8 & -1 & 7\end{bmatrix}$

(a) AB is not defined because A is 3×2 and B is 3×3

$$(b)\ BA=\begin{bmatrix}0 & -1 & 0\\ 4 & 0 & 2\\ 8 & -1 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ -3 & 4\\ 1 & 6\end{bmatrix}$$

$$=\begin{bmatrix}0(2)+(-1)(-3)+0(1) & 0(1)+(-1)(4)+0(6)\\ 4(2)+0(-3)+2(1) & 4(1)+0(4)+2(6)\\ 8(2)+(-1)(-3)+7(1) & 8(1)+(-1)(4)+7(6)\end{bmatrix}=\begin{bmatrix}3 & -4\\ 10 & 16\\ 26 & 46\end{bmatrix}$$

3 $A=\begin{bmatrix}3 & 2 & 1\end{bmatrix},\quad B=\begin{bmatrix}2\\ 3\\ 0\end{bmatrix}$

$$(a)\ AB=\begin{bmatrix}3 & 2 & 1\end{bmatrix}\begin{bmatrix}2\\ 3\\ 0\end{bmatrix}=\begin{bmatrix}3(2)+2(3)+1(0)\end{bmatrix}=\begin{bmatrix}12\end{bmatrix}$$

$$(b)\ AB=\begin{bmatrix}2\\3\\0\end{bmatrix}\begin{bmatrix}3 & 2 & 1\end{bmatrix}=\begin{bmatrix}2(3) & 2(2) & 2(1)\\3(3) & 3(2) & 3(1)\\0(3) & 0(2) & 0(1)\end{bmatrix}=\begin{bmatrix}6 & 4 & 2\\9 & 6 & 3\\0 & 0 & 0\end{bmatrix}$$

4 $A=\begin{bmatrix}2 & 0\\0 & -3\end{bmatrix},\quad B=\begin{bmatrix}-5 & 0\\0 & 4\end{bmatrix}$

$$(a)\ AB=\begin{bmatrix}2 & 0\\0 & -3\end{bmatrix}\begin{bmatrix}-5 & 0\\0 & 4\end{bmatrix}=\begin{bmatrix}-10 & 0\\0 & -12\end{bmatrix}$$

$$(b)\ BA=\begin{bmatrix}-5 & 0\\0 & 4\end{bmatrix}\begin{bmatrix}2 & 0\\0 & -3\end{bmatrix}=\begin{bmatrix}-10 & 0\\0 & -12\end{bmatrix}$$

5 $A=\begin{bmatrix}1 & 2 & 1\\ 0 & -1 & 2\\ 0 & 0 & 3\end{bmatrix},\quad B=\begin{bmatrix}2 & -1 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}$

$$(a)\ AB=\begin{bmatrix}1 & 2 & 1\\ 0 & -1 & 2\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}2 & -1 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}=\begin{bmatrix}2 & 1 & 9\\ 0 & -1 & 6\\ 0 & 0 & 12\end{bmatrix}$$

$$(b)\ BA=\begin{bmatrix}2 & -1 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ 0 & -1 & 2\\ 0 & 0 & 3\end{bmatrix}=\begin{bmatrix}2 & 5 & 3\\ 0 & -1 & 8\\ 0 & 0 & 12\end{bmatrix}$$

Write the system of linear equations in the form $\boldsymbol{Ax} = \boldsymbol{b}$ and solve this matrix equation for $\boldsymbol{x}$

1

$$\begin{aligned} -x + y &= 4 \\ -2x + y &= 0 \end{aligned}$$

$$\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Use Gauss-Jordan elimination on the augmented matrix

$$\begin{bmatrix} -1 & 1 & 4 \\ -2 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 8 \end{bmatrix}$$

Therefore, the solution is $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

② $$\begin{aligned} 2x - y - z &= 0 \\ x - 2y + 2z &= 0 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Use Gauss-Jordan elimination on the augmented matrix

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -4/3 & 0 \\ 0 & 1 & -5/3 & 0 \end{bmatrix} \Rightarrow \begin{aligned} x \quad\quad - 4/3z &= 0 \\ y - 5/3z &= 0 \end{aligned}$$

Free variable: $x = t$

Therefore, the solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ (5/4)t \\ (3/4)t \end{bmatrix} = t \begin{bmatrix} 1 \\ 5/4 \\ 3/4 \end{bmatrix}, t \in R$

Solve for X in the equation, given

$$A=\begin{bmatrix}-4 & 0\\ 1 & -5\\ -3 & 2\end{bmatrix},\quad B=\begin{bmatrix}1 & 2\\ -2 & 1\\ 4 & 4\end{bmatrix}$$

(a) $3X+2A=B$ (b) $2A-5B=3X$

(c) $X-3A+2B=O$ (d) $6X-4A-3B=O$

(a) $3X+2A=B$

$$3X=B-2A=\begin{bmatrix}1 & 2\\ -2 & 1\\ 4 & 4\end{bmatrix}-\begin{bmatrix}-8 & 0\\ 2 & -10\\ -6 & 4\end{bmatrix}=\begin{bmatrix}9 & 2\\ -4 & 11\\ 10 & 0\end{bmatrix}\Rightarrow X=\begin{bmatrix}3 & 2/3\\ -4/3 & 11/3\\ 10/3 & 0\end{bmatrix}$$

(b) $2A - 5B = 3X$

$$3X = 2A - 5B = \begin{bmatrix} -8 & 0 \\ 2 & -10 \\ -6 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ -10 & 5 \\ 20 & 20 \end{bmatrix} = \begin{bmatrix} -13 & -10 \\ 12 & -15 \\ -26 & -16 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -13/3 & -10/3 \\ 4 & -5 \\ -26/3 & -16/3 \end{bmatrix}$$

(c) $X - 3A + 2B = O$

$$X = 3A - 2B = \begin{bmatrix} -12 & 0 \\ 3 & -15 \\ -9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ -4 & 2 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} -14 & -4 \\ 7 & -17 \\ -17 & -2 \end{bmatrix}$$

(d) $6X - 4A - 3B = O$

$$6X = 4A + 3B = \begin{bmatrix} -16 & 0 \\ 4 & -20 \\ -12 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -6 & 3 \\ 12 & 12 \end{bmatrix} = \begin{bmatrix} -13 & 6 \\ -2 & -17 \\ 0 & 20 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -13/6 & 1 \\ -1/3 & -17/6 \\ 0 & 10/3 \end{bmatrix}$$

Consider the matrices below

$$X=\begin{bmatrix}1\\0\\1\end{bmatrix},\quad Y=\begin{bmatrix}1\\1\\0\end{bmatrix},\quad Z=\begin{bmatrix}2\\-1\\3\end{bmatrix},\quad W=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

(a) Find scalars a and b such that $Z=aX+bY$

(b) Show that there do not exist scalars a and b such that $W=aX+bY$

(c) Show that if $aX+bY+cW=O$, then $a=0$, $b=0$, and $c=0$

(d) Find scalars a, b, and c, not all equal to zero, such that $aX+bY+cZ=O$

(a) $$aX+bY=a\begin{bmatrix}1\\0\\1\end{bmatrix}+b\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}2\\-1\\3\end{bmatrix}\quad\Rightarrow\quad\begin{aligned}a+b&=2\\ b&=-1\\ a&=3\end{aligned}$$

The only solution to this system is: $a=3$ and $b=-1$

(b) $aX+bY=a\begin{bmatrix}1\\0\\1\end{bmatrix}+b\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix} \Rightarrow \begin{array}{rcl} a+b&=&1\\ b&=&1\\ a&=&1\end{array}$

The system is inconsistent. No values of a and b will satisfy the equation

(c) $aX+bY+cW=a\begin{bmatrix}1\\0\\1\end{bmatrix}+b\begin{bmatrix}1\\1\\0\end{bmatrix}+c\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{array}{rcl} a+b+c&=&0\\ b+c&=&0\\ a+c&=&0\end{array}$

Then $a=-c$, so $b=0$. Then $c=0$, so $a=b=c=0$

(d) $aX+bY+cZ=a\begin{bmatrix}1\\0\\1\end{bmatrix}+b\begin{bmatrix}1\\1\\0\end{bmatrix}+c\begin{bmatrix}2\\-1\\3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{array}{rcl} a+b+2c&=&0\\ b-c&=&0\\ a+3c&=&0\end{array}$

Using Gauss-Jordan elimination the solution is $a=-3t$, $b=t$ and $c=t$, where t is any real number. If $t=1$, then $a=-3$, $b=1$, and $c=1$

Determine whether the matrix is symmetric, skew-symmetric, or neither

① $A=\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$$A^T=\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}=-A$$ The matrix is skew-symmetric

② $A=\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix}$

$$A^T=\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix}=A$$ The matrix is symmetric

Show that B is the inverse of A

① $A=\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}, \quad B=\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}$

$$AB=\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

② $A=\begin{bmatrix}2 & -17 & 11\\ -1 & 11 & -7\\ 0 & 3 & -2\end{bmatrix}, \quad B=\begin{bmatrix}1 & 1 & 2\\ 2 & 4 & -3\\ 3 & 6 & -5\end{bmatrix}$

$$AB=\begin{bmatrix}2 & -17 & 11\\ -1 & 11 & -7\\ 0 & 3 & -2\end{bmatrix}\begin{bmatrix}1 & 1 & 2\\ 2 & 4 & -3\\ 3 & 6 & -5\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 1 & 2\\ 2 & 4 & -3\\ 3 & 6 & -5\end{bmatrix}\begin{bmatrix}2 & -17 & 11\\ -1 & 11 & -7\\ 0 & 3 & -2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Find the inverse of the matrix (if it exists)

1 $A=\begin{bmatrix}1 & 2\\ 3 & 7\end{bmatrix}$

$$A^{-1}=\frac{1}{7-6}\begin{bmatrix}7 & -2\\ -3 & 1\end{bmatrix}=\begin{bmatrix}7 & -2\\ -3 & 1\end{bmatrix}$$

② $A=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}$

$$A^{-1}=\begin{bmatrix}1/2 & 0\\ 0 & 1/3\end{bmatrix}$$

③ $A=\begin{bmatrix}-1 & 1\\ 3 & -3\end{bmatrix}$

$\det(A)=0 \Rightarrow A$ is singular and has no inverse

④ $A=\begin{bmatrix}1 & 1 & 1\\ 3 & 5 & 4\\ 3 & 6 & 5\end{bmatrix}$

$$[A \vdots I]=\left[\begin{array}{ccc:ccc}1 & 1 & 1 & 1 & 0 & 0\\ 3 & 5 & 4 & 0 & 1 & 0\\ 3 & 6 & 5 & 0 & 0 & 1\end{array}\right]$$

Using elementary row operations, rewrite this matrix in reduced row-echelon form

$$\left[I \vdots A^{-1}\right] = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}$$

5 $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{bmatrix}$

$$[A \vdots I] = \left[\begin{array}{ccc:ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 7 & -10 & 0 & 1 & 0 \\ 7 & 16 & -21 & 0 & 0 & 1 \end{array}\right]$$

Using elementary row operations, you cannot form the identity matrix on the left side

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 13 & 0 & -16 & 7 \\ 0 & 1 & -7 & 0 & 7 & -3 \\ 0 & 0 & 0 & 1 & 2 & -1 \end{array}\right]$$

Therefore, the matrix is singular and has no inverse

6 $$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$$

Use an inverse matrix to solve each system of linear equations

1

$$\begin{aligned} x + 2y &= -1 \\ x - 2y &= 3 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{-2-2}\begin{bmatrix} -2 & -2 \\ -1 & 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$\boldsymbol{x} = A^{-1}\boldsymbol{b} = \frac{1}{4}\begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

2

$$\begin{aligned} x + 2y + z &= 2 \\ x + 2y - z &= 4 \\ x - 2y + z &= -2 \end{aligned}$$

$$A=\begin{bmatrix}1 & 2 & 1\\ 1 & 2 & -1\\ 1 & -2 & 1\end{bmatrix} \Rightarrow A^{-1}=\frac{1}{4}\begin{bmatrix}0 & 2 & 2\\ 1 & 0 & -1\\ 2 & -2 & 0\end{bmatrix}$$

$$\boldsymbol{x}=A^{-1}\boldsymbol{b}=\frac{1}{4}\begin{bmatrix}0 & 2 & 2\\ 1 & 0 & -1\\ 2 & -2 & 0\end{bmatrix}\begin{bmatrix}2\\ 4\\ -2\end{bmatrix}=\begin{bmatrix}1\\ 1\\ -1\end{bmatrix}$$

The solution is: $x=1$, $y=1$ and $z=-1$

(a) Find $2\boldsymbol{A}-\boldsymbol{A}^2$,

(b) Conclude $\boldsymbol{A}^{-1}$

$$A=\begin{bmatrix}-1 & -2 & 0\\ 2 & 3 & 0\\ 0 & 0 & 1\end{bmatrix}$$

(a)

$$2A-A^2=2\begin{bmatrix}-1&-2&0\\2&3&0\\0&0&1\end{bmatrix}-\begin{bmatrix}-1&-2&0\\2&3&0\\0&0&1\end{bmatrix}\begin{bmatrix}-1&-2&0\\2&3&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}-2&-4&0\\4&6&0\\0&0&2\end{bmatrix}-\begin{bmatrix}-3&-4&0\\4&5&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I_3$$

(b)

$$2A-A^2=A(2I-A)=I_3 \quad\Rightarrow\quad A^{-1}=(2I-A)$$

$$A^{-1}=2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}-1&-2&0\\2&3&0\\0&0&1\end{bmatrix}=\begin{bmatrix}3&2&0\\-2&-1&0\\0&0&1\end{bmatrix}$$

Find, if possible, (a) $\boldsymbol{A}+\boldsymbol{B}$, (b) $\boldsymbol{A}-\boldsymbol{B}$, (c) $2\boldsymbol{A}$, (d) $3\boldsymbol{A}-\boldsymbol{B}$, and (e) $2\boldsymbol{B}+1/3\boldsymbol{A}$

① $A=\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix},\quad B=\begin{bmatrix}-1 & 2\\ -2 & -3\end{bmatrix}$

② $A=\begin{bmatrix}2 & 1 & 3\\ -1 & -2 & 0\end{bmatrix},\quad B=\begin{bmatrix}5 & -1\\ 2 & 3\end{bmatrix}$

③ $A=\begin{bmatrix}1 & 1 & 1\\ -1 & -1 & 5\end{bmatrix},\quad B=\begin{bmatrix}2 & -2 & 4\\ -3 & 1 & 3\end{bmatrix}$

Find, if possible, (a) $\boldsymbol{AB}$, (b) $\boldsymbol{BA}$

① $A=\begin{bmatrix}2 & -2\\ -1 & 4\end{bmatrix},\quad B=\begin{bmatrix}4 & 1\\ 2 & -2\end{bmatrix}$

② $A=\begin{bmatrix}3 & 2 & 1\\ -3 & 0 & 4\\ 4 & -2 & -4\end{bmatrix},\quad B=\begin{bmatrix}1 & 2\\ 2 & -1\\ 1 & -2\end{bmatrix}$

③ $A=\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 2\\ 0 & 0 & -2\end{bmatrix},\quad B=\begin{bmatrix}2 & 0 & 0\\ -1 & 1 & 0\\ 3 & 2 & 4\end{bmatrix}$

④ $A=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix},\quad B=\begin{bmatrix}1 & 2 & 1\end{bmatrix}$

Write the system of linear equations in the form $\boldsymbol{Ax} = \boldsymbol{b}$ and solve this matrix equation for

① $$\begin{aligned} 2x + 3y &= 5 \\ x + 4y &= 10 \end{aligned}$$

② $$\begin{aligned} x + y - 3z &= -1 \\ -x + 2y &= 1 \\ 2x - y + z &= 2 \end{aligned}$$

Find the inverse of the matrix (if it exists)

① $A = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

② $A = \begin{bmatrix} -7 & 33 \\ 4 & -19 \end{bmatrix}$

③ $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$

④ $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 5 \end{bmatrix}$

Use an inverse matrix to solve each system of linear equations

① $$\begin{aligned} 2x - y &= -3 \\ 2x + y &= 7 \end{aligned}$$

② $$\begin{aligned} x + y - 2z &= 0 \\ x - 2y + z &= 0 \\ x - y - z &= -1 \end{aligned}$$

(a) Find $A^3 - 5A^2 + 8A$,

(b) Conclude A^{-1}

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

$(A + B)^2 = A^2 + 2AB + B^2$?

$(A - B)^2 = A^2 - 2AB + B^2$?

$(A + B)(A - B) = A^2 - B^2$?