

محاضرة 3 عملي جزء 2

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جامعة المنارة

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Find a sequence of elementary matrices that can be used to write the matrix in row-echelon form

$$\textcircled{1} \quad A = \begin{bmatrix} 0 & 1 & 7 \\ 5 & 10 & -5 \end{bmatrix}$$

$$A_1 = r_{12}(A) = \begin{bmatrix} 5 & 10 & -5 \\ 0 & 1 & 7 \end{bmatrix} \Rightarrow E_1 = r_{12}(I_2) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A_2 = r_1^{(1/5)}(A_1) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 7 \end{bmatrix} \Rightarrow E_2 = r_1^{(1/5)}(I_2) = \begin{bmatrix} 0 & 1/5 \\ 1 & 0 \end{bmatrix}$$

$$\text{So, } B = E_2 E_1 A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 7 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 4 & 8 & -4 \\ -6 & 12 & 8 & 1 \end{bmatrix}$$

$$A_1 = r_{13}^{(6)}(A) = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 4 & 8 & -4 \\ 0 & 0 & 2 & 1 \end{bmatrix} \Rightarrow E_1 = r_{13}^{(6)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 6 & 0 & 1 \end{bmatrix}$$

$$A_2 = r_2^{(1/4)}(A_1) = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \Rightarrow E_2 = r_2^{(1/4)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = r_3^{(1/2)}(A_2) = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \Rightarrow E_3 = r_3^{(1/2)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$\text{So, } B = E_3 E_2 E_1 A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$

Find the inverse of the matrix using elementary matrices

$$\textcircled{1} \quad A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$$

Find a sequence of elementary row operations that can be used to rewrite A in reduced row-echelon form

$$A_1 = r_{12}(A) = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix} \Rightarrow E_1 = r_{12}(I_2) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A_2 = r_{12}^{(-3)}(A_1) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow E_2 = r_{12}^{(-3)}(I_2) = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$A_3 = r_2^{(-1/2)}(A_2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow E_3 = r_{12}^{(-1/2)}(I_2) = \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}$$

$$\text{So, } B = I_2 = E_3 E_2 E_1 A \Rightarrow A^{-1} = E_3 E_2 E_1$$

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3/2 & -1/2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1/2 & 3/2 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 6 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A_1 = r_2^{(1/6)}(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1/6 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow E_1 = r_2^{(1/6)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = r_3^{(1/4)}(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1/6 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_2 = r_3^{(1/4)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

$$A_3 = r_{31}^{(1)}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/6 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_3 = r_{31}^{(1)}(I_3) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = r_{32}^{(1/6)}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_4 = r_{32}^{(1/6)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{So, } B = I_3 = E_4 E_3 E_2 E_1 A \Rightarrow A^{-1} = E_4 E_3 E_2 E_1$$

$$\begin{aligned}
 A^{-1} &= E_4 E_3 E_2 E_1 \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 1/4 \\ 0 & 1 & 1/24 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1/4 \\ 0 & 1/6 & 1/24 \\ 0 & 0 & 1/4 \end{bmatrix}
 \end{aligned}$$

Find a sequence of elementary matrices whose product is the given nonsingular matrix

① $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$

Find a sequence of elementary row operations that can be used to rewrite A in reduced row-echelon form

$$A_1 = r_{12}(A) = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix} \Rightarrow E_1 = r_{12}(I_2) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A_2 = r_{12}^{(-3)}(A_1) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow E_2 = r_{12}^{(-3)}(I_2) = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$A_3 = r_2^{(-1/2)}(A_2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow E_3 = r_{12}^{(-1/2)}(I_2) = \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}$$

$$\text{So, } B = I_2 = E_3 E_2 E_1 A \Rightarrow A = E_1^{-1} E_2^{-1} E_3^{-1}$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 6 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A_1 = r_2^{(1/6)}(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1/6 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow E_1 = r_2^{(1/6)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = r_3^{(1/4)}(A_1) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1/6 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_2 = r_3^{(1/4)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

$$A_3 = r_{31}^{(1)}(A_2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/6 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_3 = r_{31}^{(1)}(I_3) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = r_{32}^{(1/6)}(A_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_4 = r_{32}^{(1/6)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{So, } B = I_3 = E_4 E_3 E_2 E_1 A \Rightarrow A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/6 \\ 0 & 0 & 1 \end{bmatrix}$$

Find an ***LU***-factorization of the matrix

$$\textcircled{1} \quad A = \begin{bmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{bmatrix}$$

$$A_1 = r_{12}^{(-2)}(A) = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ -3 & 1 & 0 \end{bmatrix} \Rightarrow E_1 = r_{12}^{(-2)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = r_{13}^{(1)}(A_1) = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow E_2 = r_{13}^{(1)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A_3 = r_{23}^{(-1)}(A_2) = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow E_3 = r_{23}^{(-1)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{So, } U = E_3 E_2 E_1 A \Rightarrow A = E_1^{-1} E_2^{-1} E_3^{-1} U = LU$$

$$L = E_1^{-1} E_2^{-1} E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} = LU$$

Use an ***LU***-factorization of the coefficient matrix to solve the linear system

$$\textcircled{1} \quad \begin{array}{rcl} 2x + y & = & 1 \\ y - z & = & 2 \\ -2x + y + z & = & -2 \end{array}$$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

$$A_1 = r_{13}^{(1)}(A) = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} \Rightarrow E_1 = r_{13}^{(1)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A_2 = r_{23}^{(-2)}(A_1) = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow E_2 = r_{23}^{(-2)}(I_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\text{So, } U = E_2 E_1 A \Rightarrow A = E_1^{-1} E_2^{-1} U = LU$$

$$L = E_1^{-1} E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} = LU$$

$$L\mathbf{y} = \mathbf{b}:$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \Rightarrow y_1 = 1, y_2 = 2, y_3 = -5$$

$$U\mathbf{x} = \mathbf{y}:$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} \Rightarrow x_1 = \frac{1}{3}, x_2 = \frac{1}{3}, x_3 = -\frac{5}{3}$$

Find a sequence of elementary matrices that can be used to write the matrix in row-echelon form

① $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 5 & -1 \\ 3 & -2 & -4 \end{bmatrix}$

② $A = \begin{bmatrix} 0 & 3 & -3 & 6 \\ 1 & -1 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$

Find the inverse of the matrix using elementary matrices

① $A = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$

② $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -3 & -3 \\ 4 & 0 & 3 \end{bmatrix}$

Find a sequence of elementary matrices whose product is the given nonsingular matrix

① $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

② $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{bmatrix}$

Find an ***LU***-factorization of the matrix

① $A = \begin{bmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{bmatrix}$

Use an ***LU***-factorization of the coefficient matrix to solve the linear system

$$\textcircled{1} \quad \begin{array}{rcl} 2x + y & = & 1 \\ y - z & = & 2 \\ -2x + y + z & = & -2 \end{array}$$