التحليل الرياضي ١

محاضرة 6 ميكاترونيكس

عملي

Prepared by
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1 احسب التكاملات الآتية:

- $\int \left(\sqrt{x} + \sqrt[3]{x}\right) dx$
- $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2} dt$
- $\int \frac{1 + \cos 4t}{2} dt$

2 احسب التكاملات الآتية:

- $\int_0^1 \left(x^2 + \sqrt{x}\right) dx$
- $\int_{\pi/2}^{0} \frac{1 + \cos 2t}{2} dt$
- $\int_1^8 \frac{(x^{1/3} + 1)(2 - x^{2/3})}{x^{1/3}}$

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1 احسب التكاملات الآتية:

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- $\int \frac{1 + \cos 4t}{2} dt$

الحل

$$\int \left(\sqrt{x} + \sqrt[3]{x}\right) dx$$

$$\int\left(\sqrt{x}+\sqrt[3]{x}\right)dx=\int\left(x^{1/2}+x^{1/3}\right)dx=\frac{x^{3/2}}{\frac{3}{2}}+\frac{x^{4/3}}{\frac{4}{3}}+C=\frac{2}{3}x^{3/2}+\frac{3}{4}x^{4/3}+C$$

$$\int \frac{t\sqrt{t} + \sqrt{t}}{t^2} dt$$

$$\int\frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt=\int\left(\frac{t^{3/2}}{t^2}+\frac{t^{1/2}}{t^2}\right)dt=\int\left(t^{-1/2}+t^{-3/2}\right)dt=\frac{t^{1/2}}{\frac{1}{2}}+\left(\frac{t^{-1/2}}{-\frac{1}{2}}\right)+C=2\sqrt{t}-\frac{2}{\sqrt{t}}+C$$

$$\int \frac{1 + \cos 4t}{2} dt$$

$$\int\frac{1+\cos 4t}{2}\,dt=\int\left(\frac{1}{2}+\frac{1}{2}\cos 4t\right)dt=\frac{1}{2}t+\frac{1}{2}\left(\frac{\sin 4t}{4}\right)+C=\frac{t}{2}+\frac{\sin 4t}{8}+C$$

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2 احسب التكاملات الآتية:

- $\int_0^1 (x^2 + \sqrt{x})\, dx$
- $\int_{\pi/2}^0 \frac{1 + \cos 2t}{2} dt$
- $\int_1^8 \frac{(x^{1/3} + 1)(2 - x^{2/3})}{x^{1/3}}$

الحل

$$\int_0^1 (x^2 + \sqrt{x})\, dx$$

$$\int_0^1 \left(x^2 + \sqrt{x}\right) dx = \left[\frac{x^3}{3} + \frac{2}{3}x^{3/2}\right]_0^1 = \left(\frac{1}{3} + \frac{2}{3}\right) - 0 = 1$$

$$\int_{\pi/2}^0 \frac{1 + \cos 2t}{2} dt$$

$$\int_{\pi/2}^0 \frac{1+\cos 2t}{2}\, dt = \int_{\pi/2}^0 \left(\frac{1}{2} + \frac{1}{2}\cos 2t\right) dt = \left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]_{\pi/2}^0 = \left(\frac{1}{2}(0) + \frac{1}{4}\sin 2(0)\right) - \left(\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4}\sin 2\left(\frac{\pi}{2}\right)\right) = -\frac{\pi}{4}$$

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$$\int_1^8 \frac{(x^{1/3}+1)(2-x^{2/3})}{x^{1/3}}$$

$$\int_1^8 \frac{\left(x^{1/3}+1\right)\left(2-x^{2/3}\right)}{x^{1/3}}\,dx = \int_1^8 \frac{2x^{1/3}-x+2-x^{2/3}}{x^{1/3}}\,dx = \int_1^8 \left(2-x^{2/3}+2x^{-1/3}-x^{1/3}\right) dx$$

$$= \left[2x-\tfrac{3}{5}x^{5/3}+3x^{2/3}-\tfrac{3}{4}x^{4/3}\right]_1^3$$

$$= \left(2(8)-\tfrac{3}{5}(8)^{5/3}+3(8)^{2/3}-\tfrac{3}{4}(8)^{4/3}\right)-\left(2(1)-\tfrac{3}{5}(1)^{5/3}+3(1)^{2/3}-\tfrac{3}{4}(1)^{4/3}\right)=-\frac{137}{20}$$

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طريقة التعويض

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1 احسب التكاملات الآتية:

- $\int 2x(x^2+5)^{-4}\,dx,$
- $\int (3x+2)(3x^2+4x)^4\,dx,$
- $\int \sqrt{x}\sin^2(x^{3/2}-1)\,dx,$
- $\int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}\,dx$

2 احسب التكاملات الآتية:

- $\int_0^1 t^3(1+t^4)^3\,dt$
- $\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}dv$
- $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}dx$
- $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$

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1 احسب التكاملات الآتية:

- $\int 2x(x^2+5)^{-4}\,dx,$
- $\int (3x+2)(3x^2+4x)^4\,dx,$
- $\int \sqrt{x}\sin^2(x^{3/2}-1)\,dx,$
- $\int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}\,dx$

الحل

$\int 2x(x^2+5)^{-4}\,dx,$

$$u = x^2+5 \Rightarrow du = 2x\,dx \Rightarrow \tfrac{1}{2}du = x\,dx$$

$$\int 2x(x^2+5)^{-4}dx = \int 2u^{-4}\tfrac{1}{2}du = \int u^{-4}du = -\tfrac{1}{3}u^{-3}+C = -\tfrac{1}{3}(x^2+5)^{-3}+C$$

$\int (3x+2)(3x^2+4x)^4\,dx,$

$$u = 3x^2+4x \Rightarrow du = (6x+4)dx = 2(3x+2)dx \Rightarrow \tfrac{1}{2}du = (3x+2)dx$$

$$\int (3x+2)(3x^2+4x)^4dx = \int u^4\tfrac{1}{2}du = \tfrac{1}{2}\int u^4du = \tfrac{1}{10}u^5+C = \tfrac{1}{10}(3x^2+4x)^5+C$$

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$$\int \sqrt{x}\sin^2(x^{3/2}-1)\,dx,\qquad u = x^{3/2}-1 \Rightarrow du = \tfrac{3}{2}x^{1/2}dx \Rightarrow \tfrac{2}{3}du = \sqrt{x}\,dx$$

$$\int \sqrt{x}\sin^2(x^{3/2}-1)\,dx = \int \tfrac{2}{3}\sin^2 u\,du = \tfrac{2}{3}\left(\tfrac{u}{2}-\tfrac{1}{4}\sin 2u\right)+C = \tfrac{1}{3}(x^{3/2}-1)-\tfrac{1}{6}\sin(2x^{3/2}-2)+C$$

$$\int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}\,dx$$

$$u = 2-\tfrac{1}{x} \Rightarrow du = \tfrac{1}{x^2}dx$$

$$\int \tfrac{1}{x^2}\sqrt{2-\tfrac{1}{x}}\,dx = \int \sqrt{u}\,du = \int u^{1/2}du = \tfrac{2}{3}u^{3/2}+C = \tfrac{2}{3}\left(2-\tfrac{1}{x}\right)^{3/2}+C$$

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2 احسب التكاملات الآتية:

- $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$
- $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}dx$
- $\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}dv$
- $\int_0^1 t^3(1+t^4)^3\,dt$

الحل

$\int_0^1 t^3(1+t^4)^3\,dt$

$$u=1+t^4 \Rightarrow du=4t^3dt \Rightarrow \frac{1}{4}du=t^3dt;\ t=0\Rightarrow u=1,\ t=1\Rightarrow u=2$$

$$\int_0^1 t^3(1+t^4)^3\,dt=\int_1^2 \frac{1}{4}u^3\,du=\left[\frac{u^4}{16}\right]_1^2=\frac{2^4}{16}-\frac{1^4}{16}=\frac{15}{16}$$

$\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}dv$

$$u=1+v^{3/2} \Rightarrow du=\frac{3}{2}v^{1/2}\,dv \Rightarrow \frac{20}{3}du=10\sqrt{v}\,dv;\ v=0\Rightarrow u=1,\ v=1\Rightarrow u=2$$

$$\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv=\int_1^2 \frac{1}{u^2}\left(\frac{20}{3}du\right)=\frac{20}{3}\int_1^2 u^{-2}\,du=-\frac{20}{3}\left[\frac{1}{u}\right]_1^2=-\frac{20}{3}\left[\frac{1}{2}-\frac{1}{1}\right]=\frac{10}{3}$$

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$$\int_0^1 \frac{x^3}{\sqrt{x^4+9}}dx$$

$$u = x^4 + 9 \Rightarrow du = 4x^3\ dx \Rightarrow \tfrac{1}{4}du = x^3\ dx;\ x = 0 \Rightarrow u = 9,\ x = 1 \Rightarrow u = 10$$

$$\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\ dx = \int_9^{10} \frac{1}{4} u^{-1/2}\ du = \left[\tfrac{1}{4}(2)u^{1/2}\right]_9^{10} = \tfrac{1}{2}(10)^{1/2} - \tfrac{1}{2}(9)^{1/2} = \frac{\sqrt{10}-3}{2}$$

$$\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$$

$$u = 1 + \sqrt{y} \Rightarrow du = \frac{dy}{2\sqrt{y}};\ y = 1 \Rightarrow u = 2,\ y = 4 \Rightarrow u = 3$$

$$\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} = \int_2^3 \frac{1}{u^2}\ du = \int_2^3 u^{-2}\ du = [-u^{-1}]_2^3 = \left(-\tfrac{1}{3}\right) - \left(-\tfrac{1}{2}\right) = \tfrac{1}{6}$$