



Calculus 2

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Exercices 7

Functions of Several Variables

1- find the specific function values

1. $f(x, y) = x^2 + xy^3$

a. $f(0, 0)$

c. $f(2, 3)$

b. $f(-1, 1)$

d. $f(-3, -2)$

2. $f(x, y) = \sin(xy)$

a. $f\left(2, \frac{\pi}{6}\right)$

c. $f\left(\pi, \frac{1}{4}\right)$

b. $f\left(-3, \frac{\pi}{12}\right)$

d. $f\left(-\frac{\pi}{2}, -7\right)$

3. $f(x, y, z) = \frac{x - y}{y^2 + z^2}$

a. $f(3, -1, 2)$

c. $f\left(0, -\frac{1}{3}, 0\right)$

b. $f\left(1, \frac{1}{2}, -\frac{1}{4}\right)$

d. $f(2, 2, 100)$

2- Find the domain of each function

$$f(x, y) = \ln(x^2 + y^2 - 4)$$

$$f(x, y) = x \ln(y^2 - x)$$

$$f(x, y) = \sqrt{y - x - 2}$$

$$f(x, y) = \frac{\sqrt{x^2 + y^2 - 9}}{x}$$

$$f(x, y) = \frac{(x-1)(y+2)}{(y-x)(y-x^3)}$$

$$f(x, y, z) = \frac{x}{\sqrt{9 - x^2 - y^2 - z^2}}$$

$$f(x, y) = \frac{\sqrt{x + y + 1}}{x - 1}$$

$$f(x, y, z) = \ln(z - y) + x y \sin z$$

$$f(x, y) = \sqrt{1 - x^2} + \sqrt{y^2 - 1}$$

3-Find the limit

$$\lim_{(x,y) \rightarrow (1,2)} (x^3 y^2 - x^2 y + x^2 - 2x + 3y)$$

$$\lim_{(x,y) \rightarrow (2,4)} \sqrt[3]{\frac{8xy}{2x+y}}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$$

4- If $f(x, y) = \frac{y}{x}$, dose $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist?

5- If $f(x, y) = \frac{2x^2 y}{x^4 + y^2}$, dose $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist?

$$\lim_{(x,y,z) \rightarrow (1,0,-1)} \frac{e^{x+z}}{z^2 + \cos \sqrt{xy}}$$

6-If $f(x, y) = \frac{x - x^2 + 2y + y^2}{2x - 3y}$, does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist?

7-Show that

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is continuous at every point except the origin

8-Find the limit (if it exists). If the limit does not exist, explain why

$$\lim_{(x, y) \rightarrow (1, 1)} \frac{xy - 1}{1 + xy}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{1}{x + y}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x - y}{\sqrt{x} - \sqrt{y}}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x + y}{x^2 + y}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x^2}{(x^2 + 1)(y^2 + 1)}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \ln(x^2 + y^2)$$

$$\lim_{(x, y, z) \rightarrow (0, 0, 0)} \frac{xy + yz + xz}{x^2 + y^2 + z^2}$$

$$\lim_{(x, y) \rightarrow (1, -1)} \frac{x^2 y}{1 + xy^2}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{1}{x^2 y^2}$$

$$\lim_{(x, y) \rightarrow (2, 1)} \frac{x - y - 1}{\sqrt{x - y} - 1}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x}{x^2 - y^2}$$



Thank you for your attention