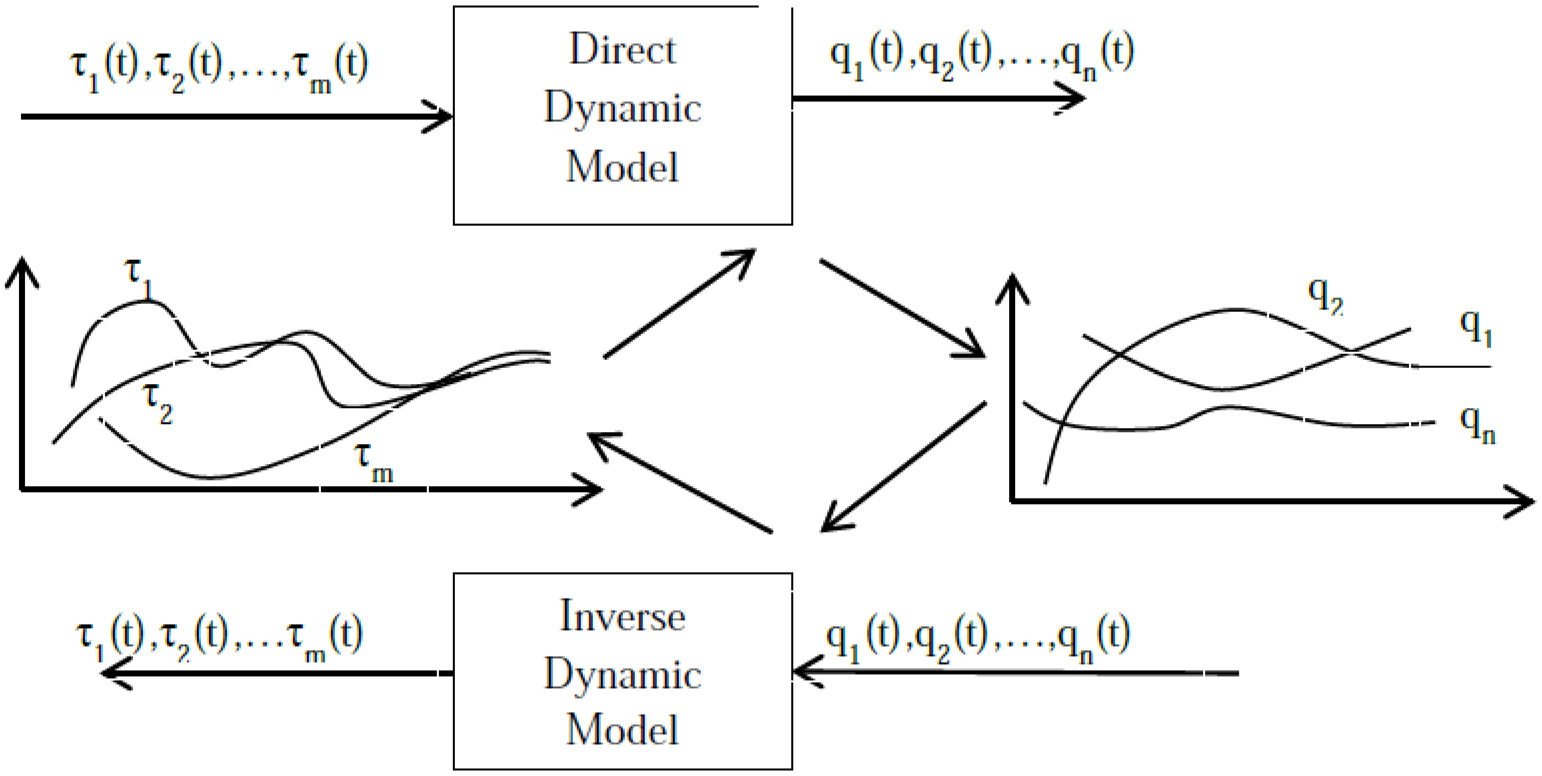
WMR Dynamic model

differential WMR model

General robot dynamic modeling

- The dynamic equations of the robot motion can be done by two methodologies:

1. Newton-Euler method
2. Lagrange method



Lagrange model for multi-link robot

$$D(q)\ddot{q} + h(q, \dot{q}) + g(q) = \tau$$

- $for\ \dot{q} \neq 0: D(q)\ is\ n \times n\ positive\ define\ matrix$
- $D(q)\ddot{q}$, represents the inertia force
- $h(q, \dot{q})$, represents the centrifugal and Coriolis force
- $g(q)$, represents gravitational force
- τ, the net force\torque applied to the robot
- $h(q, \dot{q}) = C(q, \dot{q})\dot{q} \rightarrow A = \dot{D} - 2C\ is\ n \times n\ antisymetric$
- $\rightarrow A^T = -A$

Dynamic modeling of non-holonomic robot

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} + M(q)^T \lambda = E\tau$$

- $M(q), is\ m \times n\ matrix\ of\ the\ m\ nonholonomic\ constraints$
- $M(q)\dot{q} = 0$
- $\lambda, is\ the\ vector\ of\ Lagrange\ multiplier$
- $E, the\ transformation\ matrix$

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + M(q)^T \lambda = E\tau$$

Constrained to unconstrained model

- To eliminate the constraint term $M(q)^T\lambda$ we use the $\mathrm{n}\times(n-m)\ matrix\ \mathrm{B}(q)$
- $B(q)^T M(q)^T = 0$
- $\left.\begin{matrix} M(q)\dot{q} = 0 \\ B(q)^T M(q)^T = 0 \end{matrix}\right\} \overset{?}{\Rightarrow} \exists (n-m)\ vector\ v(t): \dot{q}(t) = \mathrm{B}(q)v(t)$
- Now:

$$B(q)^T \times \{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + M(q)^T\lambda = E\tau\}$$

$$\bar{D}(q)\dot{v} + \bar{C}(q,\dot{q})v + \bar{g}(q) = \bar{E}\tau$$

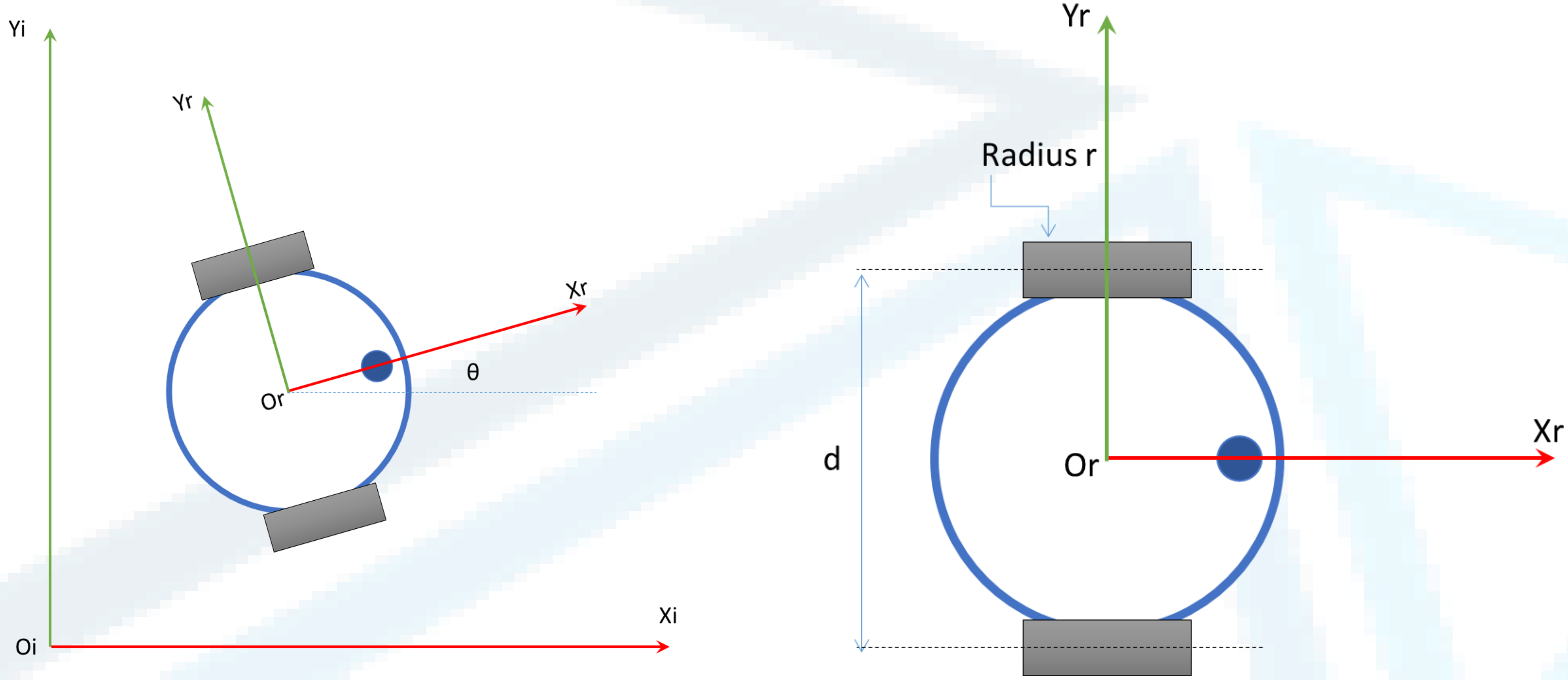
Unconstrained model

- The reduced unconstrained model describes the dynamic evolution of the n dimensional vector q(t) in terms of the dynamic evolution of the (n-m) dimensional vector ν(t).

$$\bar{D}(q)\dot{v} + \bar{C}(q,\dot{q})v + \bar{g}(q) = \bar{E}\tau$$

- $\bar{D} = B^T D B$
- $\bar{C} = B^T D \dot{B} + B^T C B$
- $\bar{g} = B^T g$
- $\bar{E} = B^T E$

Differential WMR dynamic model



Constrained model

- Since WMR moves in the horizontal plan the terms $C(q,\dot{q})$ and $g(q)$ are zero.
- The constrained dynamic model becomes:

$$D(q)\ddot{q} + M(q)^T\lambda = E\tau$$

- $\tau = [\tau_r \quad \tau_l]^T$
- $M(q) = [-\sin(\theta) \quad \cos(\theta) \quad 0]$
- $q = [x_{Or} \quad y_{Or} \quad \theta]^T$
- $D(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, E = \frac{1}{r}\begin{bmatrix} \cos(\theta) & \cos(\theta) \\ \sin(\theta) & \sin(\theta) \\ d & -d \end{bmatrix}$

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Unconstrained model

- $B(q) = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \ldots remember(\ker(M) = img(B))$

$$\bar{D}(q)\dot{v} = \bar{E}\tau$$

- ν$= [V \quad \omega]^T$
- $\bar{D} = B^T D B = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}$
- $\bar{E} = B^T E = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ d & -d \end{bmatrix}$

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ d & -d \end{bmatrix}\begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}$$

Final model

$$\dot{V} = \frac{1}{mI}(\tau_r + \tau_l)$$

$$\dot{\omega} = \frac{d}{Ir}(\tau_r - \tau_l)$$

- Always remember:

$$\left.\begin{matrix} M(q)\dot{q} = 0 \\ B(q)^T M(q)^T = 0 \end{matrix}\right\} \overset{?}{\Rightarrow} \exists (n-m)\ vector\ v(t): \dot{q}(t) = \mathrm{B}(q)v(t)$$

$$\begin{bmatrix} \dot{x_{Or}} \\ \dot{y_{Or}} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \omega \end{bmatrix} = \begin{bmatrix} V.\cos(\theta) \\ V.\sin(\theta) \\ \omega \end{bmatrix}$$

Newton-Euler WMR model

- *Assume Or is the center of gravity*

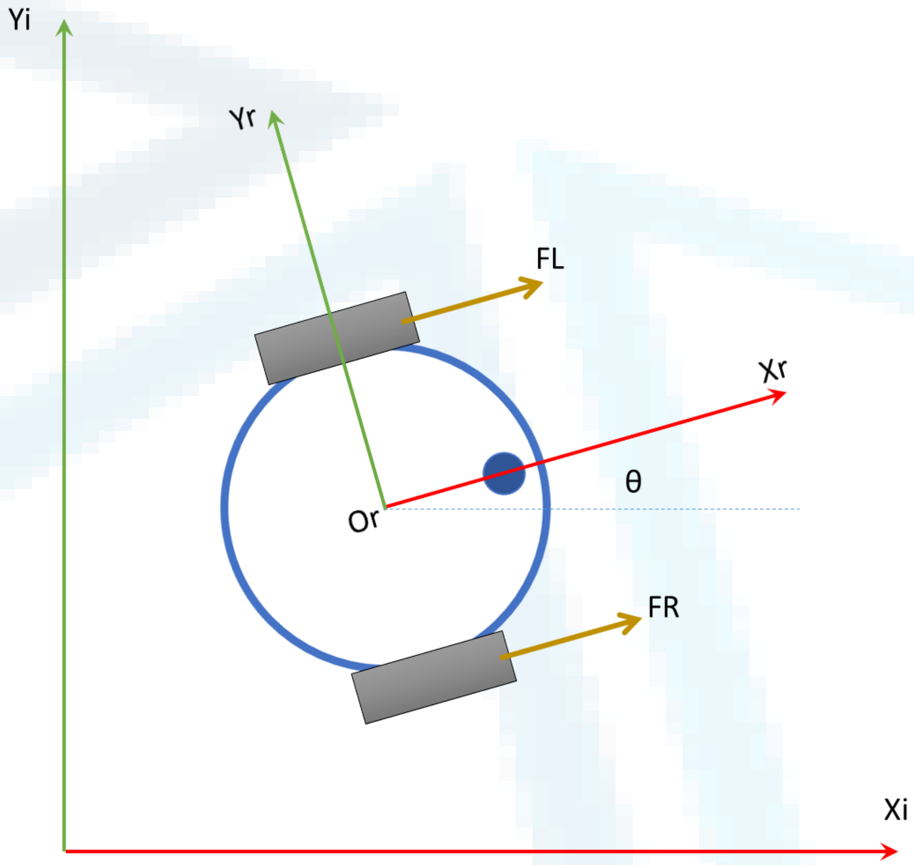
$$\sum F_{/Or} = m\dot{V}$$

$$\sum M_{/Or} = I\dot{\omega}$$

$$\tau_r = FR \times r, \tau_l = FL \times r$$

$$\sum F_{/Or} = FR + FL = \frac{1}{r}(\tau_r + \tau_l)$$

$$\sum M_{/Or} = d(FR - FL) = \frac{d}{r}(\tau_r - \tau_l)$$



Thanks

Think about the dynamic model of Bicycle WMR ……