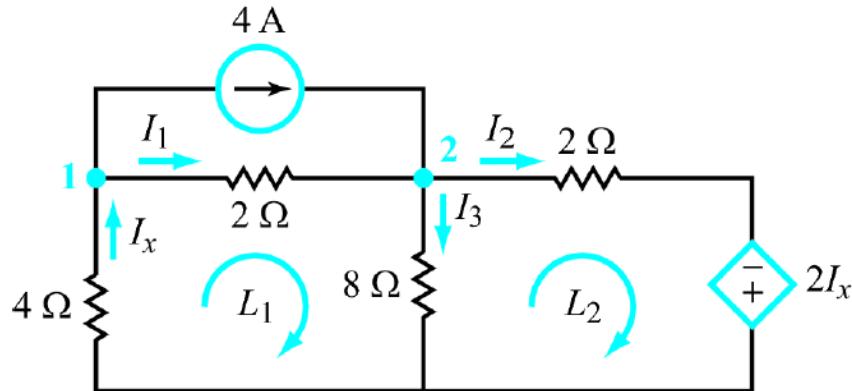


Exercise 2-6 Determine I_x in the circuit of Fig. E2-6.

Solution:



$$\text{KCL @ node 1: } I_x = I_1 + 4$$

$$\text{KCL @ node 2: } I_1 + 4 = I_2 + I_3$$

$$\text{KVL Loop 1: } 4I_x + 2I_1 + 8I_3 = 0$$

$$\text{KVL Loop 2: } -8I_3 + 2I_2 - 2I_x = 0$$

We have four equations with four unknowns. Simultaneous solution leads to

$$I_x = 1.33.$$

Exercise 2-7 Apply resistance combining to simplify the circuit of Fig. E2-7 so as to find I . All resistor values are in ohms.

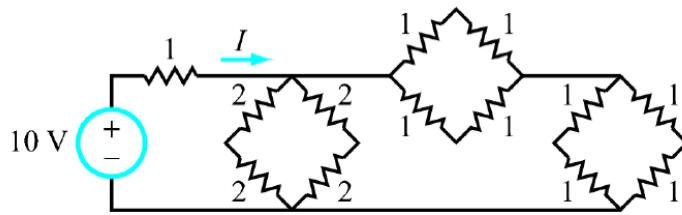
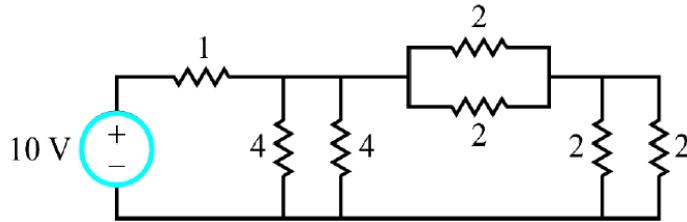
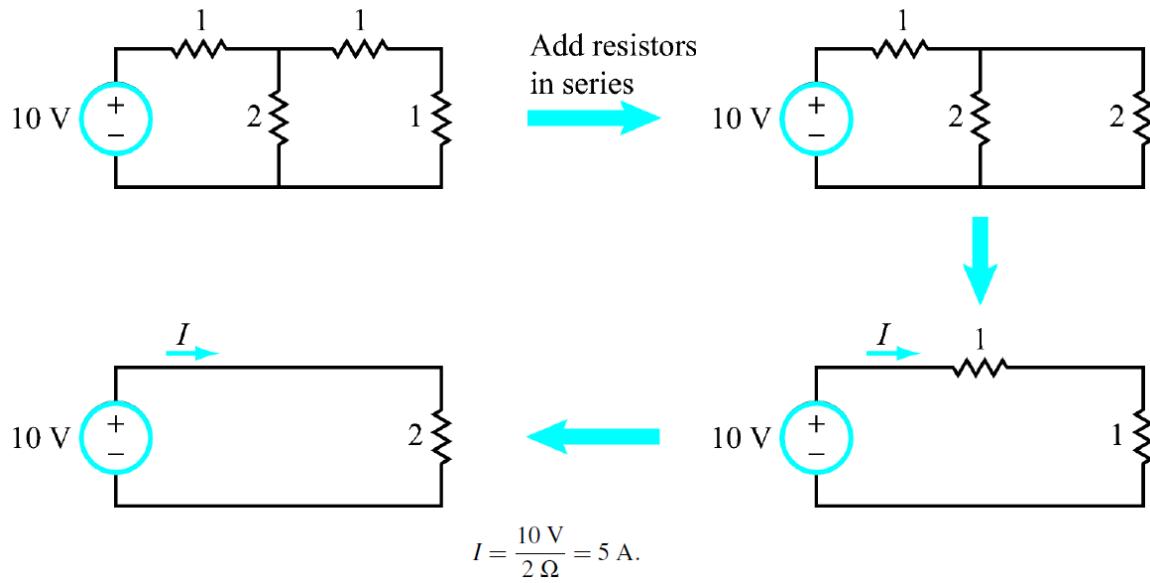


Figure E2-7

Solution: Combining all resistors that are in series will result in the following circuit:



Combining all resistors that are in parallel will result in:



Exercise 2-8 Apply source transformation to the circuit in Fig. E2-8 to find I .

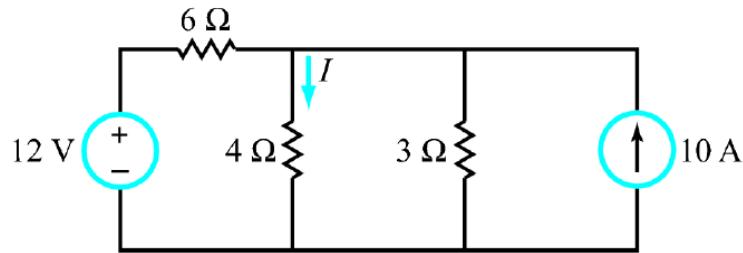
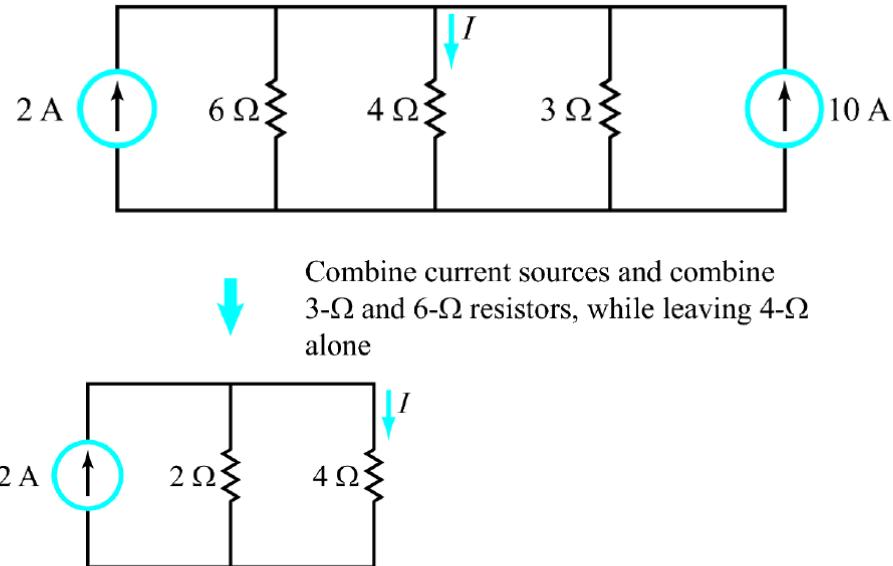


Figure E2-8

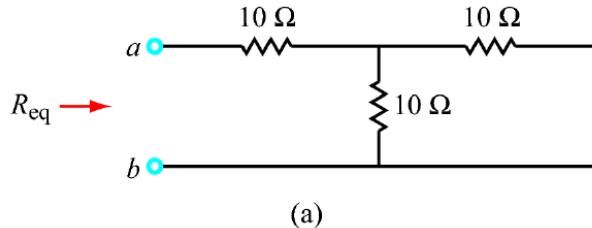
Solution: Apply source transformation to the 12-V source and 6-Ω resistor:



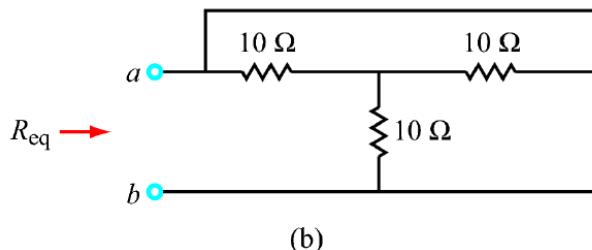
Current division gives

$$I = \frac{12 \times 2}{2+4} = 4 \text{ A.}$$

Exercise 2-9 For each of the circuits shown in Fig. E2-9, determine the equivalent resistance between terminals (a,b) .



(a)

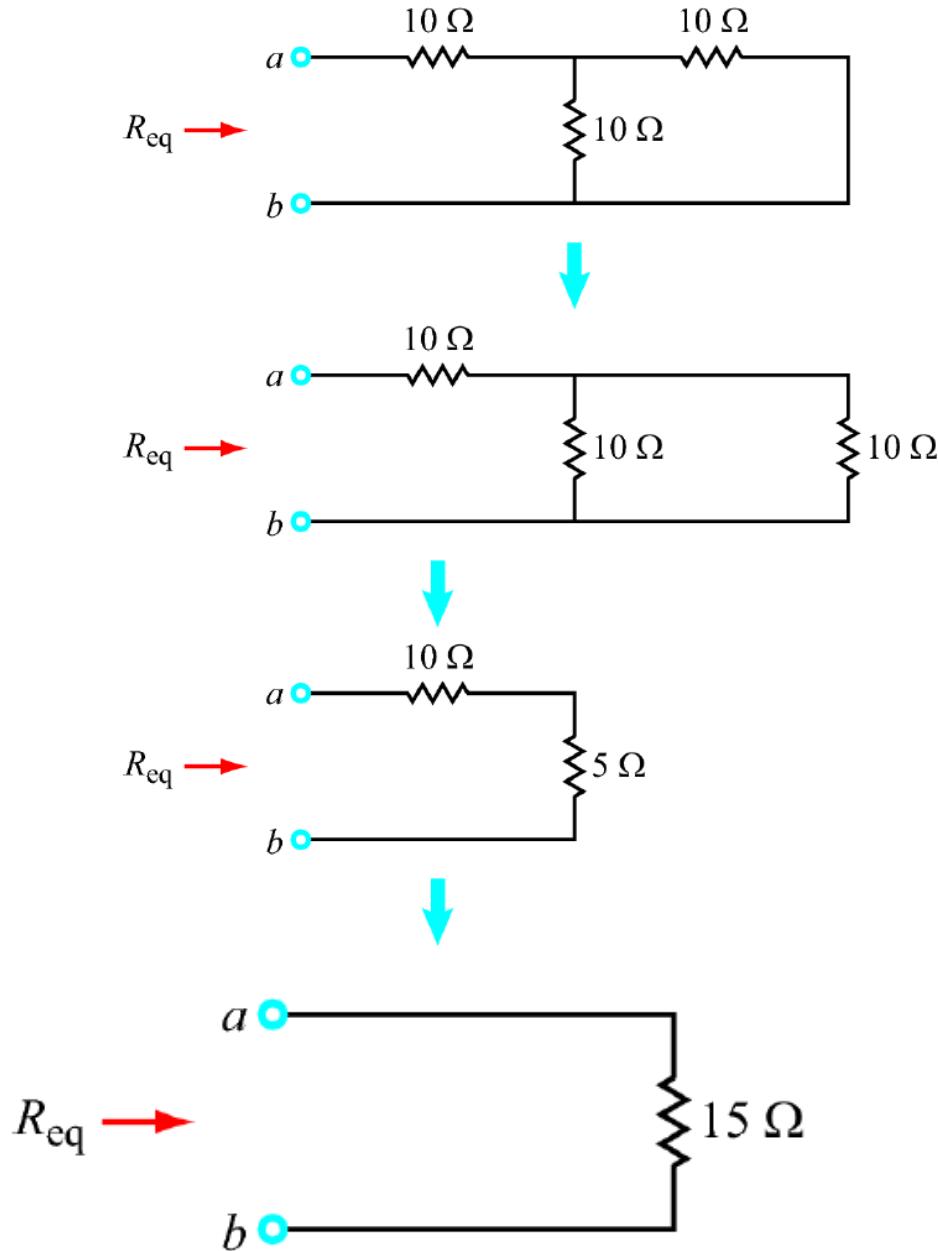


(b)

Figure E2-9

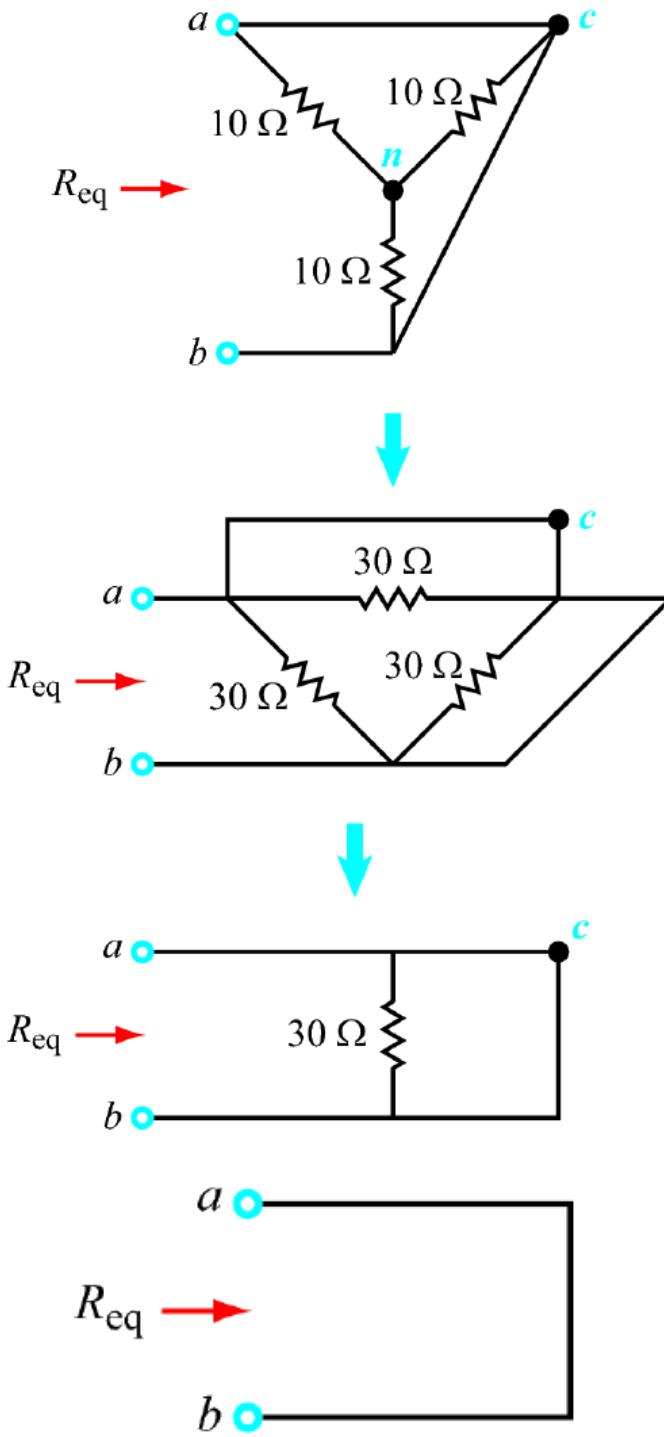
Solution:

(a)



$$R_{\text{eq}} = 15\ \Omega.$$

(b) Applying Y-Δ transformation



$$R_{\text{eq}} = 0.$$

Exercise 2-11 Determine I in the two circuits of Fig. E2-11. Assume $V_F = 0.7$ V for all diodes.

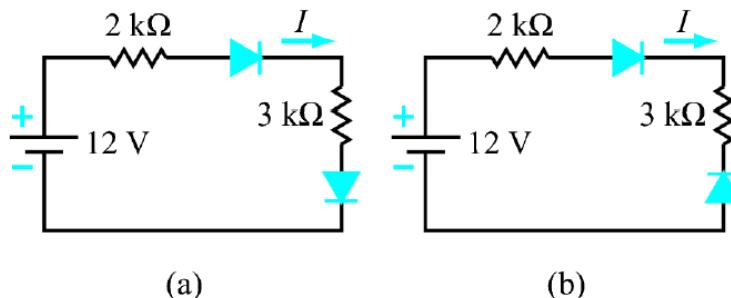


Figure E2-11

Solution:

(a) With $V_F = 0.7$ V, KVL around the loop gives

$$-12 + 2 \times 10^3 I + 0.7 + 3 \times 10^3 I + 0.7 = 0,$$

which leads to

$$I = \frac{12 - 1.4}{5 \times 10^3} = 2.12 \text{ mA.}$$

(b) Since the diodes are biased in opposition to one another, no current can flow in the circuit. Hence

$$I = 0.$$

Exercise 2-13 The circuit in Fig. E2-13 is called a resistive bridge. How does $V_x = (V_3 - V_2)$ vary with the value of potentiometer R_1 ?

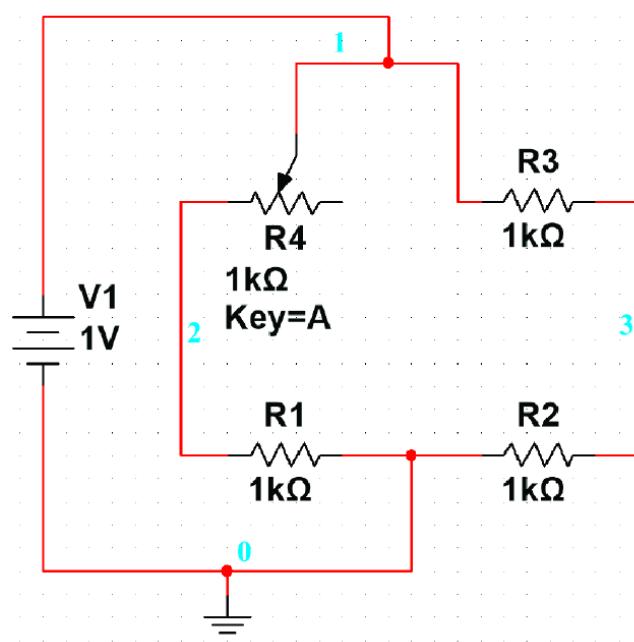


Figure E2-13

Solution: Using DC Operating Point Analysis and varying the value of the potentiometer, we obtain the following values for $(V_3 - V_2)$:

R_1 (% of $1 \text{ k}\Omega$)	$V_x = V_2 - V_3$
100%	0 mV
80%	55.6 mV
60%	125 mV
40%	214 mV
20%	333 mV
0%	500 mV

Exercise 3-1 Apply nodal analysis to determine the current I .

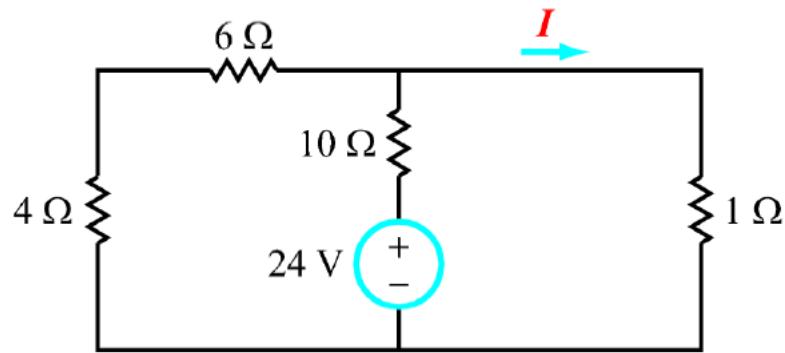
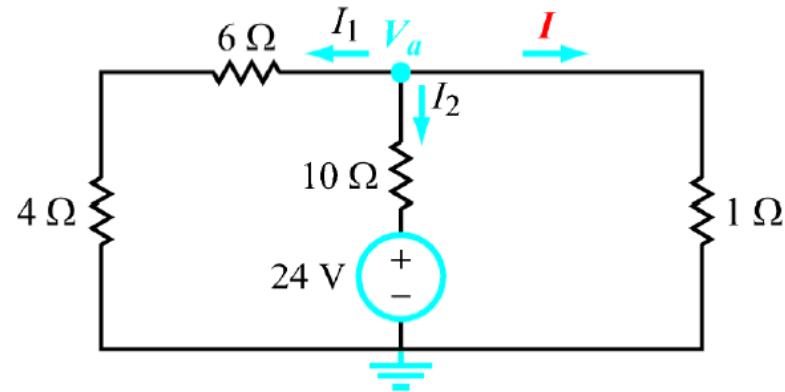


Figure E3-1

Solution:



$$I_1 + I_2 + I = 0$$

$$I_1 = \frac{V_a}{6}, \quad I_2 = \frac{V_a - 24}{10}, \quad I_3 = \frac{V_a}{1}$$

Hence,

$$\frac{V_a}{10} + \frac{V_a - 24}{10} + V_a = 0,$$

$$V_a \left(\frac{1}{10} + \frac{1}{10} + 1 \right) = \frac{24}{10},$$

which leads to

$$V_a = 2 \text{ V}, \quad I = \frac{V_a}{1} = 2 \text{ A.}$$

Exercise 3-2 Apply nodal analysis to find V_a .

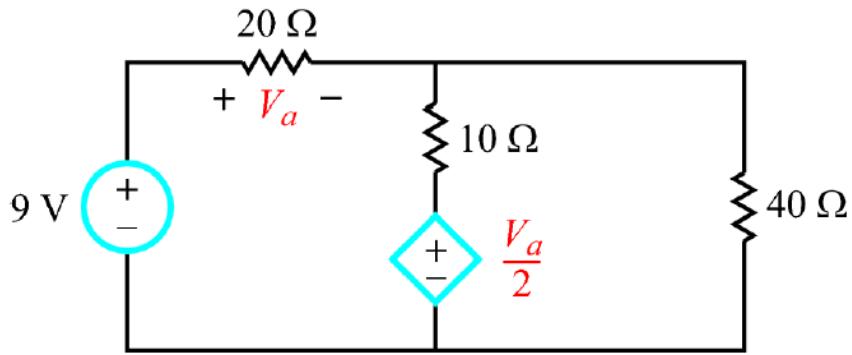
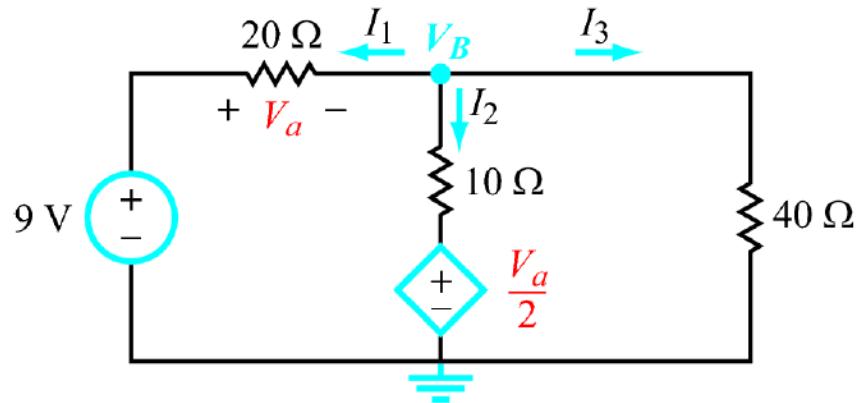


Figure E3-2

Solution:



$$I_1 + I_2 + I_3 = 0$$

$$I_1 = \frac{V_B - 9}{20}, \quad I_2 = \frac{V_B - \frac{V_a}{2}}{10}, \quad I_3 = \frac{V_B}{40}.$$

Hence,

$$\frac{V_B - 9}{20} + \frac{V_B - \frac{V_a}{2}}{10} + \frac{V_B}{40} = 0.$$

Also,

$$V_A = 9 - V_B.$$

Solution gives: $V_a = 5 \text{ V}$.

Exercise 3-3 Apply the supernode concept to determine I in the circuit of Fig. E3-3.

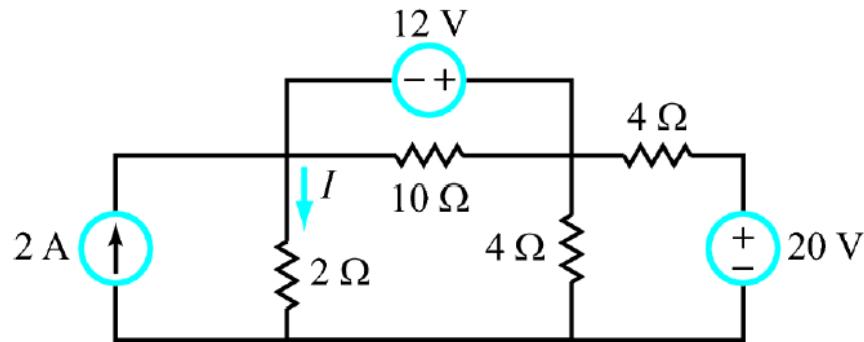
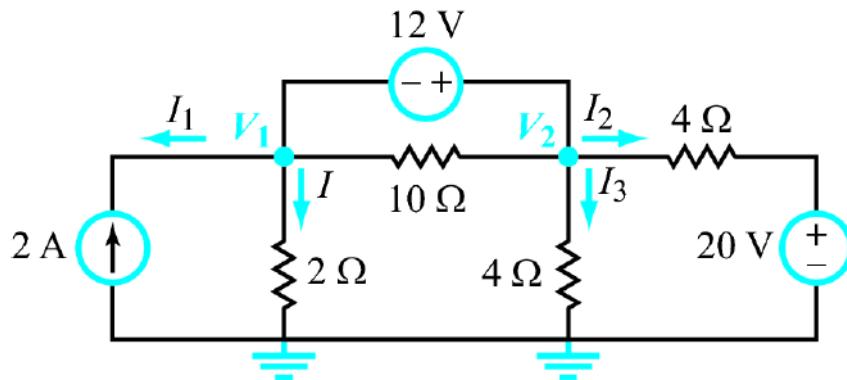


Figure E3.3

Solution:



(V_1, V_2) constitutes a supernode. Hence,

$$\begin{aligned}I_1 + I + I_2 + I_3 &= 0, \\I_1 &= -2 \text{ A}, \quad I = \frac{V_1}{2}, \\I_3 &= \frac{V_2}{4}, \quad I_2 = \frac{V_2 - 20}{4}.\end{aligned}$$

Also,

$$V_2 - V_1 = 12.$$

Solution leads to: $I = 0.5 \text{ A}$.

Exercise 3-4 Apply mesh analysis to determine I .

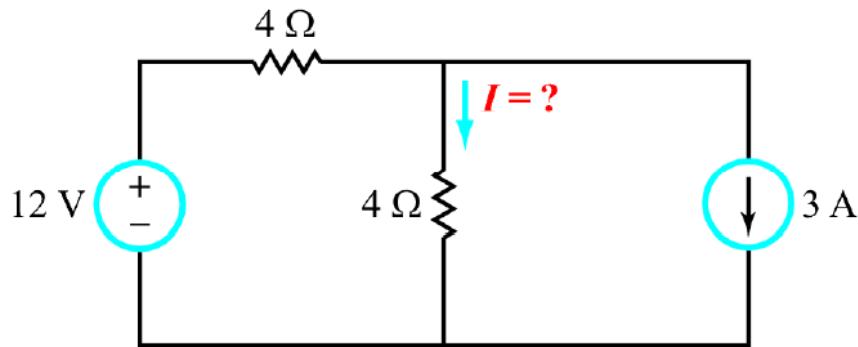
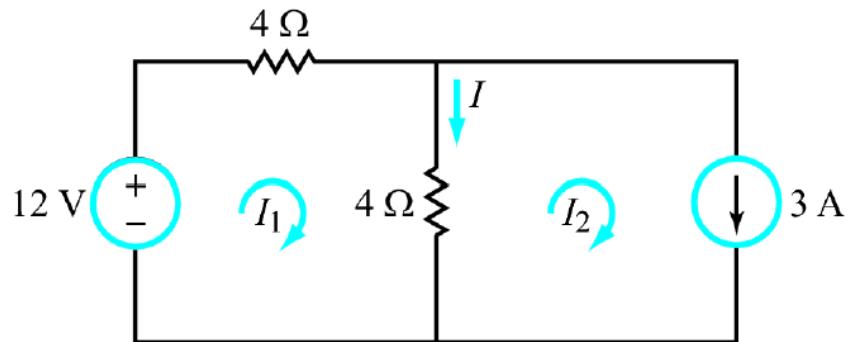


Figure E3-4

Solution:



$$\text{Mesh 1: } -12 + 4I_1 + 4(I_1 - I_2) = 0$$

$$\text{Mesh 2: } I_2 = 3 \text{ A}$$

$$4I_1 + 4I_1 - 4 \times 3 = 12$$

$$8I_1 = 24$$

$$I_1 = 3 \text{ A.}$$

$$\Rightarrow I = I_1 - I_2 = 3 - 3 = 0.$$

Exercise 3-5 Determine the current I in the circuit of Fig. E3-5.

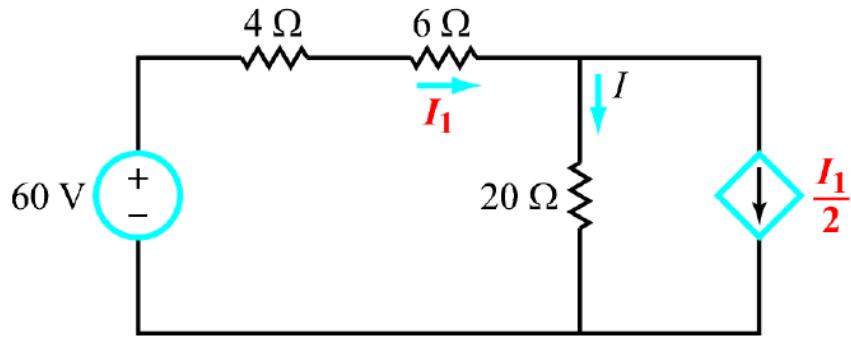
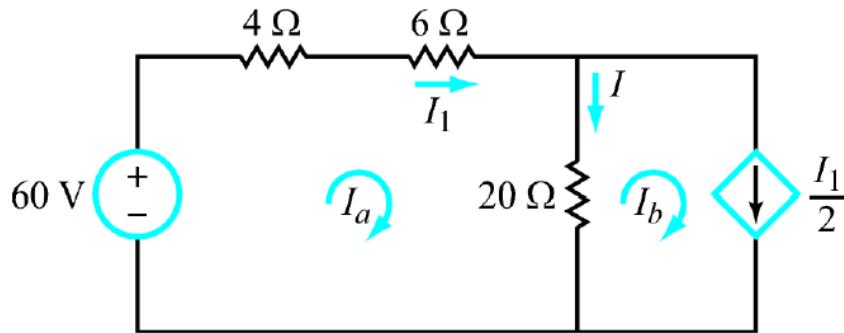


Figure E3-5

Solution:



$$\text{Mesh 1: } -60 + 10I_a + 20(I_a - I_b) = 0$$

$$\text{Mesh 2: } I_b = \frac{I_1}{2}$$

Also,

$$I_1 = I_a.$$

Hence,

$$I_b = \frac{I_a}{2},$$

$$-60 + 10I_a + 20\left(I_a - \frac{I_a}{2}\right) = 0,$$

which simplifies to

$$20I_1 = 60$$

or

$$I_a = 3 \text{ A},$$

$$I = I_a - I_b = I_a - \frac{I_a}{2} = \frac{I_a}{2} = \frac{3}{2} = 1.5 \text{ A.}$$

Exercise 3-6 Apply mesh analysis to determine I in the circuit of Fig. E3-6.

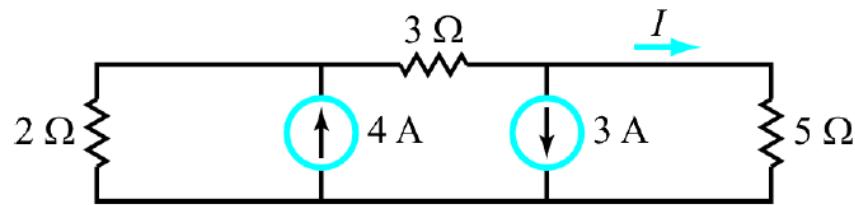
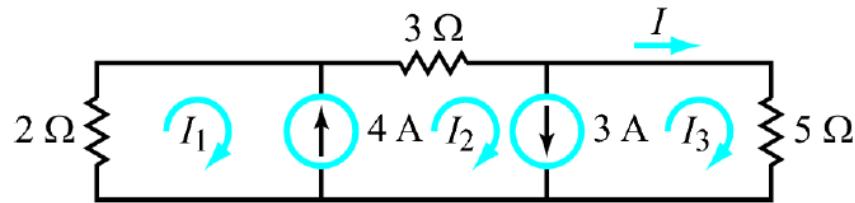


Figure E3-6

Solution:



$$\text{Outside mesh: } 2I_1 + 3I_2 + 5I_3 = 0.$$

Also,

$$I_2 - I_1 = 4\text{ A}, \quad I_2 - I_3 = 3\text{ A}.$$

Hence,

$$I_1 = I_2 - 4 = (I_3 + 3) - 4 = I_3 - 1$$

$$I_2 = I_3 + 3$$

$$2(I_3 - 1) + 3(I_3 + 3) + 5I_3 = 0$$

$$10I_3 = 2 - 9$$

$$I_3 = -0.7 \text{ A}$$

$$I = I_3 = -0.7 \text{ A.}$$

Exercise 3-9 Apply the source-superposition method to determine the current I in the circuit of Fig. E3-9.

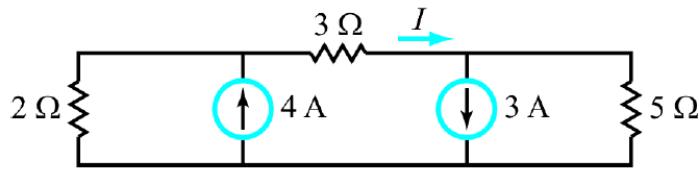
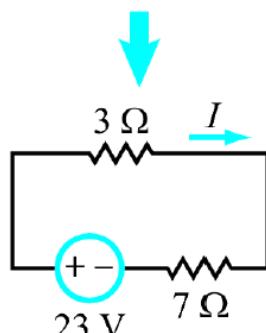
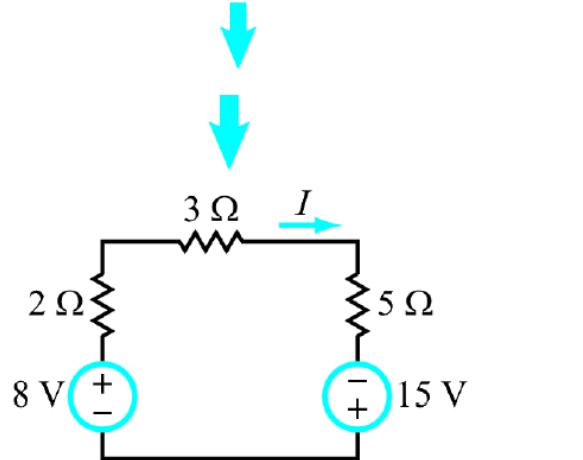
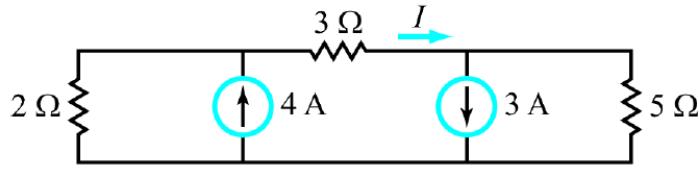


Figure E3-9

Solution:



$$I = \frac{23}{3+7} = 2.3 \text{ A.}$$

Exercise 3-10 Apply source superposition to determine V_{out} in the circuit of Fig. E3-10.

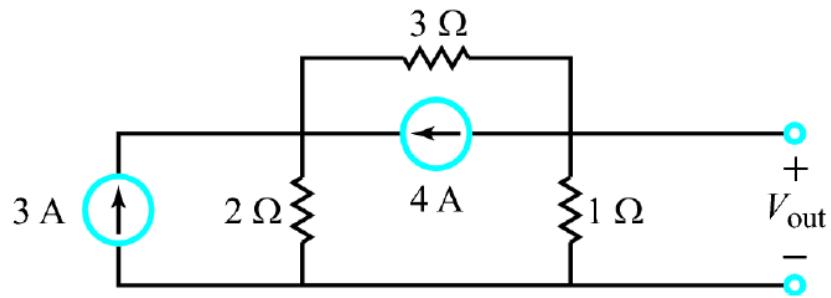
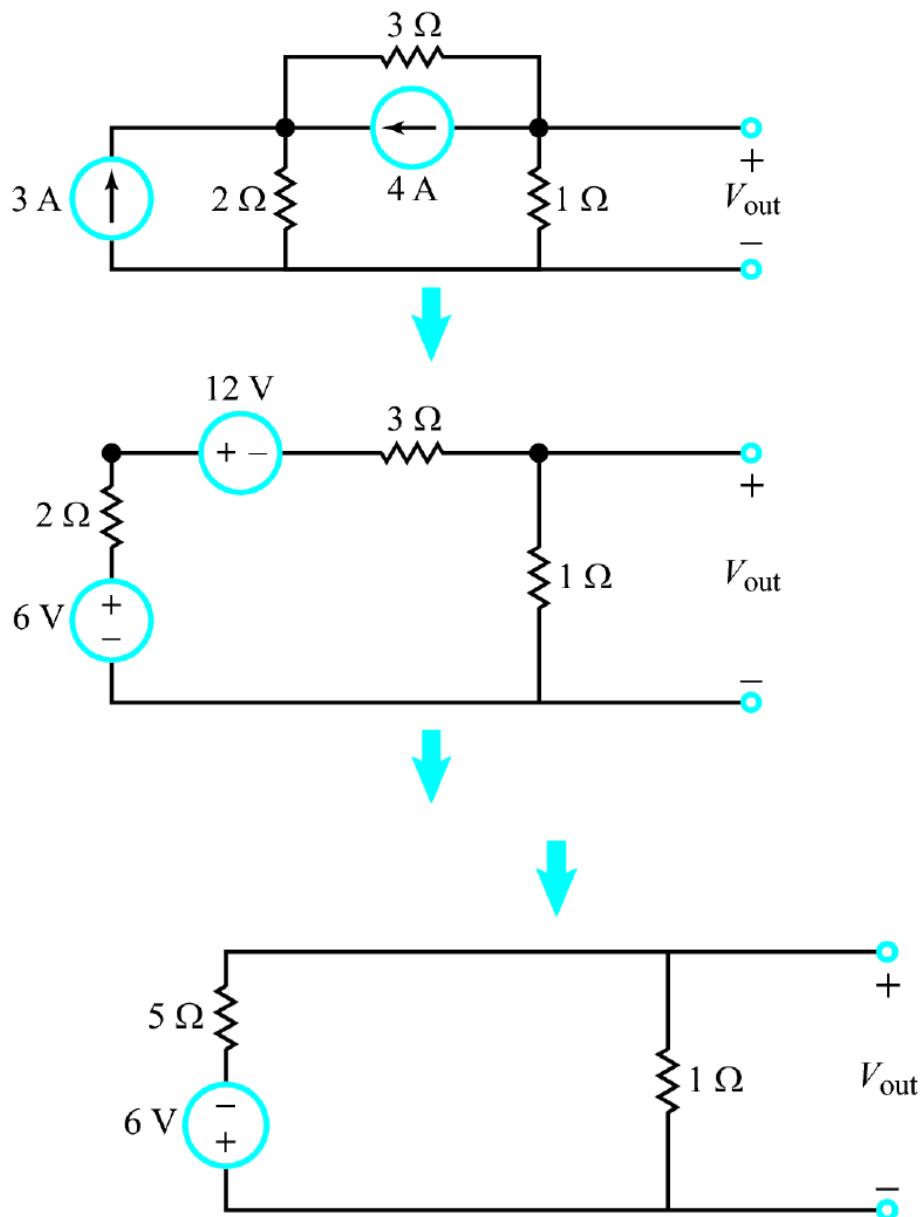


Figure E3-10

Solution:



By voltage division,

$$V_{\text{out}} = \frac{-6 \times 1}{5 + 1} = -1 \text{ V.}$$

Exercise 3-11 Determine the Thévenin-equivalent circuit at terminals (a, b) in Fig. E3-11.

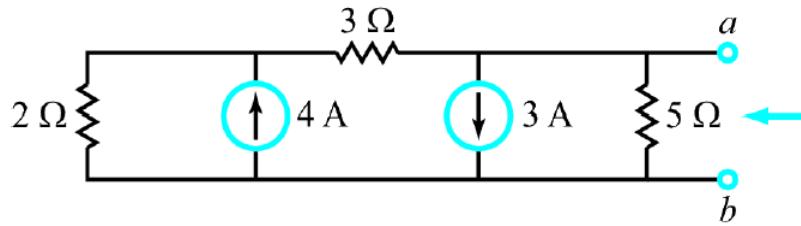
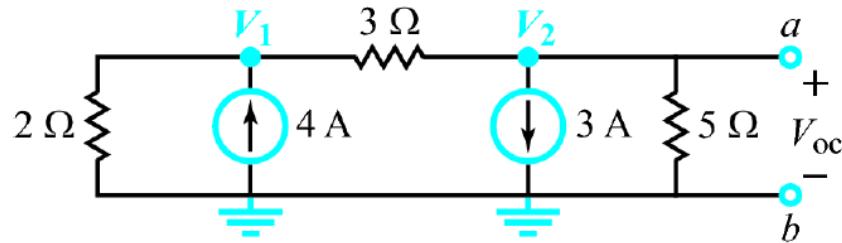


Figure E3.11

Solution:

(1) Open-circuit voltage

We apply node voltage method to determine open-circuit voltage:



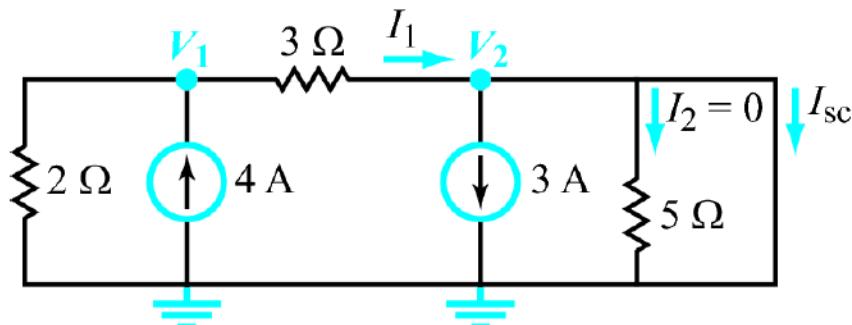
$$\begin{aligned}\frac{V_1}{2} - 4 + \frac{V_1 - V_2}{3} &= 0, \\ \frac{V_2 - V_1}{3} + 3 + \frac{V_2}{5} &= 0.\end{aligned}$$

Solution gives: $V_2 = -3.5$ V.

Hence,

$$V_{\text{Th}} = V_{\text{oc}} = -3.5 \text{ V.}$$

(2) Short-circuit current



Because of the short circuit,

$$V_2 = 0.$$

Hence at node V_1 :

$$\frac{V_1}{2} - 4 + \frac{V_1}{3} = 0$$

$$V_1 \left(\frac{1}{2} + \frac{1}{3} \right) = 4$$

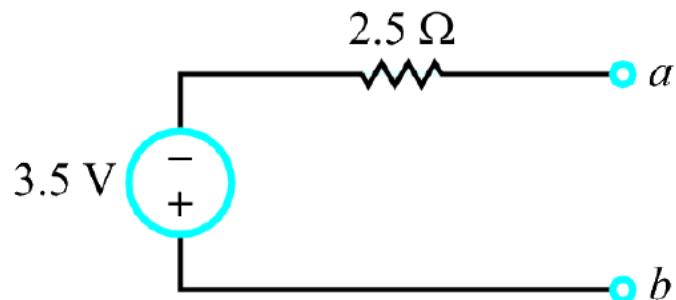
$$V_1 = \frac{24}{5} \text{ V}$$

$$I_1 = \frac{V_1}{3} = \frac{24}{5 \times 3} = \frac{8}{5} \text{ A},$$

$$I_{\text{sc}} = I_1 - 3 = \frac{8}{5} - 3 = -\frac{7}{5} = -1.4 \text{ A}$$

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_{\text{sc}}} = \frac{-3.5}{-1.4} = 2.5 \Omega.$$

Thévenin equivalent:



Exercise 3-12 Find the Thévenin equivalent of the circuit to the left of terminals (a,b) in Fig. E3-12, and then determine the current I .

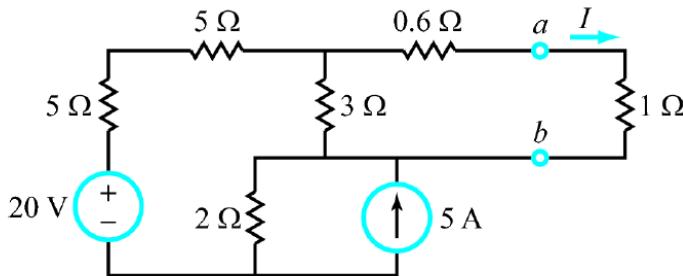
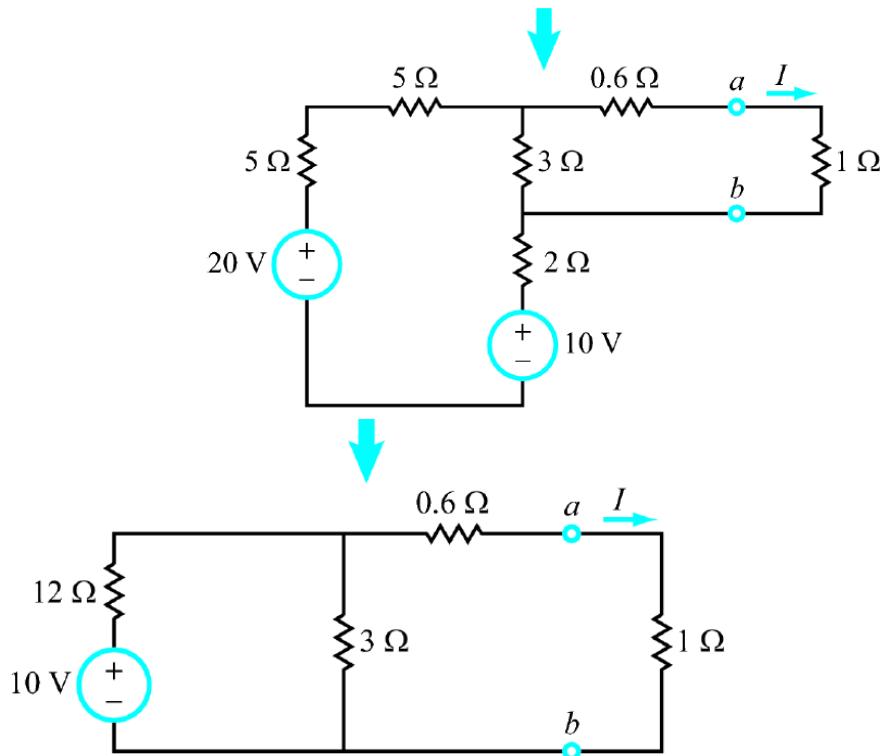


Figure E3-12

Solution: Since the circuit has no dependent sources, we will apply multiple steps of source transformation to simplify the circuit.

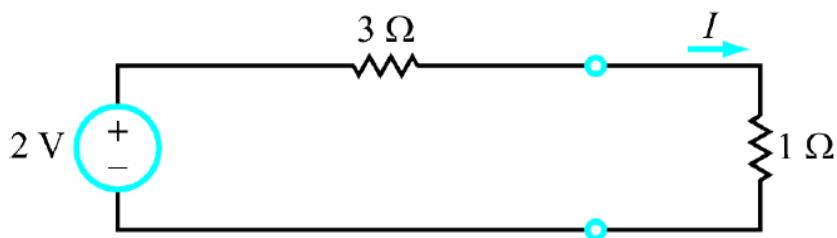


Across (a,b) ,

$$V_{Th} = V_{oc} = \frac{10 \times 3}{12 + 3} = 2 \text{ V}$$

$$\begin{aligned} R_{Th} &= 3 \parallel 12 + 0.6 \\ &= \frac{3 \times 12}{3 + 12} + 0.6 = 3 \Omega \end{aligned}$$

Hence,



$$I = \frac{2}{3+1} = 0.5 \text{ A.}$$

Exercise 3-13 Find the Norton equivalent at terminals (a, b) of the circuit in Fig. E3-13.

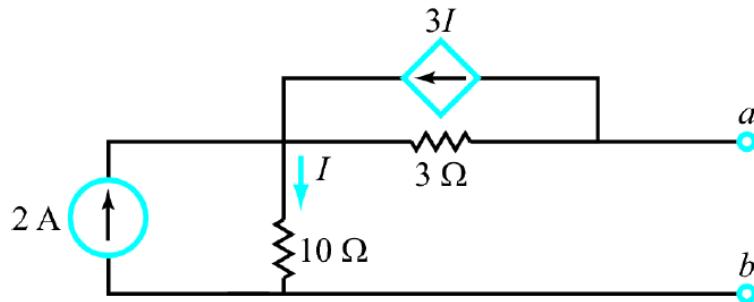
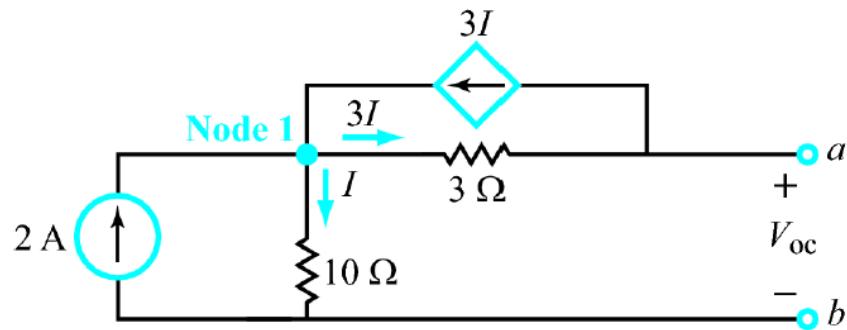


Figure E3-13

Solution: Thévenin voltage



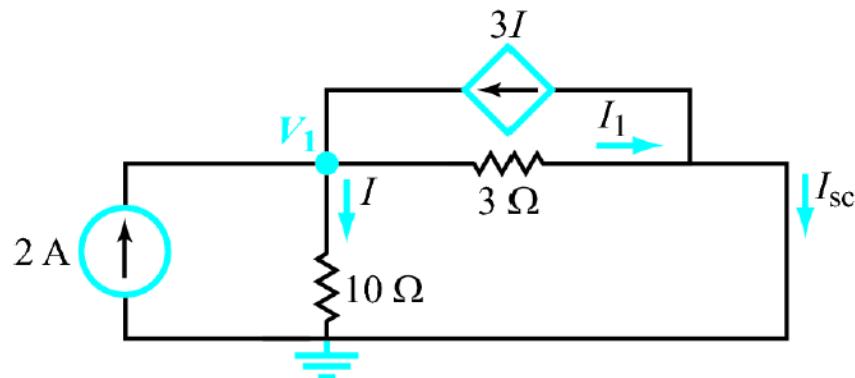
At node 1:

$$I = 2 \text{ A.}$$

Hence,

$$V_{\text{Th}} = V_{\text{oc}} = 10I - 3 \times 3I = I = 2 \text{ V.}$$

Next, we determine the short-circuit current:



At node V_1 :

$$-2 - 3I + \frac{V_1}{10} + \frac{V_1}{3} = 0.$$

Also,

$$I = \frac{V_1}{10}.$$

Hence,

$$-2 - 3I + I + \frac{10}{3}I = 0,$$

which gives

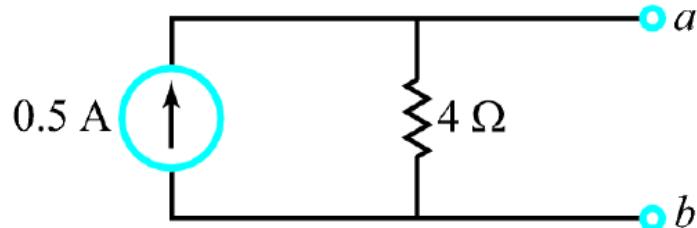
$$I = 1.5 \text{ A},$$

$$I_1 = 2 + 3I - I = 2 + 2I = 5 \text{ A},$$

$$I_{sc} = 5 - 3I = 5 - 4.5 = 0.5 \text{ A}.$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{2}{0.5} = 4 \Omega.$$

Norton circuit is:



Exercise 3-14 The bridge circuit of Fig. E3-14 is connected to a load R_L between terminals (a, b) . Choose R_L such that maximum power is delivered to R_L . If $R = 3 \Omega$, how much power is delivered to R_L ?

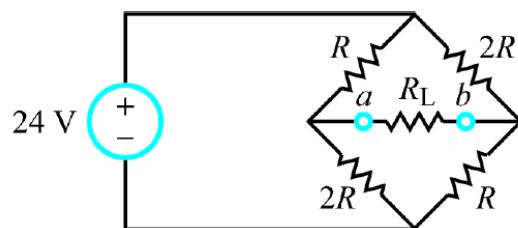
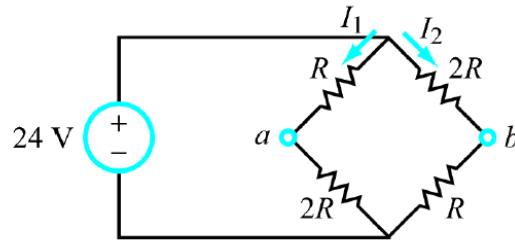


Figure E3-14

Solution: We need to remove R_L and then determine the Thévenin equivalent circuit at terminals (a, b) .
Open-circuit voltage:

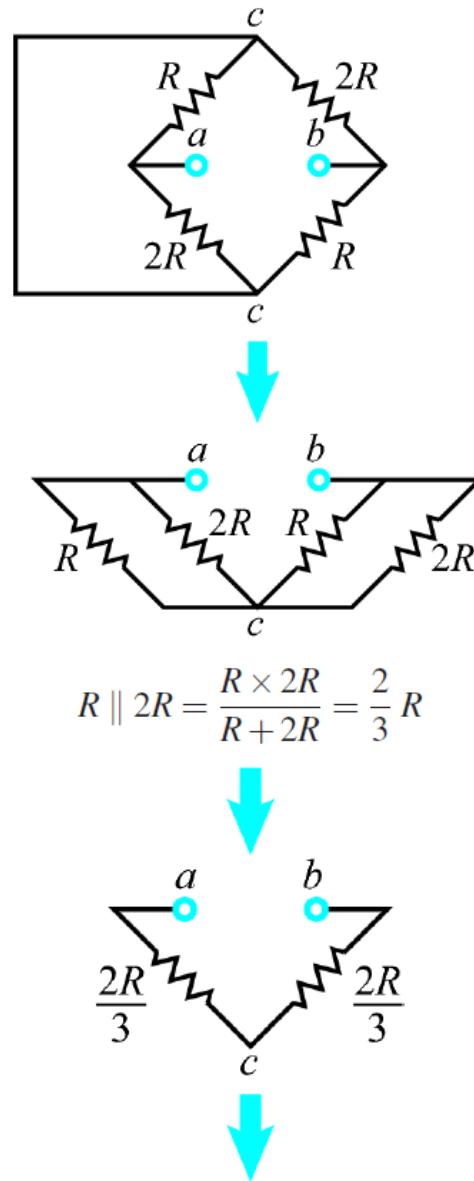


The two branches are balanced (contain same total resistance of $3R$). Hence, identical currents will flow, namely

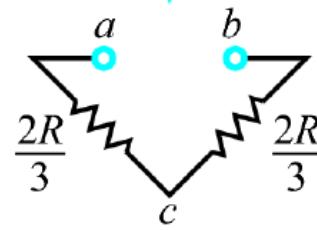
$$I_1 = I_2 = \frac{24}{3R} = \frac{8}{R}.$$

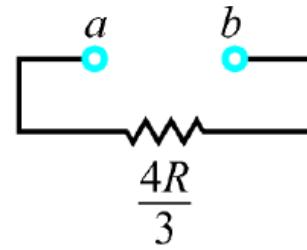
$$V_{oc} = V_a - V_b = 2RI_1 - RI_2 = RI_1 = R \frac{8}{R} = 8 \text{ V}.$$

To find R_{Th} , we replace the source with a short circuit:



$$R \parallel 2R = \frac{R \times 2R}{R+2R} = \frac{2}{3} R$$

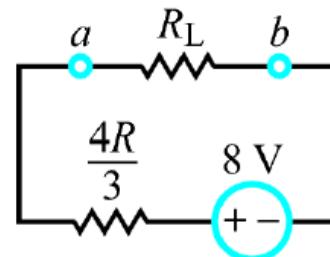




Hence,

$$R_{\text{Th}} = \frac{4R}{3},$$

and the Thévenin circuit is



For maximum power transfer with $R = 3 \Omega$, R_L should be

$$R_L = \frac{4R}{3} = \frac{4 \times 3}{3} = 4 \Omega,$$

and

$$P_{\max} = \frac{v_s^2}{4R_L} = \frac{8^2}{4 \times 4} = 4 \text{ W}.$$

Exercise 5-8 Determine the current i in the circuit of Fig. E5-8, under dc conditions.

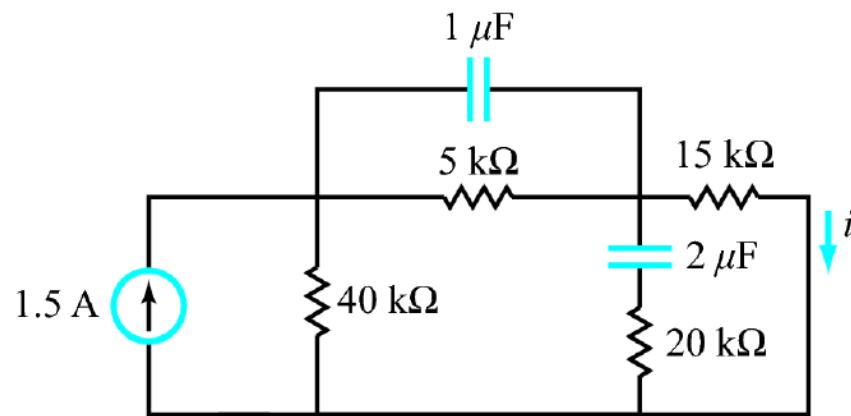
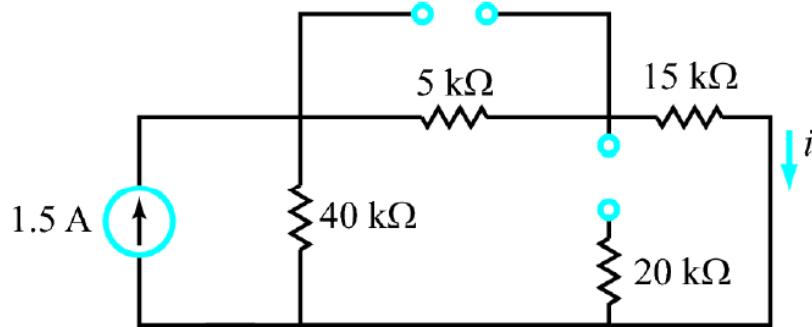


Figure E5-8

Solution: Under dc conditions, capacitors act like open circuits. Hence, the circuit becomes:



Voltage division gives

$$i = 1.5 \times \frac{40k}{40k + 15k + 5k} = 1 \text{ A.}$$

Exercise 5-9 Determine C_{eq} and $V_{\text{eq}}(0)$ at terminals (a,b) for the circuit in Fig. E5-9, given that $C_1 = 6 \mu\text{F}$, $C_2 = 4 \mu\text{F}$ and $C_3 = 8 \mu\text{F}$, and the initial voltages on the three capacitors are $v_1(0) = 5 \text{ V}$ and $v_2(0) = v_3(0) = 10 \text{ V}$.

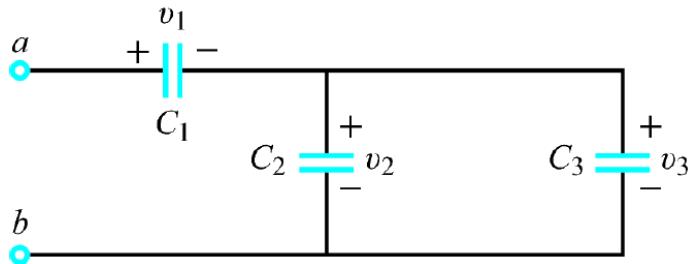


Figure E5-9

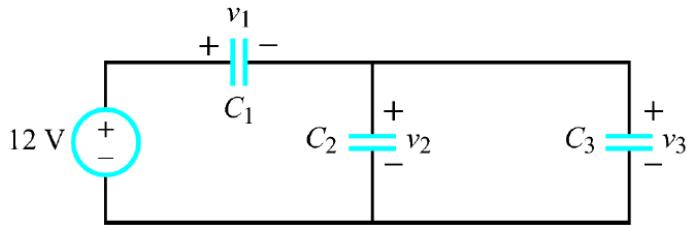
Solution:

$$\begin{aligned} C_{\text{eq}} &= \frac{C_1(C_2 \parallel C_3)}{C_1 + C_2 + C_3} \\ &= \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3} \\ &= \frac{6 \times 10^{-6}(4 \times 10^{-6} + 8 \times 10^{-6})}{(6 + 4 + 8) \times 10^{-6}} = 4 \mu\text{F}, \end{aligned}$$

$$V_{\text{eq}}(0) = v_1(0) + v_2(0) = 5 + 10 = 15 \text{ V.}$$

Exercise 5-10 Suppose the circuit of Fig. E5-9 is connected to a dc voltage source $V_0 = 12$ V. Assuming that the capacitors had no charge before they were connected to the voltage source, determine v_1 and v_2 , given that $C_1 = 6 \mu\text{F}$, $C_2 = 4 \mu\text{F}$, and $C_3 = 8 \mu\text{F}$.

Solution:



According to Eq. (5.46),

$$C_1 v_1 = (C_2 \parallel C_3) v_2,$$

or

$$v_2 = \frac{C_1 v_1}{C_2 + C_3} = \frac{6 \times 10^{-6}}{4 \times 10^{-6} + 8 \times 10^{-6}} v_1 = \frac{v_1}{2}.$$

But

$$v_1 + v_2 = 12 \text{ V}.$$

Hence,

$$v_1 = 8 \text{ V} \quad \text{and} \quad v_2 = 4 \text{ V}.$$

Exercise 5-11 Calculate the inductance of a 20-turn air-core solenoid if its length is 4 cm and the radius of its circular cross section is 0.5 cm.

Solution:

$$\begin{aligned} L &= \frac{\mu N^2 S}{\ell} = \frac{4\pi \times 10^{-7} \times 20^2 \times \pi(0.005)^2}{0.04} \\ &= 0.987 \mu\text{H}. \end{aligned}$$

Exercise 5-12 Determine currents i_1 and i_2 in the circuit of Fig. E5-12, under dc conditions.

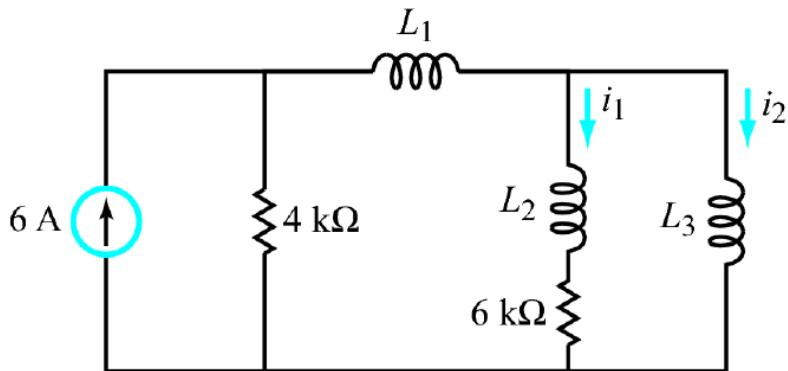
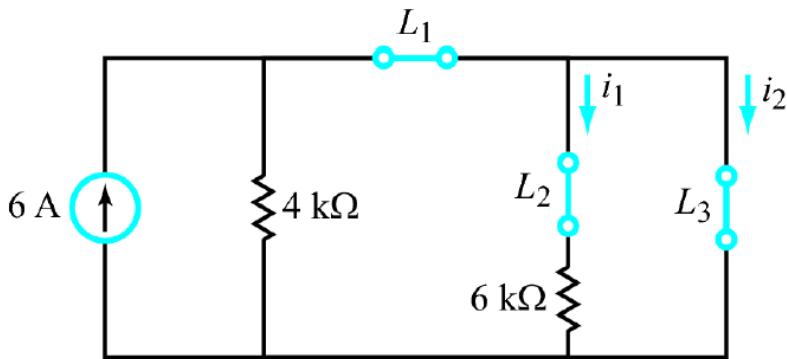


Figure E5-12

Solution: Under dc conditions, inductors act like short circuits.



The 6-A current will flow entirely through the short circuit representing L_3 . Hence,

$$i_1 = 0, \quad i_2 = 6 \text{ A}.$$

Exercise 5-13 Determine L_{eq} at terminals (a, b) in the circuit of Fig. E5-13.

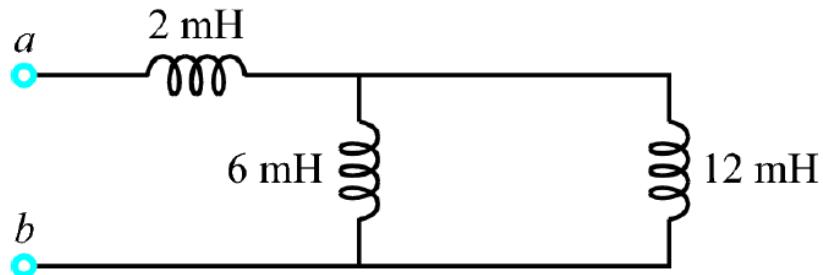


Figure E5-13

Solution:

$$\begin{aligned} L_{\text{eq}} &= 2 \text{ mH} + (6 \text{ mH} \parallel 12 \text{ mH}) \\ &= \left(2 + \frac{6 \times 12}{6+12} \right) \text{ mH} \\ &= 6 \text{ mH}. \end{aligned}$$

Exercise 5-14 If in the circuit of Fig. E5-14, $v(0^-) = 24 \text{ V}$, determine $v(t)$ for $t \geq 0$.

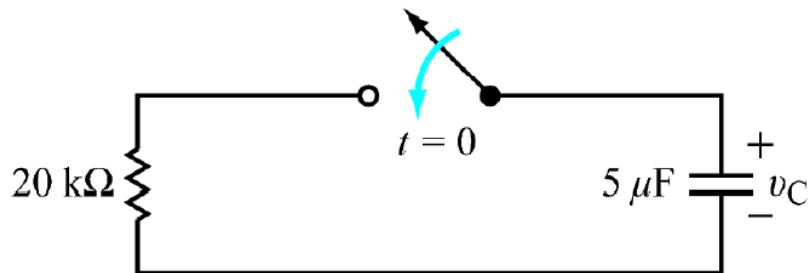


Figure E5-14

Solution:

$$\begin{aligned} v(t) &= v(0) e^{-t/\tau} \\ &= v(0) e^{-t/RC} \\ &= 24e^{-10t} \text{ V}, \quad \text{for } t \geq 0. \end{aligned}$$

Exercise 5-15 Determine $v_1(t)$ and $v_2(t)$ for $t \geq 0$, given that in the circuit of Fig. E5-15 $C_1 = 6 \mu\text{F}$, $C_2 = 3 \mu\text{F}$, $R = 100 \text{ k}\Omega$, and neither capacitor had any charge prior to $t = 0$.

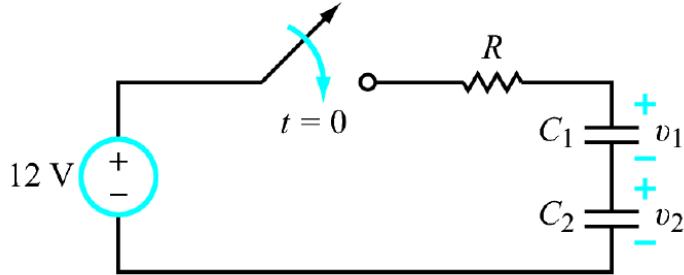


Figure E5-15

Solution:

$$v_1(0) = v_2(0) = 0 \quad [\text{given}]$$

$v_1(\infty) + v_2(\infty) = 12 \text{ V} \quad [\text{At } t = \infty, \text{ capacitors act like open circuits}]$

$$C_1 v_1(\infty) = C_2 v_2(\infty), \quad [\text{Eq. (5.46)}].$$

Hence,

$$C_1 v_1(\infty) = C_2 [12 - v_1(\infty)],$$

which leads to

$$v_1(\infty) = 12 \frac{C_2}{C_1 + C_2} = 4 \text{ V},$$

$$v_2(\infty) = 12 - 4 = 8 \text{ V}.$$

Also,

$$\tau = RC_{\text{eq}} = R \frac{C_1 C_2}{C_1 + C_2} = 0.2 \text{ s}.$$

Hence,

$$\begin{aligned} v_1(t) &= v_1(\infty) + [v_1(0) - v_1(\infty)] e^{-t/\tau} \\ &= 4(1 - e^{-5t}) \text{ V}, \quad \text{for } t \geq 0, \\ v_2(t) &= v_2(\infty) + [v_2(0) - v_2(\infty)] e^{-t/\tau} \\ &= 8(1 - e^{-5t}) \text{ V}, \quad \text{for } t \geq 0. \end{aligned}$$

Exercise 5-16 Determine $i_1(t)$ and $i_2(t)$ for $t \geq 0$, given that in the circuit of Fig. E5-16 $L_1 = 6 \text{ mH}$, $L_2 = 12 \text{ mH}$, and $R = 2 \Omega$. Assume $i_1(0^-) = i_2(0^-) = 0$.

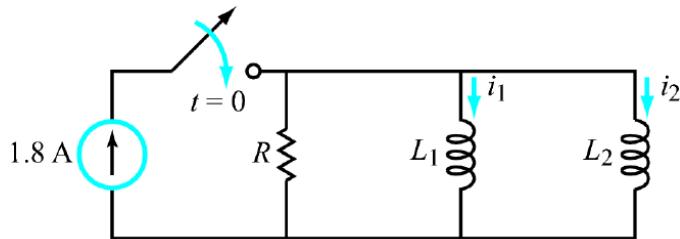


Figure E5-16

Solution:

$$\begin{aligned}
 i_1(t) &= \frac{1}{L_1} \int_0^t v(t) dt \\
 &= \frac{1.8R}{L_1} \int_0^t e^{-500t} dt \\
 &= \frac{1.8R}{L_1} \left[\frac{e^{-500t}}{-500} \right]_0^t \\
 &= \frac{1.8 \times 2}{500L_1} (1 - e^{-500t}) \\
 &= 1.2(1 - e^{-500t}) u(t) \text{ A,}
 \end{aligned}$$

$$\begin{aligned}
 i_2(t) &= \frac{1}{L_2} \int_0^t v(t) dt \\
 &= \frac{1}{12 \times 10^{-3}} \int_0^t v(t) dt \\
 &= 0.6(1 - e^{-500t}) u(t) \text{ A.}
 \end{aligned}$$

Exercise 5-17 The input signal to an ideal integrator circuit with $RC = 2 \times 10^{-3}$ s and $V_{cc} = 15$ V is given by $v_s(t) = 2 \sin 100t$ V. What is $v_{out}(t)$?

Solution:

$$v_{out}(t) = -\frac{1}{RC} \int_{t_0}^t v_i dt + v_{out}(t_0).$$

Assuming the integration started at $t_0 = 0$ at which time $v_{out}(0) = 0$,

$$\begin{aligned}
 v_{out}(t) &= -\frac{1}{2 \times 10^{-3}} \int_0^t 2 \sin 100t dt \\
 &= \frac{2}{2 \times 10^{-3} \times 100} \cos(100t)|_0^t \\
 &= 10[\cos(100t) - 1] \text{ V.}
 \end{aligned}$$

Exercise 5-18 Repeat Exercise 5-17 for a differentiator instead of an integrator.

Solution:

$$\begin{aligned}
 v_{out}(t) &= -RC \frac{d v_i}{dt} \\
 &= -2 \times 10^{-3} \frac{d}{dt}[2 \sin(100t)] \\
 &= -0.4 \cos(100t) \text{ V.}
 \end{aligned}$$

Exercise 6-1 For the circuit in Fig. E6-1, determine $v_C(0)$, $i_L(0)$, $v_L(0)$, $i_C(0)$, $v_C(\infty)$, $i_L(\infty)$.

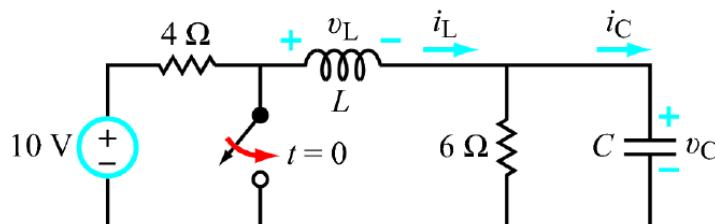
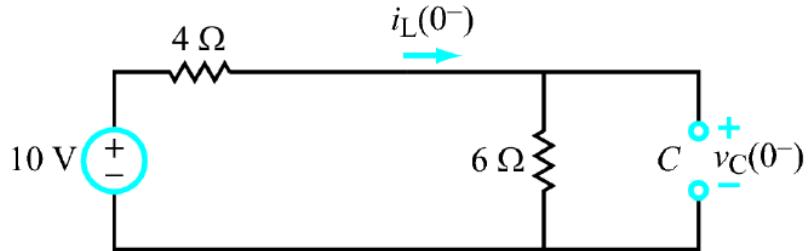


Figure E6.1

Solution:

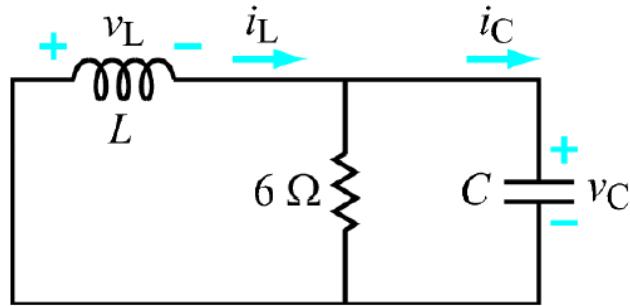
Before $t = 0$:



$$v_C(0) = v_C(0^-) = \frac{6}{4+6} 10 = 6 \text{ V},$$

$$i_L(0) = i_L(0^-) = \frac{10}{4+6} = 1 \text{ A}.$$

After $t = 0$:



$$v_L(0) = -v_C(0) = -6 \text{ V},$$

$$i_C(0) = i_L(0) - \frac{v_C(0)}{6} = 0 \text{ A},$$

$v_C(\infty) = 0 \text{ V}$ (no sources and closed loop access to resistors),

$i_L(\infty) = 0 \text{ A}$ (no sources and closed loop access to resistors).

Exercise 6-2 For the circuit in Fig. E6-2, determine $v_C(0)$, $i_L(0)$, $v_L(0)$, $i_C(0)$, $v_C(\infty)$, and $i_L(\infty)$.

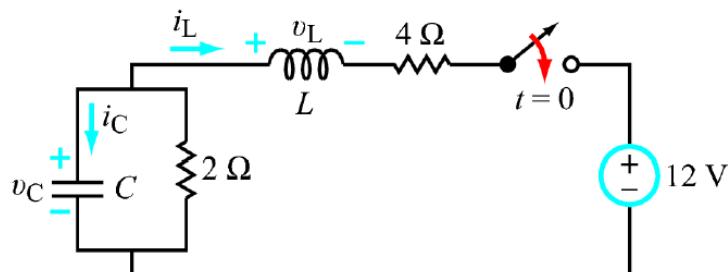
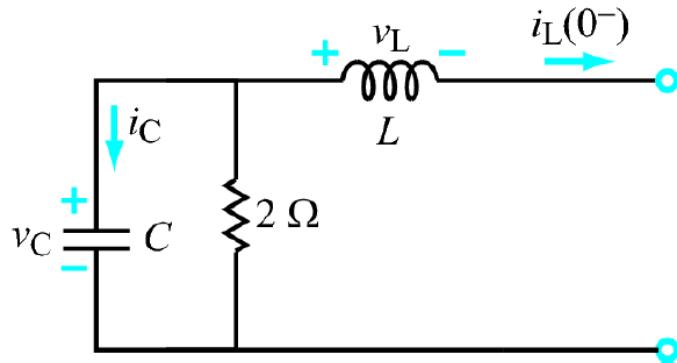


Figure E6.2

Solution:

Before $t = 0$:

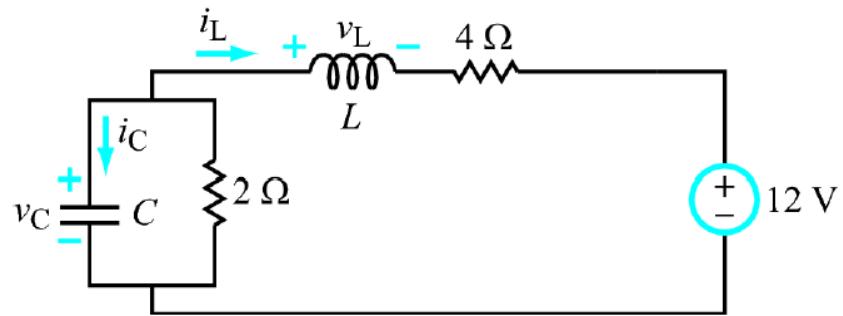


Hence:

$$v_C(0) = v_C(0^-) = 0 \text{ V} \text{ (no sources and closed loop access to resistors),}$$

$$i_L(0) = i_L(0^-) = 0 \text{ A.}$$

After $t = 0$:

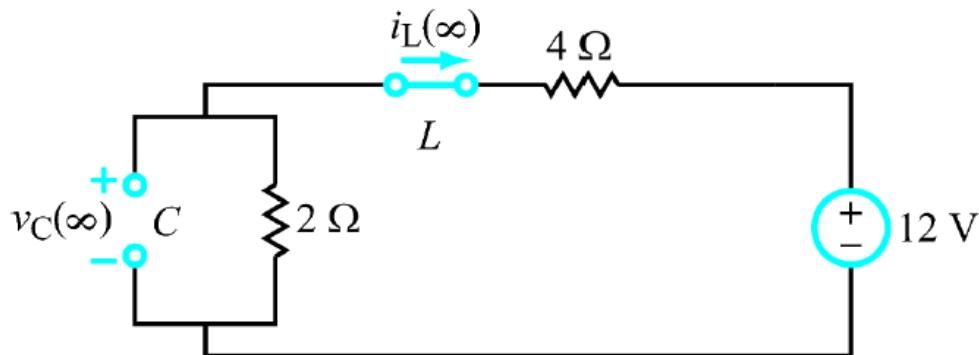


$$v_L(0) = v_C(0) - i_L(0) \times 4 - 12 = -12 \text{ V},$$

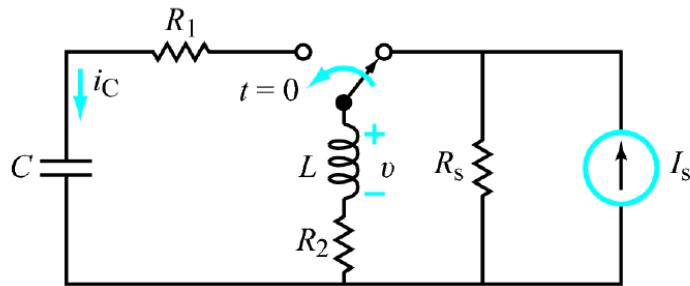
$$i_C(0) = \frac{v_C(0)}{2} = 0 \text{ A,}$$

$$v_C(\infty) = \frac{2}{2+4} 12 \text{ V} = 4 \text{ V,}$$

$$i_L(\infty) = \frac{v_C(\infty) - 12}{4} = -2 \text{ A.}$$

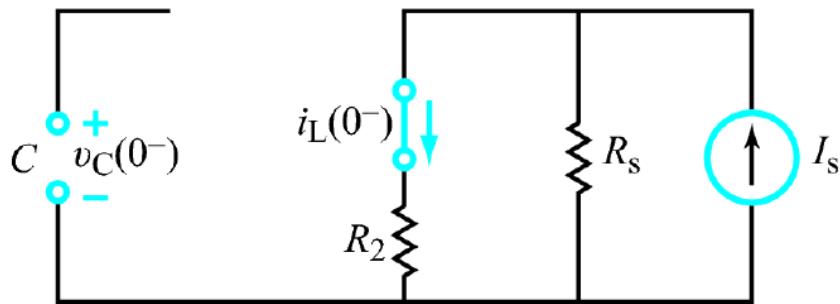


Exercise 6-3 After interchanging the locations of L and C in Fig. 6-9(a), repeat Example 6-4 to determine $v_c(t)$ across C .



Solution:

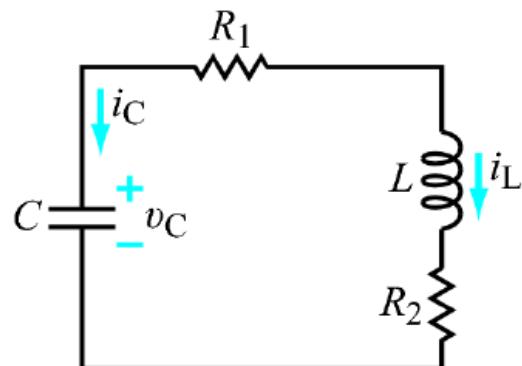
Before $t = 0$:



$v_C(0) = v_C(0^-) = 0 \text{ V}$ (assume capacitor initially uncharged),

$$i_L(0) = i_L(0^-) = \frac{R_s}{R_2 + R_s} I_s = \frac{10}{0.2 + 10} 2 = 1.961 \text{ A.}$$

After $t = 0$:



$$i_C(0) = -i_L(0) = -1.961 \text{ A},$$

$$v'_C(0) = \frac{i_C(0)}{C} = -\frac{1.961}{5 \times 10^3} = -392.2 \text{ V/s},$$

$$R = R_1 + R_2 = 2.01 \Omega.$$

Since R , L , and C are the same as in Example 6-4:

$$\alpha = \frac{R}{2L} = 201 \text{ Np/s},$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 200 \text{ rad/s},$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -181 \text{ Np/s},$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -221 \text{ Np/s}.$$

Apply new initial conditions:

$$v_C(0) = A_1 + A_2 = 0,$$

$$v'_C(0) = s_1 A_1 + s_2 A_2 = -392.2,$$

which leads to

$$A_1 = -\frac{392.2}{s_1 - s_2} = -\frac{392.2}{-181 - (-221)} = -9.79 \text{ V},$$

$$A_2 = -A_1 = 9.79 \text{ V},$$

$$v_C(t) = (A_1 e^{s_1 t} + A_2 e^{s_2 t}) u(t),$$

$$v_C(t) = 9.79(e^{-221t} - e^{-181t}) u(t) \text{ V}.$$

Exercise 6-4 The switch in Fig. E6.4 is moved to position 2 after it had been in position 1 for a long time. Determine: (a) $v_C(0)$ and $i_C(0)$, and (b) $i_C(t)$ for $t \geq 0$.

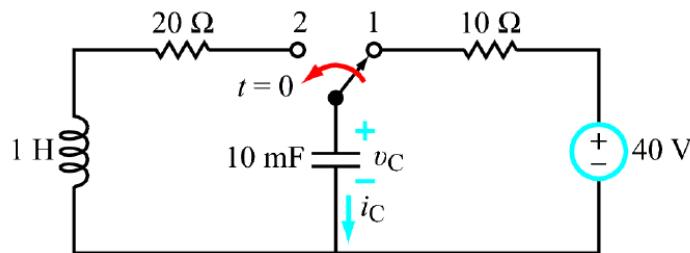
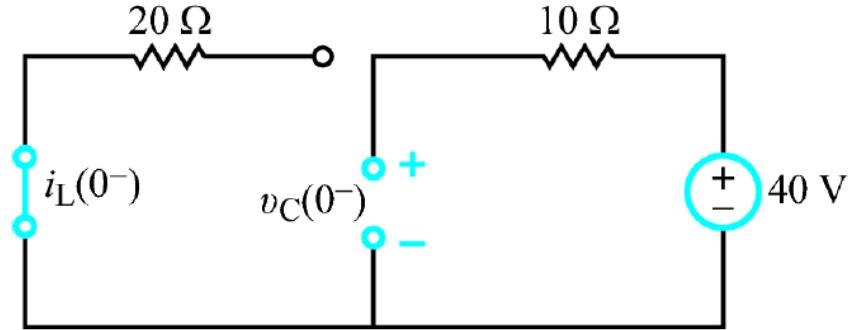


Figure E6.4

Solution:

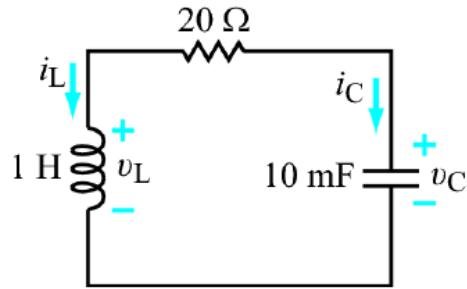
Before $t = 0$:



$$v_C(0) = v_C(0^-) = 40 \text{ V},$$

$$i_L(0) = i_L(0^-) = 0 \text{ A}.$$

After $t = 0$:



$$i_C(0) = -i_L(0) = 0 \text{ A},$$

$$v'_C(0) = \frac{i_C(0)}{C} = 0,$$

$$\alpha = \frac{R}{2L} = \frac{20}{2 \times 1} = 10,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 0.01}} = 10.$$

Since $\alpha = \omega_0$, the circuit is critically damped. Apply initial conditions:

$$v_C(t) = (B_1 + B_2 t) e^{-\alpha t} u(t),$$

$$v'_C(t) = [-(B_1 + B_2 t) \alpha e^{-\alpha t} + B_2 e^{-\alpha t}] u(t),$$

$$v'_C(t) = [(1 - \alpha t) B_2 - \alpha B_1] e^{-\alpha t} u(t),$$

$$v_C(0) = B_1,$$

$$B_1 = v_C(0) = 40,$$

$$v'_C(0) = B_2 - \alpha B_1,$$

$$B_2 = v'_C(0) + \alpha B_1,$$

$$B_2 = 0 + 10 \times 40 = 400,$$

$$i_C(t) = C v'_C(t),$$

$$= C[(1 - \alpha t) B_2 - \alpha B_1] e^{-\alpha t} u(t),$$

$$= 0.01[(1 - 10t)400 - 10 \times 40] e^{-10t} u(t),$$

$$= -40t e^{-10t} u(t) \text{ A}.$$

Exercise 6-5 The circuit in Fig. E6.5 is a replica of the circuit in Fig. E6.4, but with the capacitor and inductor interchanged in location. Determine: (a) $i_L(0)$ and $v_L(0)$, and (b) $i_L(t)$ for $t \geq 0$.

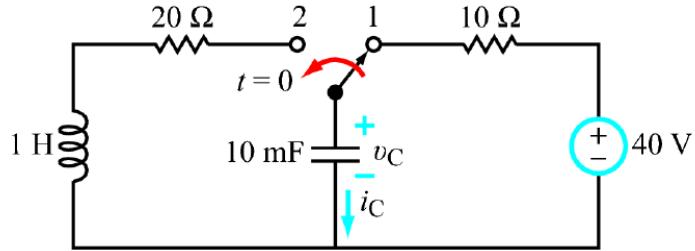
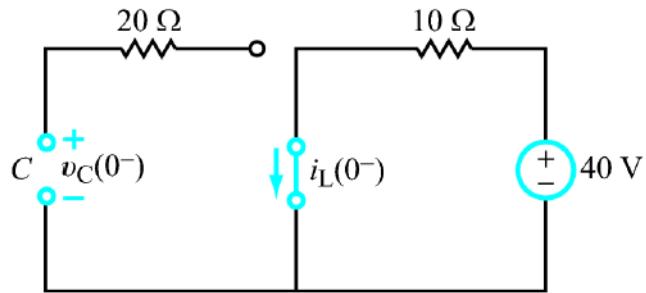


Figure E6.5

Solution:

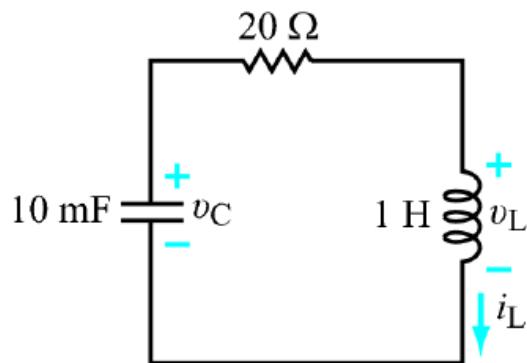
Before $t = 0$:



$$v_C(0) = v_C(0^-) = 0 \text{ V},$$

$$i_L(0) = i_L(0^-) = \frac{40}{10} = 4 \text{ A}.$$

After $t = 0$:



Since the capacitor is initially a short circuit:

$$v_L(0) = -20i_L(0) = -20 \times 4 = -80 \text{ V},$$

$$\alpha = \frac{R}{2L} = \frac{20}{2 \times 1} = 10,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 0.01}} = 10.$$

The circuit is critically damped.

$$i_L(t) = (B_1 + B_2 t)e^{-\alpha t} u(t),$$

$$i'_L(t) = [(1 - \alpha t)B_2 - \alpha B_1]e^{-\alpha t} u(t),$$

$$B_1 = i_L(0) = 4,$$

$$B_2 = i'_L(0) + \alpha B_1 = \frac{v_L(0)}{L} + \alpha B_1,$$

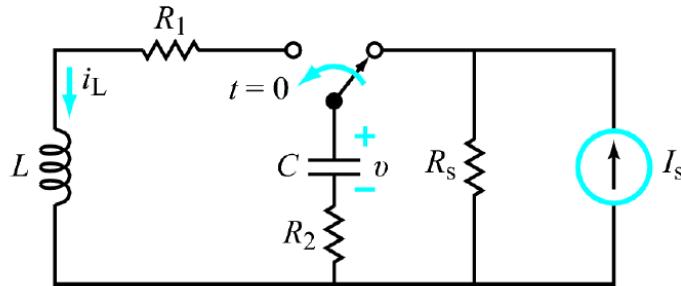
$$= -\frac{80}{1} + 10 \times 4 = -40.$$

Hence

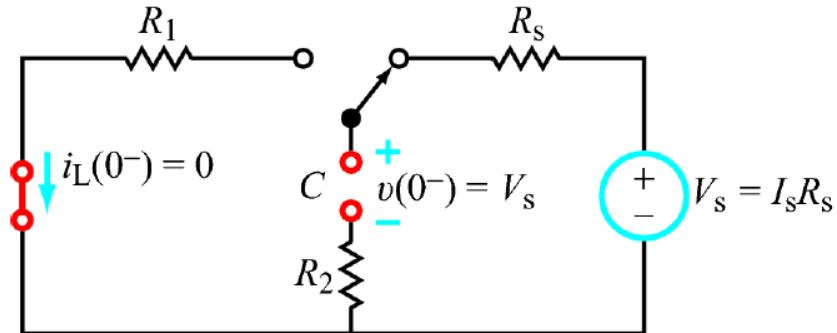
$$i_L(t) = 4(1 - 10t)e^{-10t} u(t) \text{ A.}$$

Exercise 6-6 Repeat Example 6-4 after replacing the 8 V source with a short circuit and changing the value of R_1 to 1.7Ω .

Solution:



Before $t = 0$:

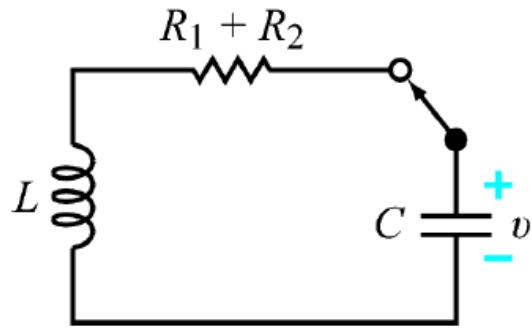


where we used source transformation on (I_s, R_s) . From the circuit

$$v(0) = v(0^-) = I_s R_s = 20 \text{ V},$$

$$i_L(0) = i_L(0^-) = 0 \text{ A.}$$

After $t = 0$:



$$v'(0) = \frac{i_C(0)}{C} = -\frac{i_L(0)}{C} = 0,$$

$$\alpha = \frac{R}{2L} = \frac{R_1 + R_2}{2L} = \frac{1.7 + 0.2}{2 \times 0.005} = 190,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.005 \times 0.005}} = 200,$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{200^2 - 190^2} = 62.45,$$

$$v(t) = (D_1 \cos \omega_d t + D_2 \sin \omega_d t) e^{-\alpha t} u(t),$$

$$v(0) = D_1,$$

$$D_1 = v(0) = 20,$$

$$D_2 = \frac{\alpha v(0)}{\omega_d} = \frac{190 \times 20}{62.45} = 60.85,$$

$$v(t) = (20 \cos 62.45t + 60.85 \sin 62.45t) e^{-190t} u(t) \text{ V.}$$

Exercise 6-7 Determine the initial and final values for i_L in the circuit of Fig. E6.7, and provide an expression for $i_L(t)$.

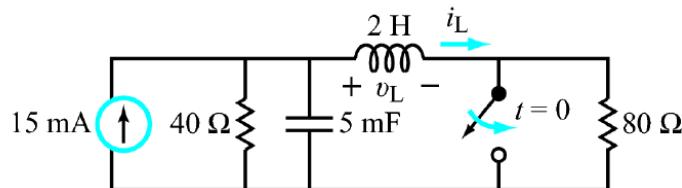
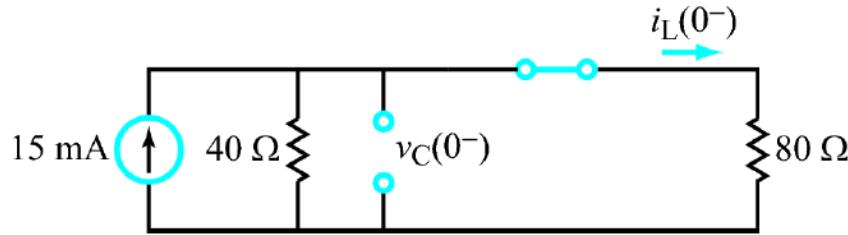


Figure E6.7

Solution:

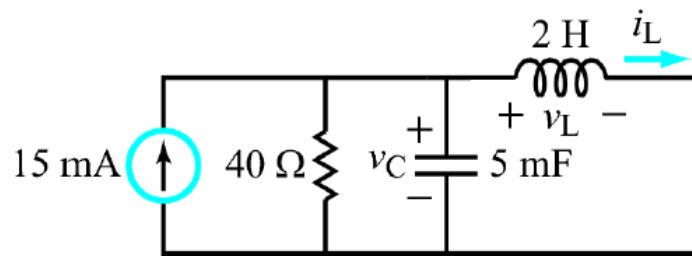
Before $t = 0$:



$$v_C(0) = 0.015(40 \Omega \parallel 80 \Omega) = 0.015 \frac{40 \times 80}{40 + 80} = 0.4,$$

$$i_L(0) = \frac{v_C(0)}{80} = \frac{0.4}{80} = 0.005.$$

After $t = 0$:



$$i'_L(0) = \frac{v_L(0)}{L} = \frac{v_C(0)}{L} = \frac{0.4}{2} = 0.2,$$

$$i_L(\infty) = 0.015 \quad (L \text{ acts like a short circuit at } t = \infty)$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 40 \times 0.005} = 2.5,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 0.005}} = 10 \text{ rad/s},$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9.68 \text{ rad/s.}$$

$$i_L(t) = [i_L(\infty) + e^{-\alpha t} (D_1 \cos \omega_d t + D_2 \sin \omega_d t)],$$

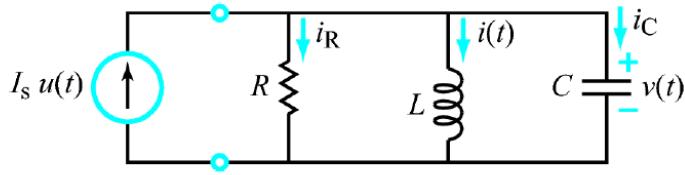
$$D_1 = i_L(0) - i_L(\infty) = 0.005 - 0.015 = -0.010,$$

$$D_2 = \frac{i'_L + \alpha[i_L(0) - i_L(\infty)]}{\omega_d} = 0.01808.$$

Hence,

$$i_L(t) = \{15 - [10 \cos 9.68t - 18.08 \sin 9.68t]e^{-2.5t}\} \text{ mA.}$$

Exercise 6-8 In the parallel RLC circuit shown in Fig. 6-13(b), how much energy will be stored in L and C at $t = \infty$?



Solution: At $t = \infty$, L is a short circuit:

$$\begin{aligned} v_C(\infty) &= 0, \\ w_C(\infty) &= 0, \\ i_L(\infty) &= I_s, \\ w_L(\infty) &= \frac{1}{2} L i_L^2(\infty) = \frac{1}{2} L I_s^2. \end{aligned}$$

Exercise 6.9 Develop an expression for $i_C(t)$ in the circuit of Fig. E6.14 for $t \geq 0$.

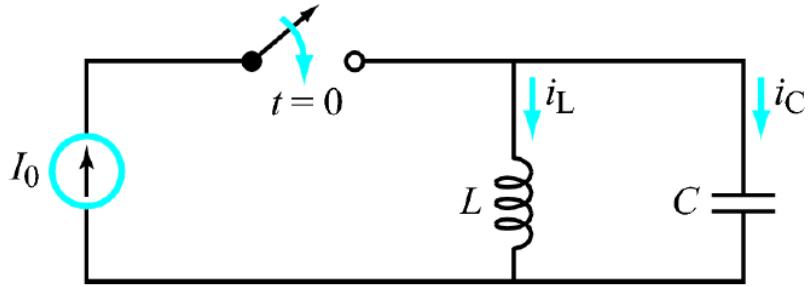


Figure E6.9

Solution:

Before $t = 0$:

$$\begin{aligned} v_C(0) &= 0, \\ i_L(0) &= 0. \end{aligned}$$

After $t = 0$:

$$\begin{aligned} i'_L(0) &= \frac{v_L(0)}{L} = \frac{v_C(0)}{L} = 0, \\ i_L(\infty) &= I_0, \\ \alpha &= \frac{1}{2RC} = \frac{1}{2 \times \infty \times C} = 0, \\ \omega_0 &= \frac{1}{\sqrt{LC}}. \end{aligned}$$

Since α is less than ω_0 , the circuit is underdamped:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0,$$

$$D_1 = i_L(0) - i_L(\infty) = 0 - I_0 = -I_0,$$

$$D_2 = \frac{i'_L(0) + \alpha D_1}{\omega_d} = \frac{0 - 0 \times I_0}{\omega_0} = 0,$$

$$i_L(t) = i_L(\infty) + [D_1 \cos \omega_d t + D_2 \sin \omega_d t] e^{-\alpha t},$$

$$i_L(t) = I_0 - I_0 \cos \omega_0 t = I_0(1 - \cos \omega_0 t),$$

$$\begin{aligned} i_C(t) &= I_0 - i_L(t) = I_0 - (I_0 - I_0 \cos \omega_0 t) \\ &= I_0 \cos \omega_0 t. \end{aligned}$$

Hence, without a resistor in the circuit, the circuit behaves like an oscillator.

Exercise 6.10 For the circuit in Fig. E6.10, determine $i_C(t)$ for $t \geq 0$.

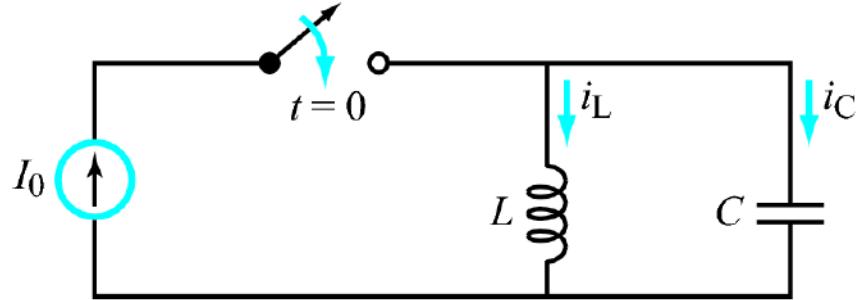


Figure E6.10

Solution:

Before $t = 0$, there are no sources:

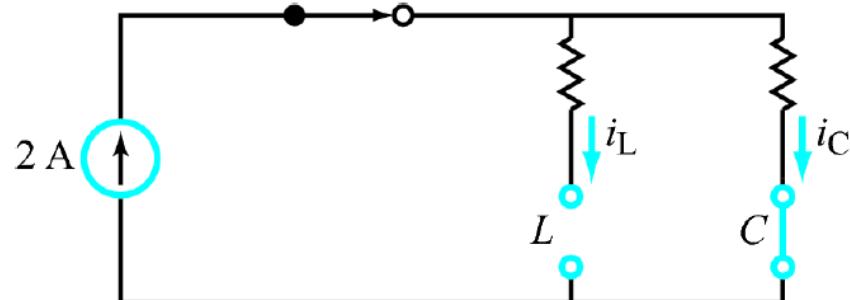
$$v_C(0) = 0,$$

$$i_L(0) = 0.$$

At $t = 0$:

$$i_C(0) = 2 - i_L(0) = 2,$$

$$i'_L(0) = \frac{v_L(0)}{L} = \frac{R i_C(0)}{L} = \frac{3 \times 2}{2} = 3.$$



After $t = 0$:

$$R i_L(t) + L \frac{di_L}{dt} = R i_C(t) + v_C(t),$$

$$i_L(t) = 2 - i_C(t),$$

$$\frac{di_L}{dt} = -\frac{di_C}{dt},$$

$$R[2 - i_C(t)] - L \frac{di_C}{dt} = R i_C(t) + v_C(t),$$

$$2R - L \frac{di_C}{dt} = 2R i_C(t) = v_C(t),$$

$$-LC \frac{d^2 i_C}{dt^2} = 2RC \frac{di_C}{dt} + C \frac{dv_C}{dt},$$

$$\frac{d^2 i_C}{dt^2} + \frac{2R}{L} \frac{di_C}{dt} + \frac{1}{LC} i_C(t) = 0,$$

$$i_C'' + \frac{2R}{L} i_C' + \frac{1}{LC} i_C = 0,$$

$$i_C'(0) = -i_L'(0) = -3,$$

$$i_C(\infty) = 0,$$

$$a = \frac{2R}{L} = \frac{2 \times 3}{2} = 3,$$

$$b = \frac{1}{LC} = \frac{1}{2 \times 0.02} = 25,$$

$$\alpha = \frac{a}{2} = 1.5,$$

$$\omega_0 = \sqrt{b} = 5.$$

The circuit is underdamped:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{25 - 1.5^2} = 4.77,$$

$$D_1 = i_C(0) - i_C(\infty) = 2 - 0 = 2,$$

$$D_2 = \frac{i_C'(0) + \alpha D_1}{\omega_d} = \frac{-3 + 1.5 \times 2}{4.77} = 0,$$

$$i_C(t) = [i_C(\infty) + (D_1 \cos \omega_d t + D_2 \sin \omega_d t) e^{-\alpha t}] u(t),$$

$$i_C(t) = (2e^{-\alpha t} \cos 4.77t) u(t) \text{ A.}$$