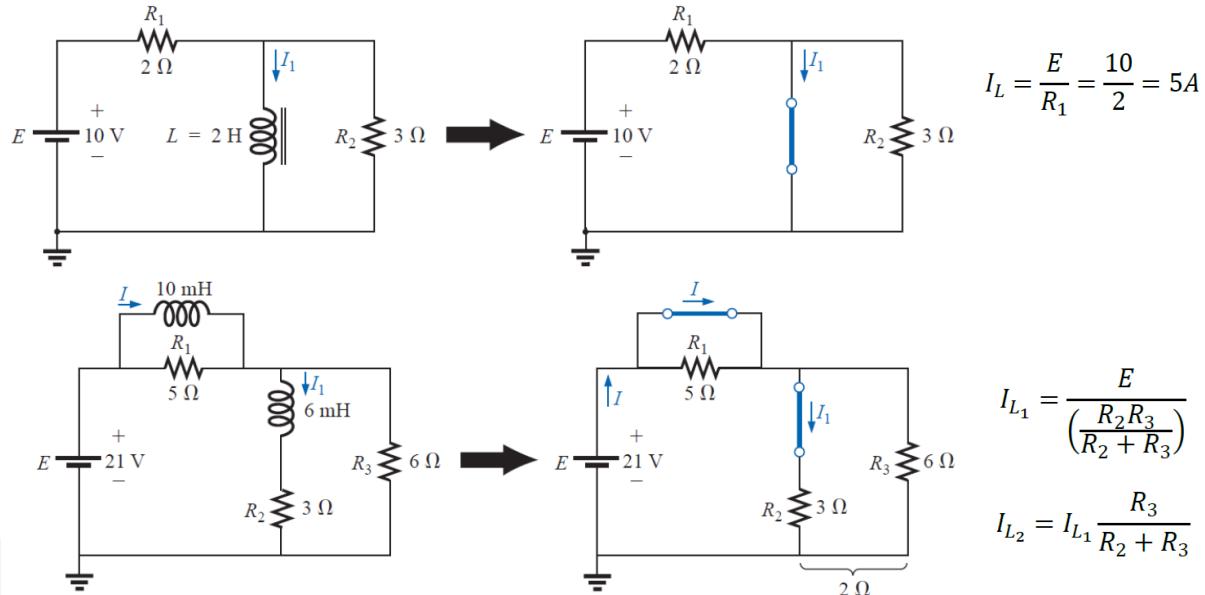


Basic RL and RC Circuits

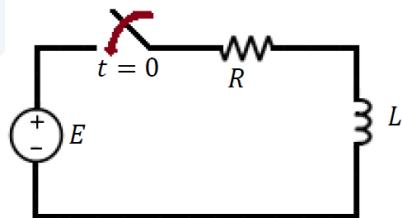
The RL circuit with D.C (steady state)

The inductor is short time at $t = \infty$

Calculate the inductor current for circuits shown below.



R-L TRANSIENTS: STORAGE CYCLE



$$-E + Ri + L \frac{di}{dt} = 0$$

$$Ri + L \frac{di}{dt} = E$$

$$L \frac{di}{dt} = E - Ri$$

$$L di = (E - Ri) dt$$

$$\frac{L di}{(E - Ri)} = dt$$

$$\int \frac{L di}{(E - Ri)} = \int dt$$

$$-\frac{L}{R} \ln(E - Ri) = t + k$$

at $t = 0, i = 0$, therefore $-\frac{L}{R} \ln(E) = k$

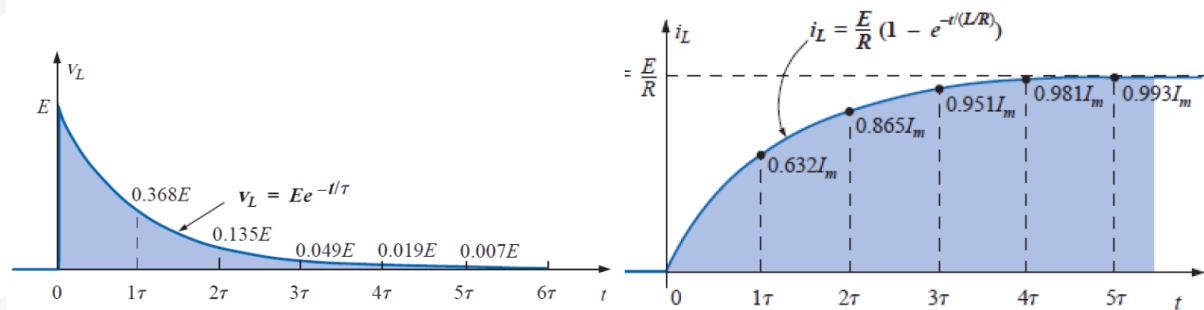
And

$$\begin{aligned}
 \frac{L}{R} \ln(E - Ri) &= t - \frac{L}{R} \ln(E) \\
 -\frac{L}{R} \ln(E - Ri) + \frac{L}{R} \ln(E) &= t \\
 -\frac{L}{R} \left(\ln \left(\frac{E - Ri}{E} \right) \right) &= t \\
 \frac{E - Ri}{E} &= e^{-\frac{R}{L}t} \\
 i &= \frac{E}{R} \left(1 - e^{-\frac{R}{L}t} \right) = \frac{E}{R} \left(1 - e^{-\frac{t}{\tau}} \right)
 \end{aligned}$$

$$\tau = \frac{L}{R} \quad (\text{seconds, s})$$

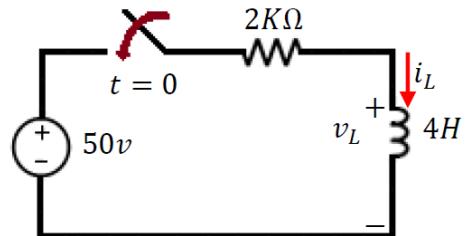
$$i_L = I_m(1 - e^{-t/\tau}) = \frac{E}{R}(1 - e^{-t/(L/R)})$$

$$V_L = Ee^{-t/\tau}$$



Example:

Find the mathematical expression for the transient behaviour of i_L and v_L .



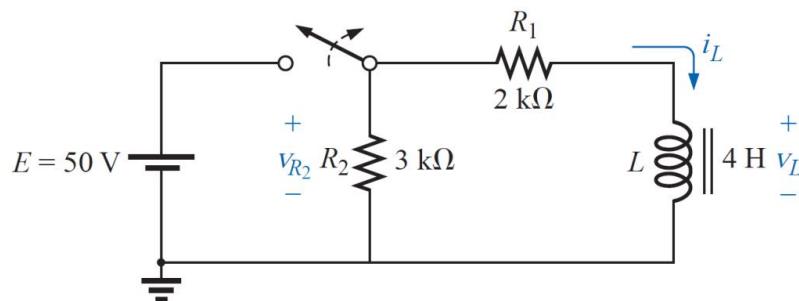
$$\tau = \frac{L}{R} = \frac{4}{2 \times 10^3} = 2ms$$

$$i_L = \frac{E}{R} \left(1 - e^{-\frac{t}{\tau}} \right) = \frac{50}{2 \times 10^3} (1 - e^{-500t}) = 25(1 - e^{-500t})mA$$

$$v_L = E e^{-\frac{t}{\tau}} = 50 e^{-500t} V$$

Example:

For the circuit shown below, calculate the mathematical expression of i_L , v_L , v_{R_1} , v_{R_2} before and after the storage phase has been complete and the switch is open.



1-switch on

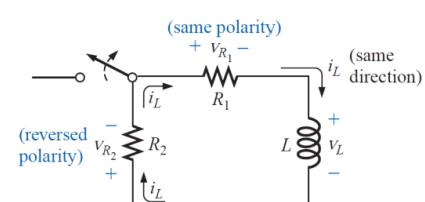
$$\tau = \frac{L}{R_{eq}} = \frac{4}{2 \times 10^3} = 2ms$$

$$i_L = \frac{E}{R_1} \left(1 - e^{-\frac{t}{\tau}} \right) = \frac{50}{2 \times 10^3} (1 - e^{-500t}) = 25(1 - e^{-500t})mA$$

$$v_L = E e^{-\frac{t}{\tau}} = 50 e^{-500t} V$$

$$v_{R_1} = i_L R_1 = \frac{E}{R_1} R_1 \left(1 - e^{-\frac{t}{\tau}} \right) = 50(1 - e^{-500t})V$$

$$v_{R_2} = E = 50 V$$



2-switch off

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to the resistor R_2 , which provides a complete path for the current i_L . The voltage v across the inductor will reverse polarity and have a magnitude determined by

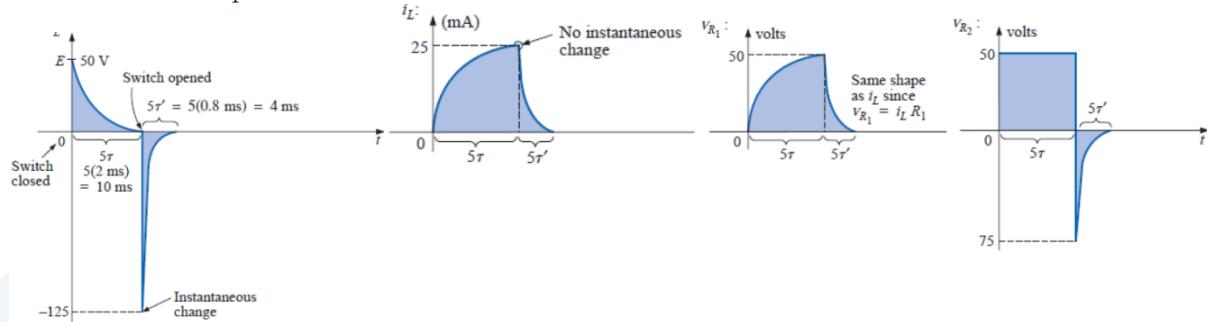
$$\dot{i} = \frac{L}{R_{eq}} = \frac{4}{2 \times 10^3 + 3 \times 10^3} = 0.8 \text{ ms}$$

$$i_L = \frac{E}{R_1} e^{-\frac{t}{\tau}} = \frac{50}{2 \times 10^3} e^{-\frac{t}{\tau}} = 25e^{-\frac{t}{0.8 \times 10^{-3}}} \text{ mA}$$

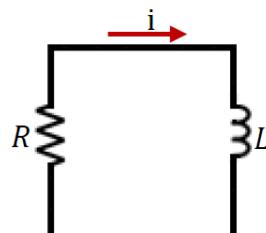
$$v_L = -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2)e^{-\frac{t}{\tau}} = -E \left(1 + \frac{R_2}{R_1}\right) e^{-\frac{t}{0.8 \times 10^{-3}}} = -50 \left(1 + \frac{3}{2}\right) e^{-\frac{t}{0.8 \times 10^{-3}}} = -75e^{-\frac{t}{0.8 \times 10^{-3}}} V$$

$$v_{R_1} = i_L R_1 = \frac{E}{R_1} R_1 e^{-\frac{t}{\tau}} = 50e^{-\frac{t}{0.8 \times 10^{-3}}} v$$

$$v_{R_2} = -i_L R_2 = -\frac{E}{R_1} R_2 e^{-\frac{t}{\tau}} = -\frac{50}{2} 3e^{-\frac{t}{0.8 \times 10^{-3}}} t = -75e^{-\frac{t}{0.8 \times 10^{-3}}} t V$$



The Source Free RL circuit



Using KVL, leads

$$Ri + L \frac{di}{dt} = 0$$

This equation represents a differential equation and can be solved by several different methods

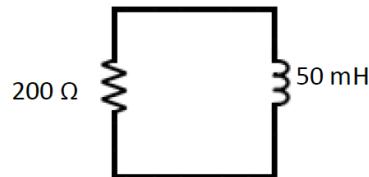
$$\frac{di}{i} = -\frac{R}{L} dt$$

Since the current is I_0 at $t = 0$ and $i(t)$ at time t , we may equate the two definite integrals with are obtained by integrating each side between the corresponding limits

$$\begin{aligned} \int_{I_0}^{i(t)} \frac{di}{i} &= -\frac{R}{L} \int_0^t dt \\ i(t) &= I_0 e^{-\frac{R}{L}t} \\ v(t) &= -v_L e^{-\frac{R}{L}t} \end{aligned}$$

Example:

If the inductor has a current 2A at t=0, find an expression for $i_L(t)$ valid for $t > 0$, and its value at t=200μs.



$$i(t) = I_0 e^{-\frac{R}{L}t} = 2e^{-\frac{200}{50 \times 10^{-3}}t} = 2e^{-4000t} \text{ A}$$

At t=200μs

$$i(200\mu\text{s}) = 2e^{-4000 \times 200 \times 10^{-6}} = 2e^{-0.8} = 898.7 \text{ mA}$$

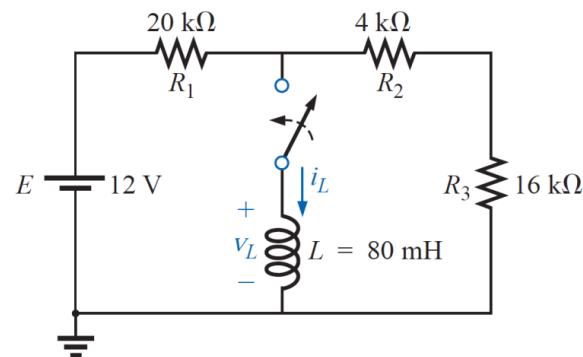
THÉVENIN EQUIVALENT:

Example:

For the network of Figure below

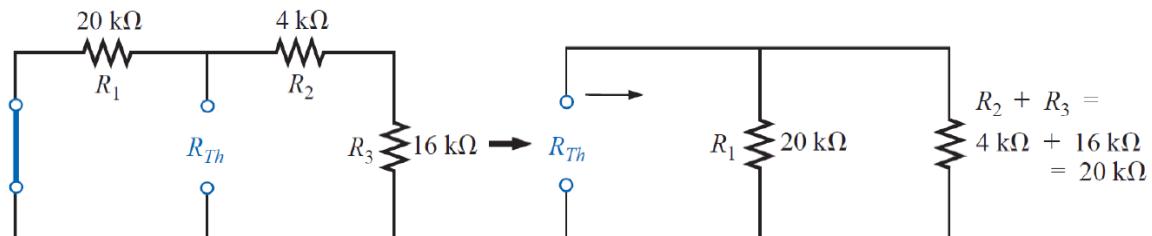
a. Find the mathematical expression for the transient behavior of the current i_L and the voltage v_L after the closing of the switch ($I_i = 0$ mA).

b. Draw the resultant waveform for each.



Solutions:

a. Applying Thévenin's theorem to the 80-mH inductor ,yields



$$R_{th} = \frac{(4 + 16) \times 20}{(4 + 16) + 20} = 10 \text{ K}\Omega$$

Applying the voltage divider rule to determine Thevenin voltage

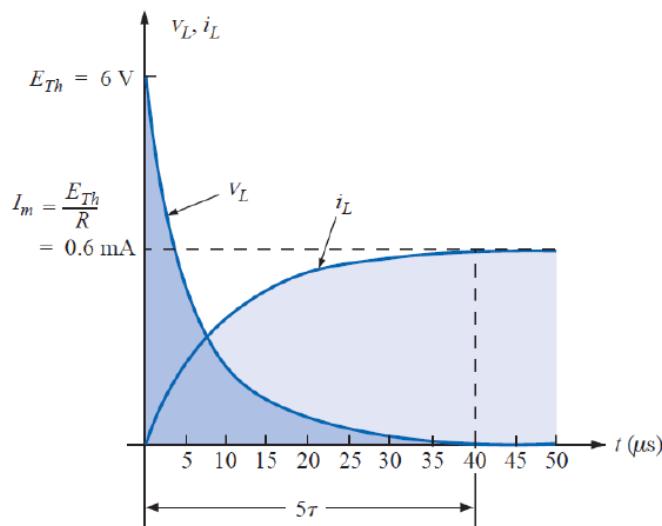
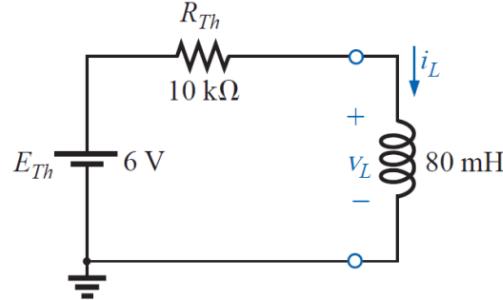
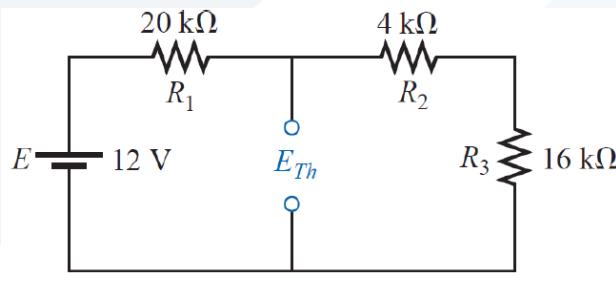
$$E_{th} = 12 \frac{(4 + 16)}{(4 + 16) + 20} = 6 V$$

$$\tau = \frac{L}{R_{th}} = \frac{80 \times 10^{-3}}{10 \times 10^3} = 8 \mu s$$

$$i_L = \frac{E_{th}}{R_{th}} \left(1 - e^{-\frac{t}{\tau}} \right) = \frac{6}{10 \times 10^3} \left(1 - e^{-\frac{t}{8 \times 10^{-6}}} \right)$$

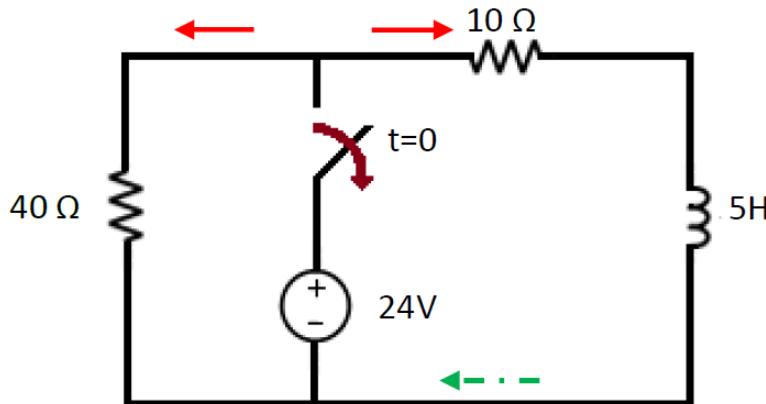
$$i_L = 0.6 \left(1 - e^{-\frac{t}{8 \times 10^{-6}}} \right) mA$$

$$v_L = E_{th} e^{-\frac{t}{\tau}} = 6 e^{-\frac{t}{8 \times 10^{-6}}} V$$



Example:

Find the voltage across 40Ω resistor at $t=200\text{ms}$.



$$I_l = \frac{24}{10} = 2.4 \text{ A}$$

$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5 \text{ s}$$

At $t=0$

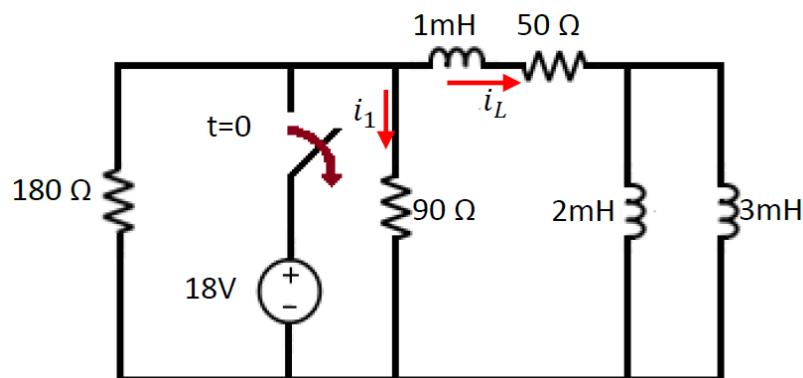
$$\tau = \frac{L}{R_{eq}} = \frac{5}{10 + 40} = 0.1 \text{ s}$$

$$i(t) = I_0 e^{-\frac{R}{L}t} = 2.4 e^{-10t}$$

$$V_{40} = i(t)R = -2.4 \times 40 \times e^{-10t} = -96e^{-10t} \text{ V}$$

Example:

Determine both i_1 and i_L for $t > 0$.



$$L_{eq} = \frac{2 \times 3}{2 + 3} + 1 = 2.2mH$$

$$R_{eq} = \frac{180 \times 90}{180 + 90} + 50 = 110\Omega$$

$$I_L = \frac{18}{50} = 360 \text{ mA}$$

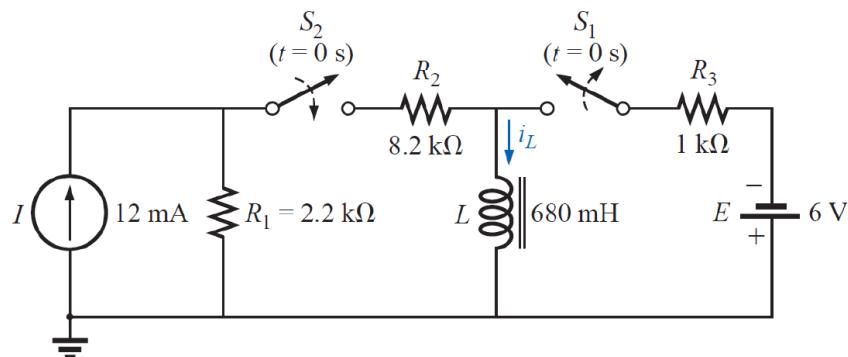
$$I_L(t) = 360 e^{-\frac{R_{eq}}{L_{eq}}t} = 360 e^{-\frac{110}{2.2 \times 10^{-3}}t} = 360 e^{-50000t} \text{ mA}$$

$$I_1(t) = -360 \frac{180}{180 + 90} e^{-50000t} = -240 e^{-50000t} \text{ mA}$$

H.W

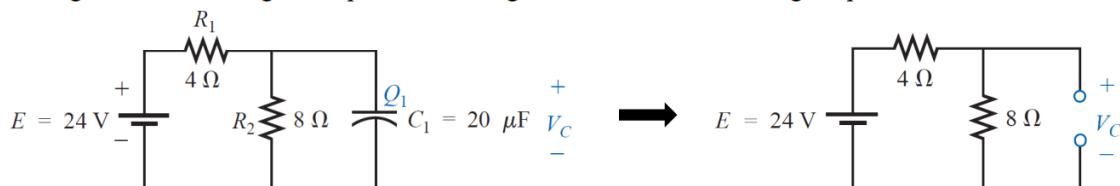
The switch S_1 of following Figure has been closed for a long time. At $t = 0$ s, S_1 is opened at the same instant S_2 is closed to avoid an interruption in current through the coil.

- Find the initial current through the coil. Pay particular attention to its direction.
- Find the mathematical expression for the current i_L following the closing of the switch S_2 .
- Sketch the waveform for i_L .



RC Circuits

Find the voltage across and charge on capacitor C_1 of Figure below after it has charged up to its final value.



Solution :

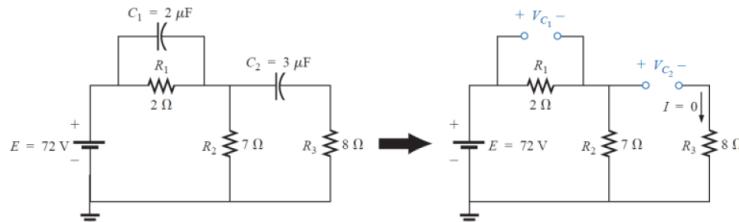
the capacitor is effectively an open circuit for dc after charging up to its final value

$$V_C = E \frac{R_2}{R_1 + R_2} = 24 \frac{8}{12} = 16V$$

$$Q_1 = V_C C_1 = 16 \times 20 \times 10^{-6} = 320 \mu C$$

Example:

Find the voltage across and charge on each capacitor of the network of Figure below after each has charged up to its final value.



Solution

$$V_{C_1} = E \frac{R_1}{R_1 + R_2} = 72 \frac{2}{9} = 16V$$

$$V_{C_2} = E \frac{R_2}{R_1 + R_2} = 72 \frac{7}{9} = 56V$$

$$Q_1 = V_{C_1} C_1 = 16 \times 2 \times 10^{-6} = 32 \mu C$$

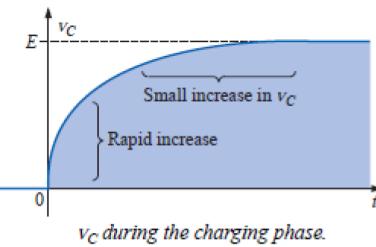
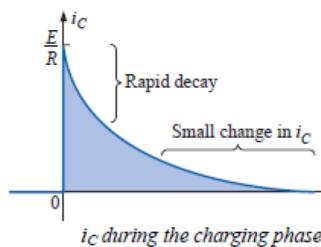
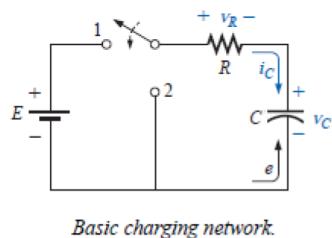
$$Q_2 = V_{C_2} C_2 = 56 \times 3 \times 10^{-6} = 168 \mu C$$

ENERGY STORED BY A CAPACITOR

The ideal capacitor does not dissipate any of the energy supplied to it. It stores the energy in the form of an electric field between the conducting surfaces.

$$W_C = \frac{1}{2} CV^2 \quad (\text{J})$$

TRANSIENTS IN CAPACITIVE NETWORKS: CHARGING PHASE



$$\begin{aligned} -E + I_R R + V_C &= 0 \\ -E + RC \frac{dV_C}{dt} + V_C &= 0 \\ RC \frac{dV_C}{dt} &= E - V_C \\ RC \frac{dV_C}{(E - V_C)} &= dt \\ \int RC \frac{dV_C}{(E - V_C)} &= \int dt \\ -RC \ln(E - V_C) &= t + K \end{aligned}$$

At $t=0$, $V_C = 0$, therefore

$$\begin{aligned}
 -RCl\ln(E) &= K \\
 -RCl\ln(E - V_C) &= t - RCl\ln(E) \\
 -RCl\ln(E - V_C) + RCl\ln(E) &= t \\
 -RCl\ln\left(\frac{E - V_C}{E}\right) &= t \\
 \ln\left(\frac{E - V_C}{E}\right) &= -\frac{t}{RC} \\
 \frac{E - V_C}{E} &= e^{-\frac{t}{RC}} \\
 V_C &= E - Ee^{-\frac{t}{RC}} = E \left(1 - e^{-\frac{t}{RC}}\right) \\
 \tau &= RC
 \end{aligned}$$

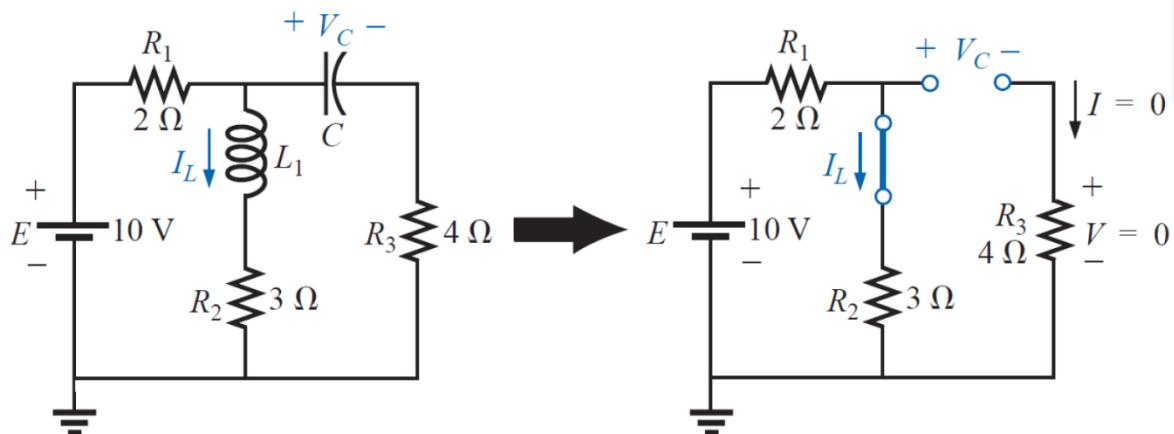
And

$$I_C = I_C = I_0 e^{-\frac{t}{RC}} = \frac{E}{R} e^{-\frac{t}{RC}}$$

RLC Circuits

Example:

Find the current I_L and the voltage V_C for the network



Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10}{5} = 2 A$$

$$V_L = E \frac{R_2}{R_1 + R_2} = 10 \frac{3}{5} = 6 V$$

H.W

Find the currents I_1 and I_2 and the voltages V_1 and V_2 for the network

