Tension and Compression in Bars

1. Stress الإجهاد
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Objectives: *Mechanics of Materials* investigates the stressing and the deformations of structures subjected to applied loads, starting by the simplest structural members, namely, bars in tension or compression.

يدرس ميكانيك المواد إجهادات وتشوهات الجمل الإنشائية (الهياكل الحاملة) الناتجة عن الحمولات الخارجية، مبتدأً بالعناصر الأبسط أي القضبان (العناصر الطولية) المشدودة أو المضغوطة.

In order to treat such problems, the kinematic relations and a constitutive law are needed to complement the equilibrium conditions which are known from Engineering Mechanics (Statics).

تقوم هذه الدراسة على:

(1) معادلات التوازن التي دُرست في الميكانيك الهندسي (علم السكون)

(2) العلاقات الكينماتيكية التي ستدرس وهي تصف التشوهات كمياً أي تحدد شكل ومقدار تغيرات الشكل الجيومتري.

(3) قوانين سلوك مادة الجملة وهي كما ستُعرض لاحقاً، قوانين تجريبية تعرّف السلوك الميكانيكي لمادة الهيكل الحامل.

The kinematic relations represent the geometry of the deformation, whereas the behavior of the material is described by the constitutive law. The students will learn how to apply these equations and how to solve determinate as well as statically indeterminate problems. يعالج الطلبة مسائل مقررة سكونياً وأخرى غير مقررة سكونياً ؟؟

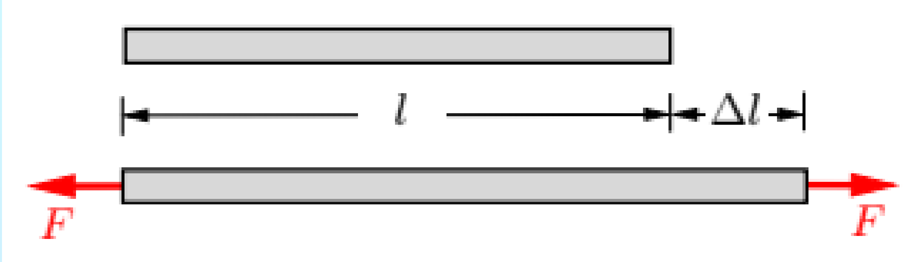
2 Strain التشوه

Let us first consider a bar with a constant cross-sectional area which has the undeformed length l.

نبدأ أولاً بالنظر إلى قضيب بمقطع ثابت وطول أولي غير مشوه قدره: l .

Under the action of tensile forces (Fig.) it gets slightly longer.

تحت تأثير قوتي شد كما في الشكل سيتطاول القضيب قليلاً.



The elongation is denoted by Δl and is assumed to be much smaller than the original length l.

نرمز لهذا التطاول Δl ونفترض أنه مقدار صغير جداً مقارنة مع الطول l.

As a measure of the amount of deformation, it is useful to introduce, in addition to the elongation, the ratio between the elongation and the original (undeformed) length:

نقيس التشوه الطولي بالنسبة بين التطاول والطول الأصلي

$$\varepsilon = \frac{\Delta l}{l}$$

The <u>dimensionless</u> quantity ε is called *strain*. التشوه الطولي النسبي وهنا اختصارا التشوه، مقدار لابعدي (دون واحدات)

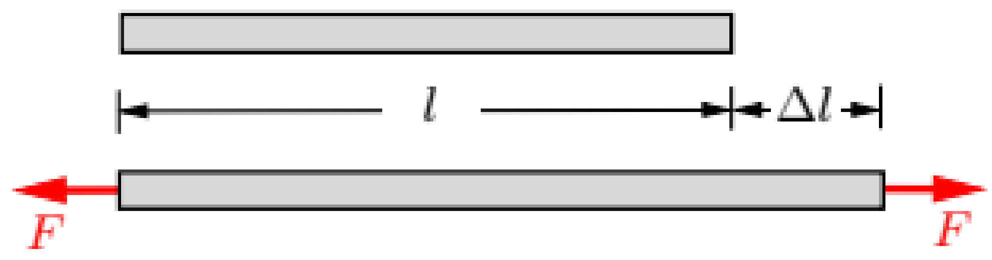
Example: If, for example, a bar of the length $l = 1$ m undergoes an elongation of $\Delta l = 0.5$ mm then we have $\varepsilon = 0.5 \times 10^{-3}$. This is a strain of 0.05%.

If the bar gets longer ($\Delta l > 0$) the strain is <u>positive</u>; it is <u>negative</u> in the case of a shortening $(\Delta l < 0)$
.
إذا ازداد الطول أي كان $(\Delta l > 0)$ يكون التشوه موجباً، وهذا يتوافق مع إجهاد الشد الموجب.
أما إذا نقص الطول أي كان $(\Delta l > 0)$ يكون التشوه سالباً، وهذا يتوافق مع إجهاد الضغط السالب..

In what follows we will consider only small deformations

فيما يلي سنفترض أن التشوهات صغيرة دوماً

$$|\Delta l| \ll l \text{ or } |\varepsilon| \ll 1.$$

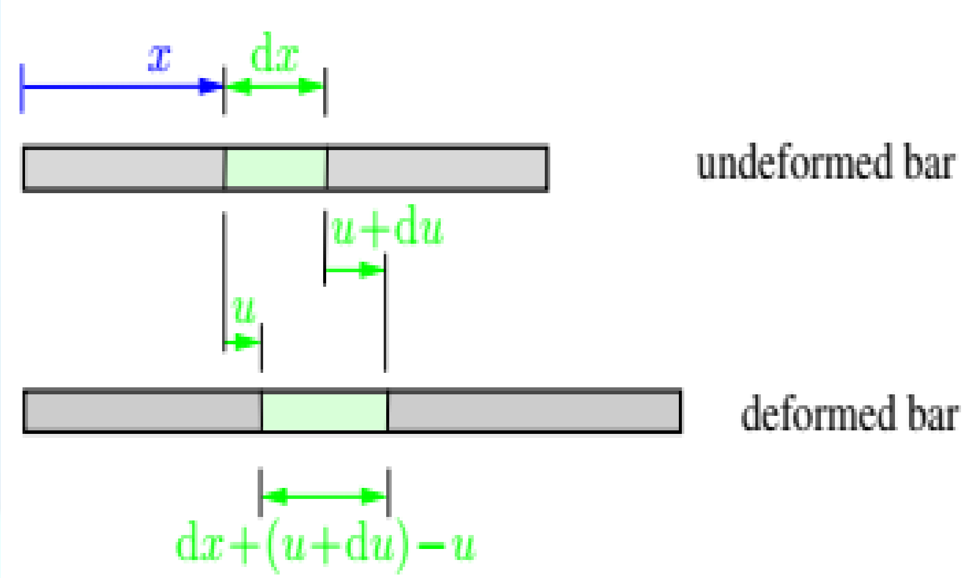


The above definition $\varepsilon = \frac{\Delta l}{l}$ for the strain is valid only if ε is constant over the bar length.

يصح التعريف السابق فقط إذا كان التشوه ثابتاً على كامل الطول.

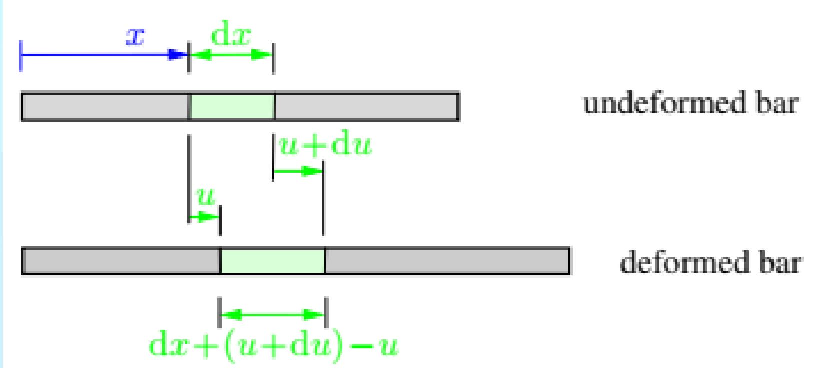
If the cross-sectional area is not constant or if the bar is subjected to volume forces acting along its axis, the strain may depend on the location.

Instead of the whole we consider an element of the bar (Fig.). It has the length dx in the undeformed state. Its left end is located at x, the right end at $x + dx$.



Instead of the whole we consider an element of the bar (Fig.). It has the length dx in the undeformed state. Its left end is located at x, the right end at $x + dx$.

If the bar is elongated, the cross sections undergo displacements in the x-direction which are denoted by u. They depend on the location: $u = u(x)$.



Thus, the displacements are u at the left end of the element and $u + du$ at the right end.

The length of the elongated element is $dx + (u + du) - u = dx + du$.

Hence, the elongation of the element is given by du. Now the local strain can be defined as the ratio between the elongation and the undeformed length of the element: $\varepsilon(x) = \frac{du}{dx}$

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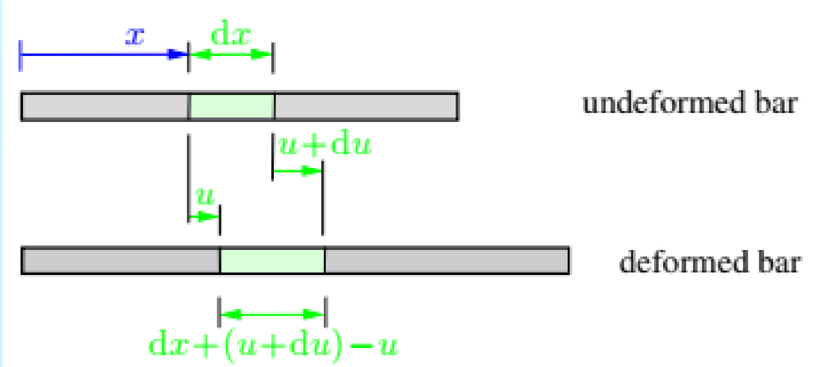


If the displacement $u(x)$ is known, the strain $\varepsilon(x)$ can be determined through differentiation. Reversely, if $\varepsilon(x)$ is known, the displacement $u(x)$ is obtained through integration.

The displacement $u(x)$ and the strain $\varepsilon(x)$ describe the geometry of the deformation. Therefore they are called *kinematic quantities*. So the equation $\varepsilon(x) = \frac{du}{dx}$ is referred to as a kinematic relation.

$$External\ Loads$$

Statics $\quad \sigma(x) = \frac{N(x)}{A(x)} \quad \longleftrightarrow \quad \varepsilon(x) = \frac{du}{dx},$ kinematic

3 Constitutive Law

Stresses are quantities derived from statics; they are a measure for the stressing in the material .

On the other hand, strains are kinematic quantities; they measure the deformation of a body.

However, the deformation depends on the load which acts on the body. Therefore, the stresses and the strains are not independent.

$External\ Loads$

Statics $\sigma(x) = \dfrac{N(x)}{A(x)}$ *constitutive law* قانون السلوك المادي $\varepsilon(x) = \dfrac{du}{dx},$ kinematic

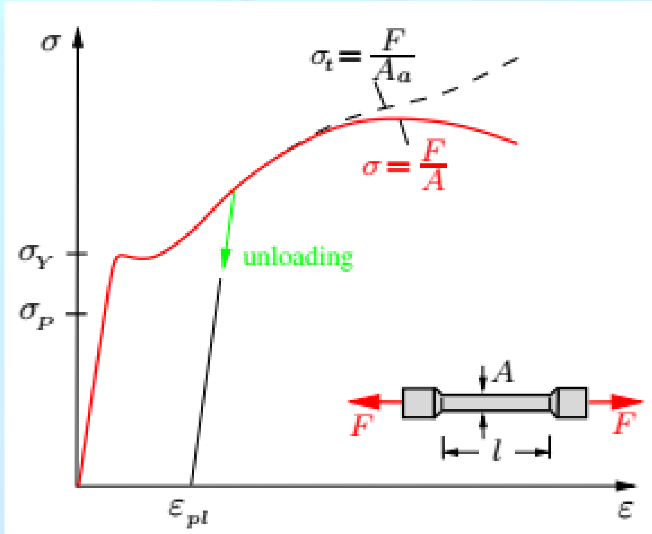
The physical relation that connects these quantities is called *constitutive law*.

It describes the behavior of the material of the body under a load. It depends on the material and can be obtained only with the aid of experiments.

One of the most important experiments to find the relationship between stress and strain is the tension or compression test. Here, a small specimen of the material is placed into a testing machine and elongated or shortened.

The force F applied by the machine onto the specimen can be read on the dial of the machine; it causes the normal stress $\sigma = F/A$. The change Δl of the length l of the specimen can be measured and the strain $\varepsilon = \Delta l/l$ can be calculated.

The graph of the relationship between stress and strain is shown schematically (not to scale) for a steel specimen in Fig.



This graph is referred to as *stress-strain diagram*. One can see that for small values of the strain the relationship is linear (straight line) and the stress is proportional to the strain.

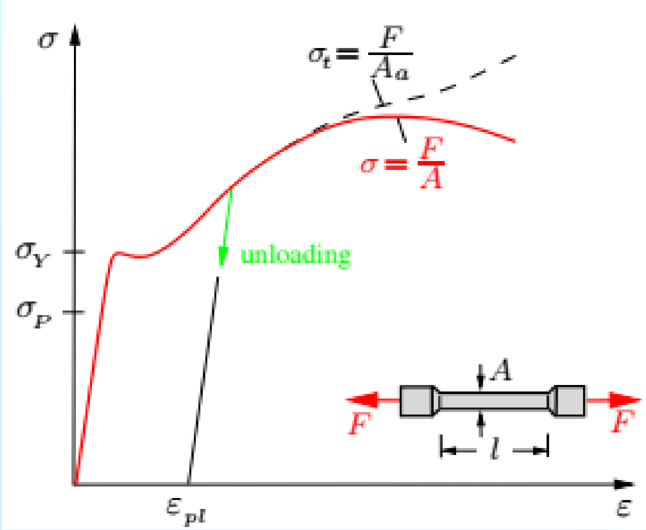
This behavior is valid until the stress reaches the *proportional limit* σ_P. If the stress exceeds the proportional limit the strain begins to increase more rapidly and the slope of the curve decreases.

This continues until the stress reaches the *yield stress* σ_Y . From this point of the stress-strain diagram the strain increases at a practically constant stress: the material begins to *yield*. Note that many materials do not exhibit a pronounced yield point.

At the end of the yielding the slope of the curve increases again which shows that the material can sustain an additional load. This phenomenon is called *strain hardening*.

Experiments show that an elongation of the bar leads to a reduction of the cross-sectional area A. This phenomenon is referred to as *lateral contraction*.

Whereas the cross-sectional area decreases uniformly over the entire length of the bar in the case of small stresses, it begins to decrease locally at very high stresses.



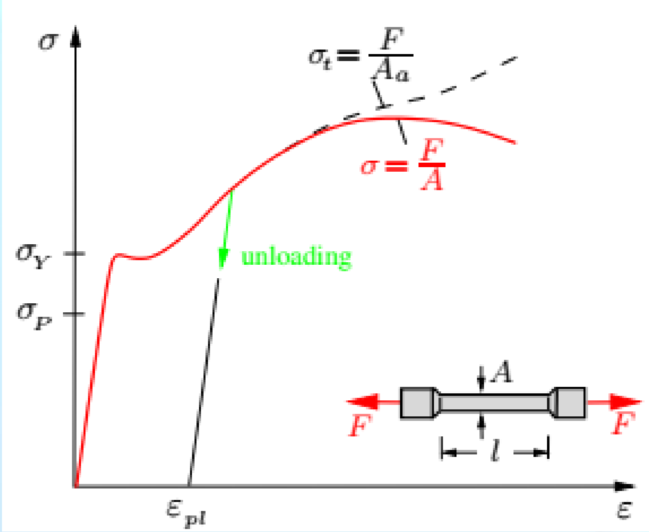
This phenomenon is called *necking*. Since the actual cross section A_a may then be considerably smaller than the original cross section A, the stress $\sigma = F/A$ does not describe the real stress any more.

It is therefore appropriate to introduce the stress $\sigma_t = F/Aa$ which is called *true stress* or *physical stress*. It represents the true stress in the region where necking takes place. The stress $\sigma = F/A$ is referred to as *nominal* or *conventional* or *engineering stress*. The Fig. shows both stresses until fracture occurs.

Consider a specimen being first *loaded* by a force which causes the stress σ. Assume that σ is smaller than the yield stress σ_Y , i.e., $\sigma < \sigma_Y$.

Subsequently, the load is again removed. Then the specimen will return to its original length: the strain returns to zero.

In addition, the curves during the loading and the unloading coincide. This behavior of the material is called *elastic* ; the behavior in the region $\sigma \leq \sigma_P$ is referred to as *linearly elastic*.



Now assume that the specimen is loaded beyond the yield stress, i.e., until a stress $\sigma > \sigma_Y$ is reached. Then the curve during the unloading is a straight line which is parallel to the straight line in the linear-elastic region, see Fig. If the load is completely removed the strain does not return to zero: a *plastic strain* ε_{pl} remains after the unloading. This material behavior is referred to as *plastic*.

In the following we will always restrict ourselves to a linearly elastic material behavior. For the sake of simplicity we will refer to this behavior shortly as elastic, i.e., Then we have the linear relationship between the stress and the strain.

$$\sigma = E\varepsilon$$

The proportionality factor E is called *modulus of elasticity* or *Young's modulus* (Thomas Young, 1773–1829). The constitutive law $\sigma = E\varepsilon$ is called *Hooke's law* after Robert Hooke (1635–1703). Note that Robert Hooke could not present this law in this form since the notion of stress was introduced only in 1822 by Augustin Louis Cauchy.

The modulus of elasticity has the same value for tension and compression. But, σ must be less than the proportional limit σ_P which may be different for tension or compression.

The modulus of elasticity E is a constant which depends on the material and which can be determined with the aid of a tension test. It has the dimension of force/area (which is also the dimension of stress); it is given, for example, in the unit MPa.

Next Table shows the values of E for several materials at room temperature. Note that these values are just a guidance since the modulus of elasticity depends on the composition of the material and on the temperature.

A tensile or a compressive force, respectively, causes the strain: $\varepsilon = \sigma/E$

Changes of the length and thus strains are not only caused by forces but also by changes of the temperature. Experiments show that the *thermal strain* ε_T is proportional to the change ΔT of the temperature if the temperature of the bar is changed uniformly across its section and along its length: $\varepsilon_T = \alpha_T \Delta T$

The proportionality factor α_T is called *coefficient of thermal expansion*. It is a material constant and is given in the unit $1/°\mathrm{C}$. Next Table shows several values of α_T and E.

If the change of the temperature is not the same along the entire length of the bar (if it depends on the location) then $\varepsilon_T = \alpha_T \Delta T$ represents the local strain $\varepsilon_T(x) = \alpha_T\, \Delta T(x)$.

If a bar is subjected to a stress σ as well as to a change ΔT of the temperature, the total strain ε is obtained through a superposition $\varepsilon = \frac{\sigma}{E} + \alpha_T \Delta T$

This relation can also be written in the form $\sigma = E(\varepsilon - \alpha_T \Delta T)$.

Table of Material Constants

Material	E in MPa	α_T in $1/°\mathrm{C}$
Steel	$2.1 \cdot 10^5$	$1.2 \cdot 10^{-5}$
Aluminium	$0.7 \cdot 10^5$	$2.3 \cdot 10^{-5}$
Concrete	$0.3 \cdot 10^5$	$1.0 \cdot 10^{-5}$
Wood (in fibre direction)	$0.7 \ldots 2.0 \cdot 10^4$	$2.2 \ldots 3.1 \cdot 10^{-5}$
Cast iron	$1.0 \cdot 10^5$	$0.9 \cdot 10^{-5}$
Copper	$1.2 \cdot 10^5$	$1.6 \cdot 10^{-5}$
Brass	$1.0 \cdot 10^5$	$1.8 \cdot 10^{-5}$