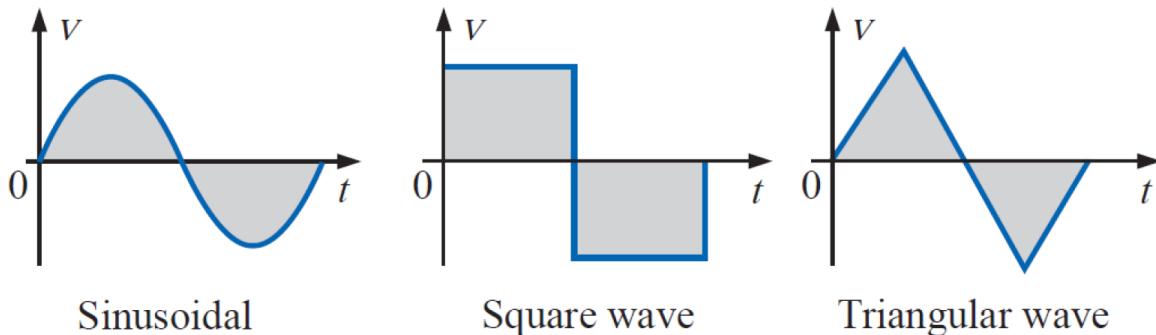


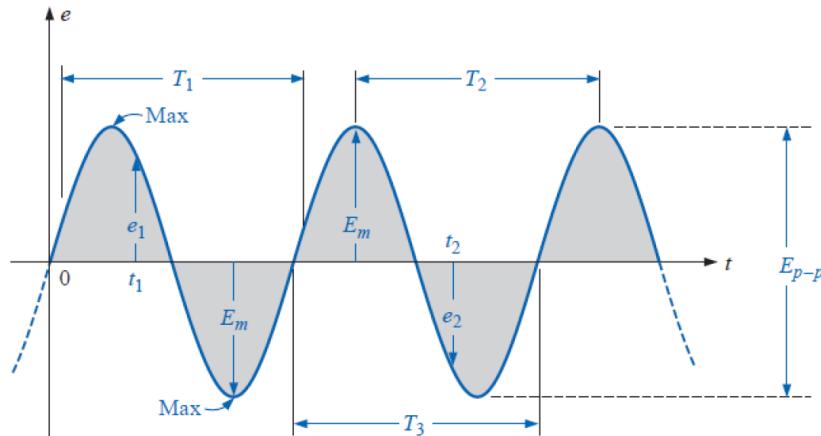
Sinusoidal Alternating Waveforms

Introduction

The term *alternating* indicates only that the waveform alternates between two prescribed levels in a set time sequence



The vertical scaling of the sinusoidal waveform is in volts or amperes and the horizontal scaling is *always* in units of time and can be represented as.



Instantaneous value, Peak amplitude, Peak value, Peak-to-peak value and Period.

Frequency (f): The number of cycles that occur in 1 s.

1 hertz (Hz) = 1 cycle per second (c/s)

$$f = \frac{1}{T}$$

Example:

Find the period of a periodic waveform with a frequency of

a. 60 Hz.

b. 1000 Hz.

Solution

a- $T = \frac{1}{f} = \frac{1}{60} = 16.67 \text{ ms}$

b- $T = \frac{1}{f} = \frac{1}{1000} = 1 \text{ ms}$

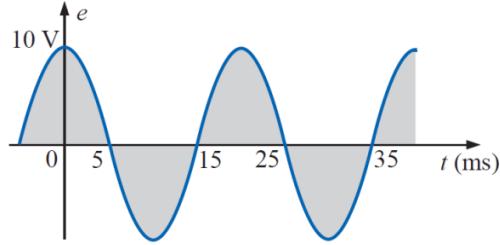
Example:

Determine the frequency of the waveform of following Fig

Solution:

From the figure, $T = (25 \text{ ms} - 5 \text{ ms}) = 20 \text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50 \text{ Hz}$$



Example:

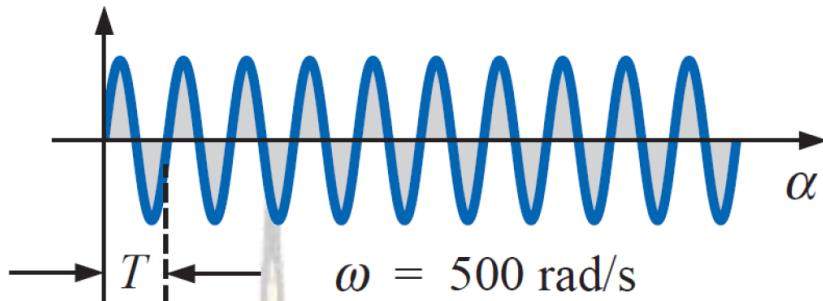
Determine the frequency and period of the sine wave of following Figure.

Solution

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{500} = 12.57 \text{ ms}$$

$$f = \frac{1}{T} = 79.58 \text{ Hz}$$

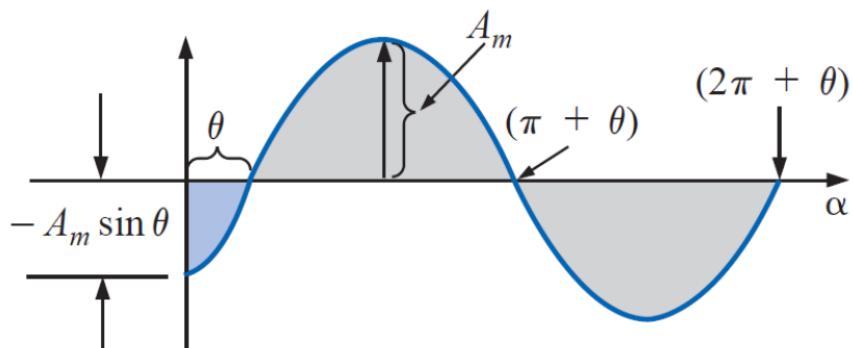


PHASE RELATIONS

If the waveform is shifted to the right or left of 0° , the expression becomes

$$A_m \sin(\omega t \pm \theta)$$

where θ is the angle in degrees or radians that the waveform has been shifted.



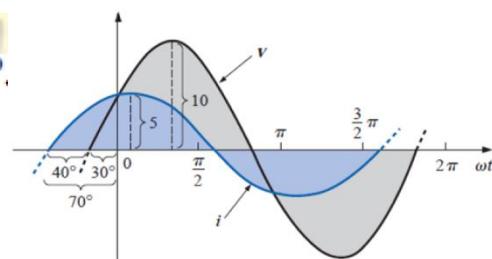
Example:

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30)$

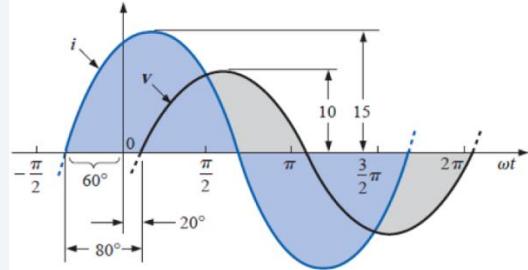
$$i = 5 \sin(\omega t + 70)$$

i leads v by 40° , or v lags i by 40° .



b. $i = 15 \sin(\omega t + 60)$
 $v = 10 \sin(\omega t - 20)$

i leads v by 80° , or v lags i by 80° .



AVERAGE VALUE

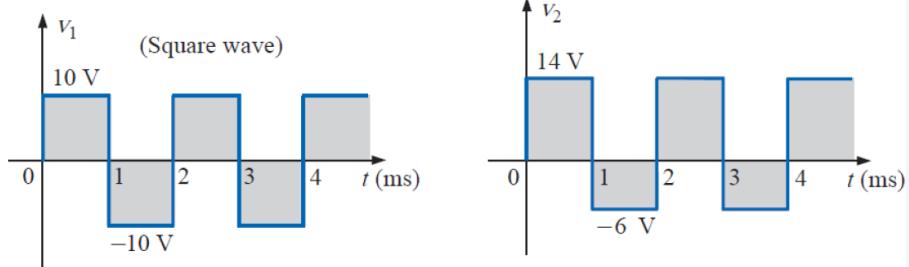
For half wave rectifier

$$V_{av} = \frac{1}{\pi} \int_0^{\pi} v(t) dt$$

$$V_{av} = \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t d\omega t = \frac{2V_m}{\pi} = 0.636V_m$$

Example:

Determine the average value of the waveforms



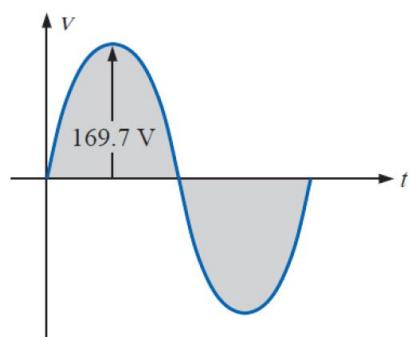
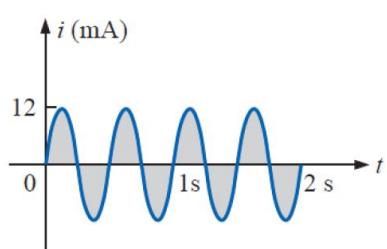
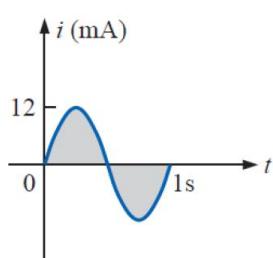
EFFECTIVE Root Mean Square (rms) VALUES

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v^2(t) dt}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \sin^2 \omega t d\omega t} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

Example:

Find the rms values of the sinusoidal waveform in each part



RESPONSE OF BASIC R, L, AND C ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Resistor

$$\text{Let } v(t) = V_m \sin \omega t$$

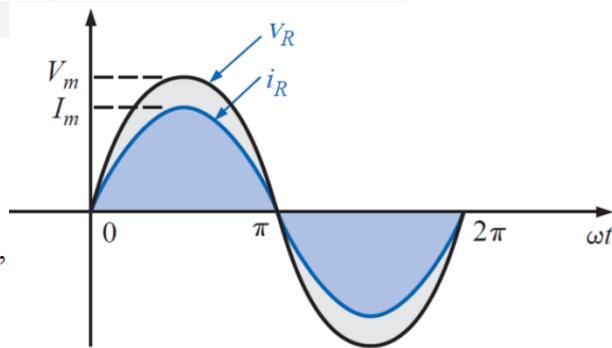
$$i(t) = \frac{v(t)}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

In addition, for a given $i(t) = I_m \sin \omega t$,

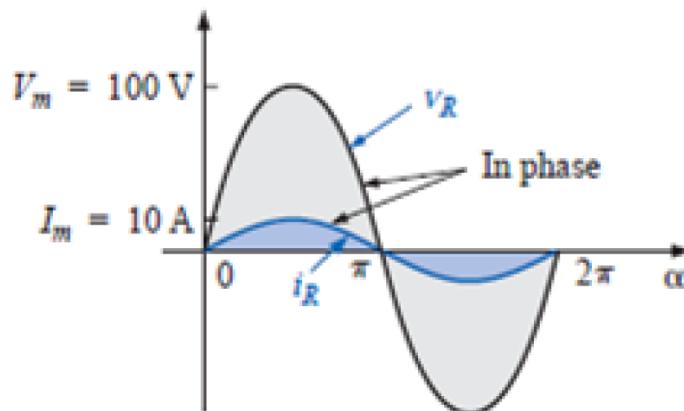
$$v(t) = RI_m \sin \omega t = V_m \sin \omega t$$

$$V_m = RI_m$$



Example:

The voltage across 10Ω resistor is $100 \sin 377t$, sketch the curves for the voltage and current.



Example:

The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10Ω . Sketch the curves for v and i .

a. $v = 100 \sin 377t$

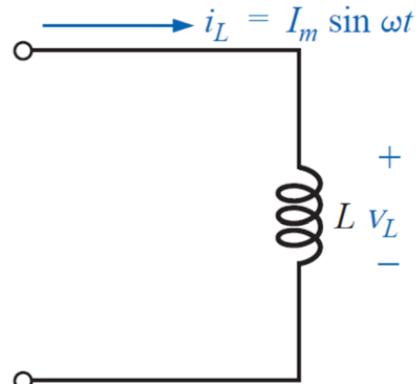
b. $v = 25 \sin(377t + 60^\circ)$

Inductor

$$v_L = L \frac{di_L}{dt} = L\omega I_m \cos \omega t$$

$$v_L = V_m \cos \omega t = V_m \sin(\omega t + 90^\circ)$$

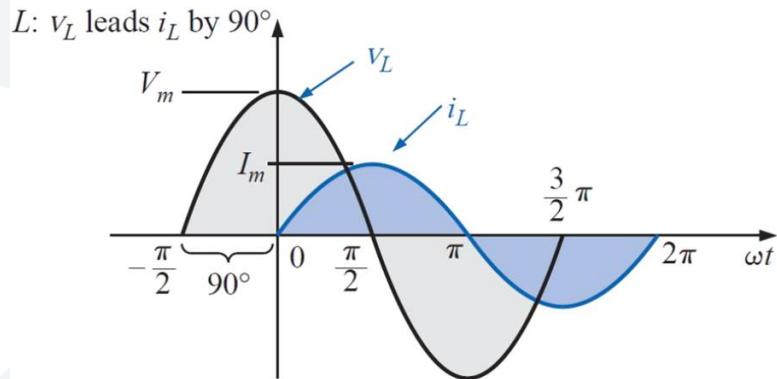
$$V_m = L\omega I_m$$



for an inductor, v_L leads i_L by 90° , or i_L lags v_L by 90° .

Reactance X_L

$$X_L = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$



Example:

The voltage across a 0.5-H coil is provided below. What is the sinusoidal expression for the current?

$$v = 100 \sin 20t$$

$$X_L = \omega L = 20 \times 0.5 = 10 \Omega$$

$$i = \frac{V_m}{X_L} \sin(20t - 90) = 10 \sin(20t - 90) A$$

Example:

The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$

b. $i = 7 \sin(377t - 70^\circ)$

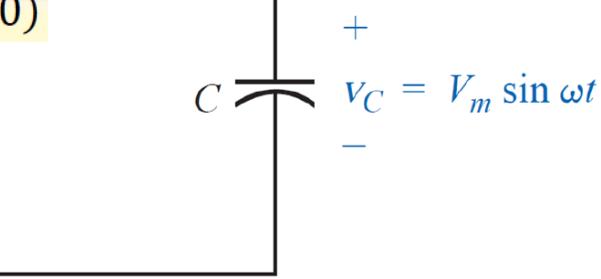
Capacitor

$$i_C = C \frac{dv_C}{dt} = C\omega V_m \cos \omega t$$

$$i_C = ?$$

$$i_C = I_m \cos \omega t = I_m \sin(\omega t + 90)$$

$$I_m = \omega C V_m$$

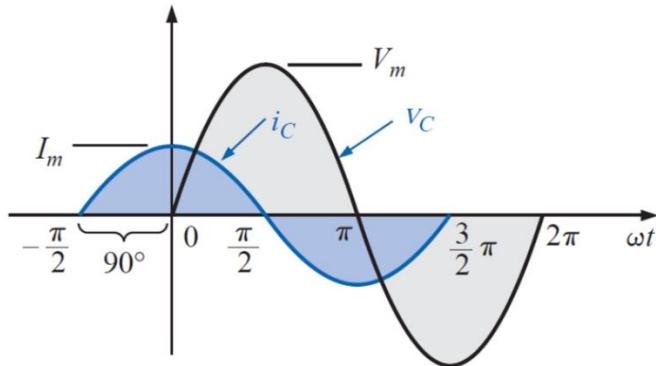


Reactance X_C

$$X_C = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

for a capacitor, i_C leads v_C by 90° , or v_C lags i_C by 90° .

C: i_C leads v_C by 90° .



Example:

The voltage across a $1-\mu\text{F}$ capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

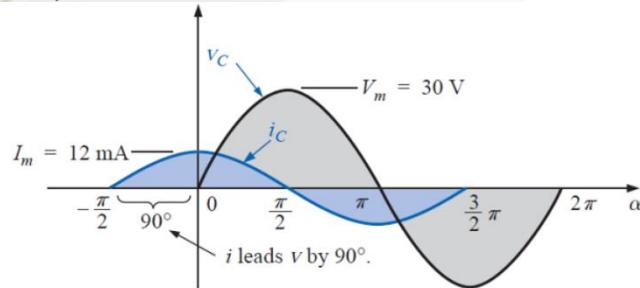
$$v = 30 \sin 400t$$

Solution

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \times 1 \times 10^{-6}} = 2500\Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{30}{2500} = 12\text{mA}$$

$$v = 12 \sin(400t + 90)$$



Example:

The current through a $100-\text{mF}$ capacitor is $i = 40 \sin(500t - 60^\circ)$. Find the sinusoidal expression for the voltage across the capacitor.

Example:

For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of C , L , or R if sufficient data are provided :

a. $v = 100 \sin(\omega t - 40^\circ)$

$i = 20 \sin(\omega t - 40^\circ)$

b. $v = 1000 \sin(377t - 10^\circ)$

$i = 5 \sin(377t - 80^\circ)$

c. $v = 500 \sin(157t - 30^\circ)$

$i = 1 \sin(157t - 120^\circ)$

d. $v = 50 \cos(\omega t - 20^\circ)$

$i = 5 \sin(\omega t - 110^\circ)$

AVERAGE POWER and power factor

Let we have

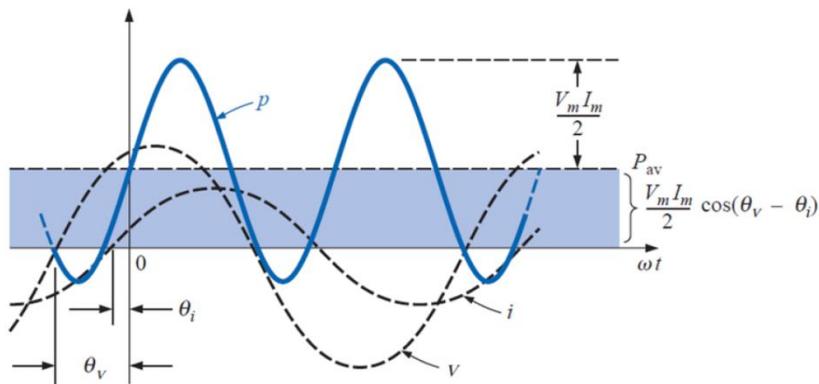
$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$

then the power is defined by

$$p = vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i)$$

$$p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) - \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)$$



$$p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m I_m}{2} \cos \theta$$

$$\text{power factor} = F_p = \cos \theta = \frac{p}{\frac{V_m I_m}{2}} = \frac{p}{\frac{V_m I_m}{\sqrt{2} \sqrt{2}}} = \frac{p}{V_{eff} I_{eff}}$$

Resistor

$$p = \frac{V_m I_m}{2} \cos(0) = \frac{V_m I_m}{2} = \frac{V_m I_m}{\sqrt{2} \sqrt{2}} = V_{eff} I_{eff} = \frac{V_{eff} V_{eff}}{R} = \frac{V_{eff}^2}{R} = I_{eff}^2 R$$

Inductor

$$p = \frac{V_m I_m}{2} \cos(90) = 0$$

$$\text{Capacitor} \quad p = \frac{V_m I_m}{2} \cos(90) = 0$$

Example:

Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5 \sin(\omega t + 40^\circ)$$

$$v = 10 \sin(\omega t + 40^\circ)$$

Example:

Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40^\circ)$

$$i = 20 \sin(\omega t + 70^\circ)$$

b. $v = 150 \sin(\omega t - 70^\circ)$

$$i = 3 \sin(\omega t - 50^\circ)$$

Example:

Determine the power factors of the following loads, and indicate whether they are leading or lagging:

a. $v = 50 \sin(\omega t - 20^\circ)$

$$i = 2 \sin(\omega t + 40^\circ)$$

b. $v = 120 \sin(\omega t + 80^\circ)$

$$i = 5 \sin(\omega t + 30^\circ)$$

c. $I_{eff} = 5A, V_{eff} = 20V$ and $p = 100W$

COMPLEX NUMBERS

RECTANGULAR FORM

The format for the **rectangular form** is

$$C = X + jY$$

$$X = Z \cos\theta$$

$$Y = Z \sin\theta$$

Example:

Sketch the following complex numbers in the complex plane:

a. $C = 3 + j4$

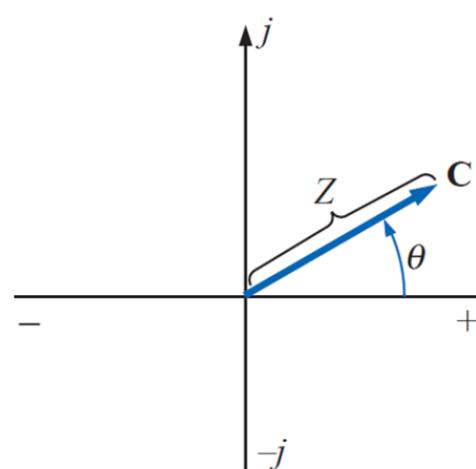
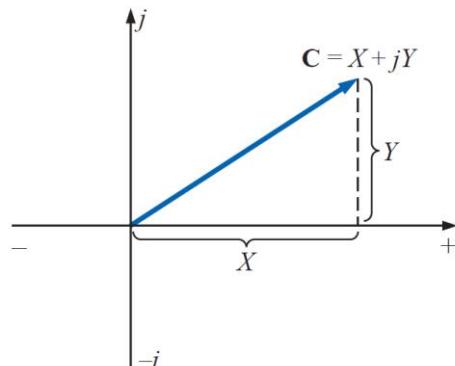
b. $C = 0 - j6$

c. $C = -10 - j20$

POLAR FORM

$$C = Z\angle\theta$$

$$Z = \sqrt{X^2 + Y^2} \quad \theta = \tan^{-1} \frac{Y}{X}$$



Example:

Sketch the following complex numbers in the complex plane:

- $C = 5\angle 30^\circ$
- $C = 7\angle -120^\circ$
- $C = -4.2\angle 60^\circ$

Example:

Convert the following from rectangular to polar form:

- $C = 3 + j 4$
- $C = -6 + j 3$

Example:

Convert the following from polar to rectangular form:

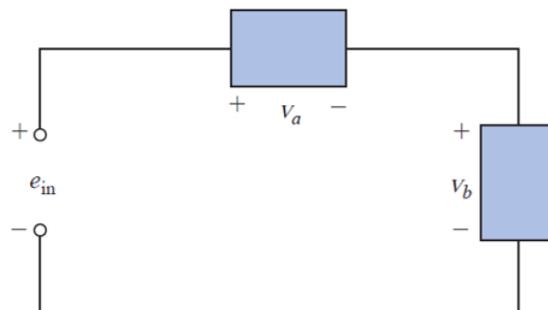
- $C = 10\angle 40^\circ$
- $C = 10\angle 230^\circ$

Example:

Find the input voltage of the circuit shown below when the frequency is 60 Hz

$$v_a = 50 \sin(377t + 30)$$

$$v_b = 30 \sin(377t + 60)$$



Solution:

$$v_a = \frac{50}{\sqrt{2}} \angle 30 = 35.35 \angle 30 V = 30.61 V + j 17.68 V$$

$$v_b = \frac{30}{\sqrt{2}} \angle 60 = 21.21 \angle 60 V = 10.61 V + j 18.37 V$$

Applying Kirchhoff's voltage law, we have

Rectangular form

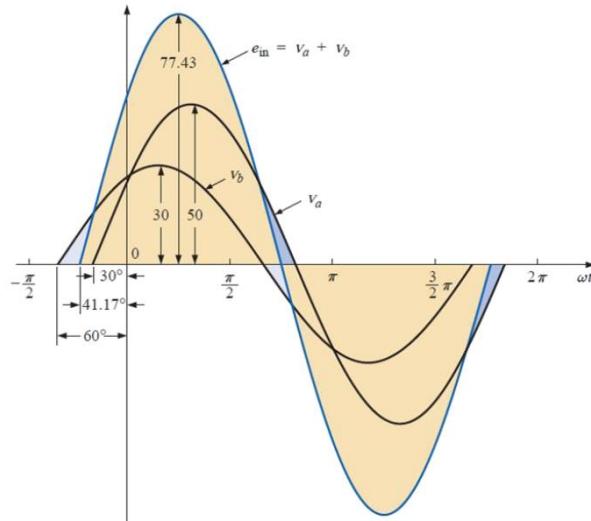
$$E_{in} = v_a + v_b = 30.61 V + j 17.68 + 10.61 V + j 18.37 = 41.22 V + j 36.05 V$$

Polar form

$$E_{in} = 54.76 V \angle 41.17^\circ$$

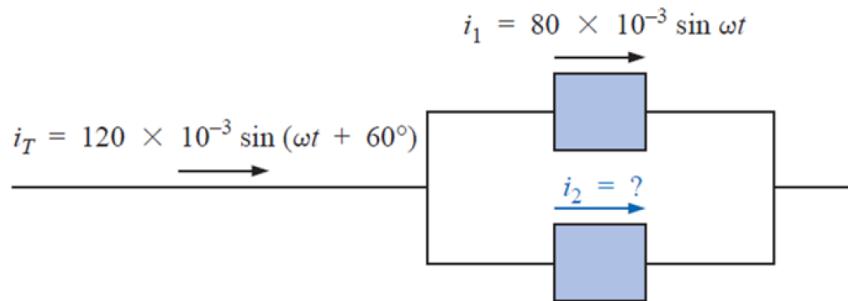
Time domain

$$E_{in} = \sqrt{2}(54.76) \sin(377t + 41.17) = 77.43 \sin(377t + 41.17)$$



Example:

Determine the current i_2 for the network



Solution:

Applying Kirchhoff's current law, we obtain

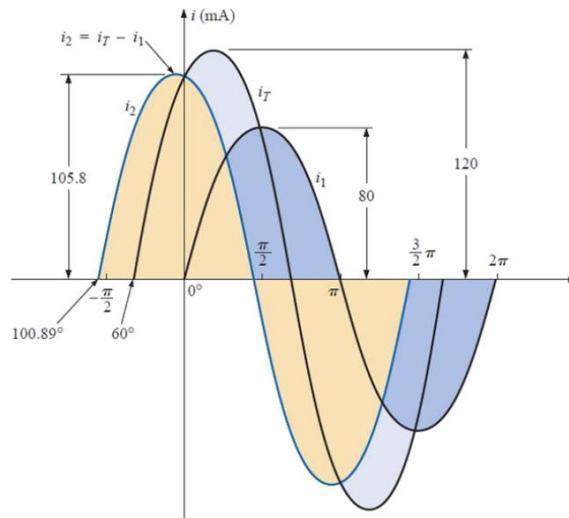
$$i_T = i_1 + i_2$$

$$i_2 = i_T - i_1$$

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60) = \frac{120 \times 10^{-3}}{\sqrt{2}} \angle 60 = 84.84 \angle 60 \text{ mA} = 42.42 + j 73.47 \text{ mA}$$

$$i_1 = 80 \times 10^{-3} \sin \omega t = \frac{80 \times 10^{-3}}{\sqrt{2}} \angle 0 = 56.56 \angle 0 \text{ mA} = 56.56 + j 0 \text{ mA}$$

$$i_2 = i_T - i_1 = -14.14 + j 73.47 \text{ mA} = 74.82 \text{ mA} \angle 100.89^\circ = 105.8 \times 10^{-3} \sin(\omega t + 100.89^\circ) \text{ A}$$



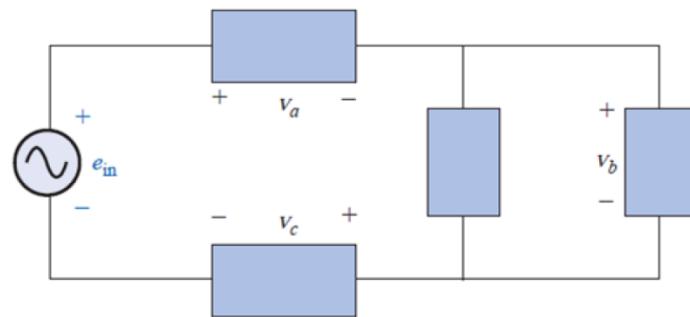
H.W

Find the sinusoidal expression for the applied voltage e for the system

$$v_a = 60 \sin(\omega t + 30)$$

$$v_b = 30 \sin(\omega t - 30)$$

$$v_c = 40 \sin(\omega t + 120)$$

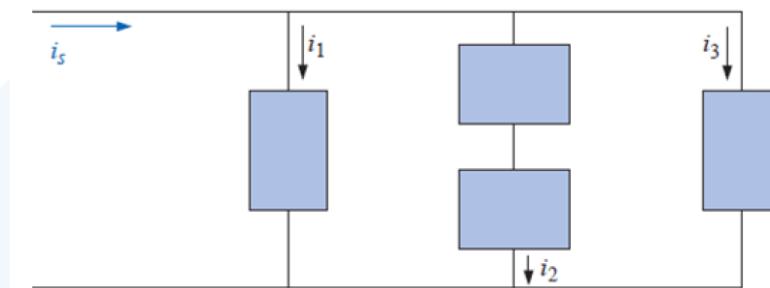


Find the sinusoidal expression for the current i_s for the system

$$i_1 = 120 \times 10^{-3} \sin(377t + 180)$$

$$i_2 = 120 \times 10^{-3} \sin(377t)$$

$$i_3 = 2i_1$$



MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

$$j = \sqrt{-1}$$

$$j^2 = -1$$

$$\frac{1}{j} = -j$$

Complex Conjugate

The **conjugate** or **complex conjugate** of a complex number can be found by simply changing the sign of the imaginary part in the rectangular form or by using the negative of the angle of the polar form. For example, the conjugate of $C = 2 + j3$ is $2 - j3$

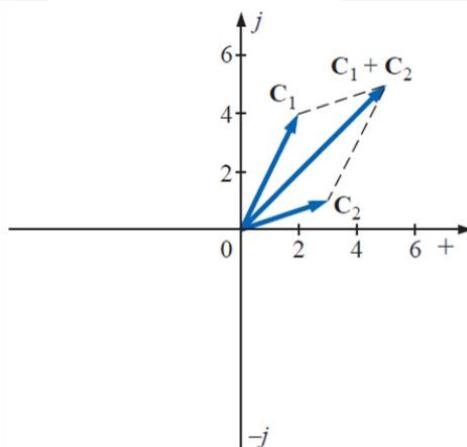
The conjugate of $C = 2\angle 30^\circ$ is $2\angle -30^\circ$

Addition

To add two or more complex numbers, simply add the real and imaginary parts separately. For example, if

$$C_1 = X_1 + jY_1 \text{ and } C_2 = X_2 + jY_2$$

$$\text{Then } C_1 + C_2 = X_1 + X_2 + j(Y_1 + Y_2)$$



Example:

$$\text{Add } C_1 = 2 + j4 \text{ and } C_2 = 3 + j1$$

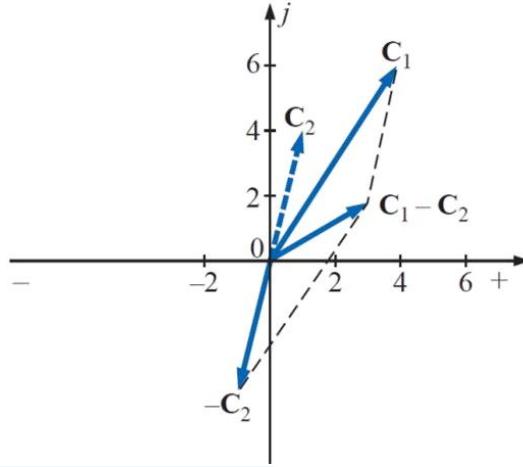
$$\text{Add } C_1 = 3 + j6 \text{ and } C_2 = -6 + j3$$

Subtraction

In subtraction, the real and imaginary parts are again considered separately. For example, if

$$C_1 = X_1 + jY_1 \text{ and } C_2 = X_2 + jY_2$$

$$\text{Then } C_1 - C_2 = X_1 - X_2 + j(Y_1 - Y_2)$$



Example:

Subtract $C_2 = 1 + j4$ and $C_1 = 4 + j6$

Subtract $C_2 = -2 + j5$ and $C_1 = 3 + j3$

Multiplication

To multiply two complex numbers in *rectangular* form, multiply the real and imaginary parts of one in turn by the real and imaginary parts of the other. For example, if

$$C_1 = X_1 + jY_1 \text{ and } C_2 = X_2 + jY_2$$

$$\text{Then } C_1 \times C_2 = X_1X_2 - Y_1Y_2 + j(X_1Y_2 + X_2Y_1)$$

Example:

Find $C_1 \cdot C_2$ if

$$C_1 = 2 + j3 \text{ and } C_2 = 5 + j10$$

In *polar* form, the magnitudes are multiplied and the angles added algebraically. For example, for $C_1 = Z_1\angle\theta_1$ and $C_2 = Z_2\angle\theta_2$

$$\text{Then } C_1 \cdot C_2 = Z_1Z_2\angle(\theta_1 + \theta_2)$$

Example:

Find $C_1 \cdot C_2$ if $C_1 = 5\angle20$ and $C_2 = 10\angle30$

Division

To divide two complex numbers in *rectangular* form, multiply the numerator and denominator by the conjugate of the denominator and the resulting real and imaginary parts collected. That is, if

$$C_1 = X_1 + jY_1 \text{ and } C_2 = X_2 + jY_2$$

Then

$$\frac{C_1}{C_2} = \frac{X_1 + jY_1}{X_2 + jY_2} = \frac{X_1 + jY_1}{X_2 + jY_2} \times \frac{X_2 - jY_2}{X_2 - jY_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j \frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}$$

Example:

Find C_1 / C_2 if

$$C_1 = 1 + j4 \text{ and } C_2 = 4 + j5$$