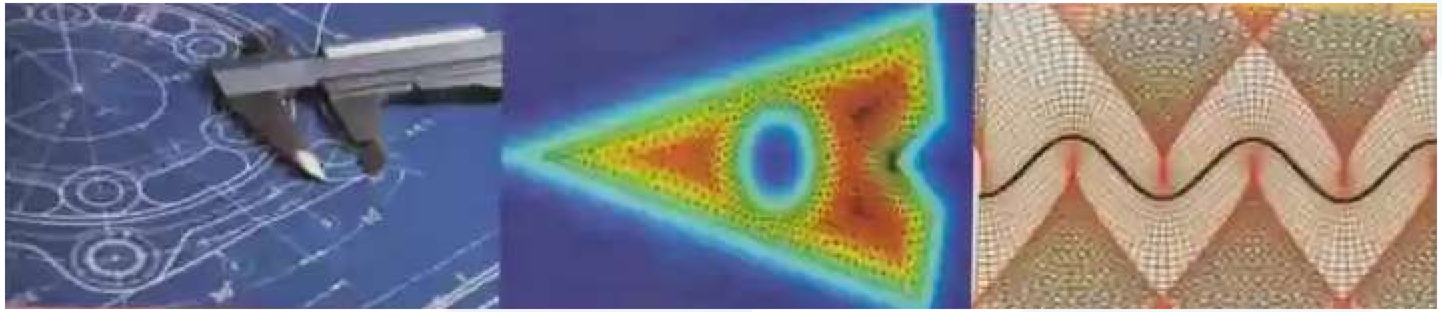
CEDC301: Mathematics Engineering

Exercises 4: Series and Residues: Part A



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1. Find the circle and radius of convergence of the given power series

$$\sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{i}{1+i}\right)^{k} z^{k} \qquad \sum_{k=1}^{\infty}\frac{1}{k^{2}(3+4i)^{k}}(z+3i)^{k} \qquad \sum_{k=0}^{\infty}(-1)^{k}\left(\frac{1+2i}{2}\right)^{k}(z+2i)^{k}$$

2. Show that the power series $\sum_{k=1}^{\infty}\frac{(z-i)^{k}}{k2^{k}}$ is not absolutely convergent on its circle of convergence. Determine at least one point on the circle of convergence at which the power series converges

3. Expand the given function in a Maclaurin series

$$f(z)=\frac{i}{(z-i)(z-2i)} \qquad f(z)=\frac{z-7}{z^{2}-2z-3}$$

3. Expand the given function in a Taylor series centered at the indicated point. Give the radius of convergence of each series

$$f(z) = \frac{1}{1+z},\ z_0 = -i \qquad f(z) = \frac{z-1}{z-3},\ z_0 = 1 \qquad f(z) = \frac{1+z}{1-z},\ z_0 = i$$

4. (a) Suppose the principal branch of the logarithm $f(z) = \operatorname{Log} z = \ln|z| + i \operatorname{Arg} z$ is expanded in a Taylor series with center $z_0 = -1 + i$. Explain why $R = 1$ is the radius of the largest circle centered at $z_0 = -1 + i$ within which f is analytic.

(b) Show that within the circle $|z - (-1 + i)| = 1$ the Taylor series for f is

$$\operatorname{Log} z = \frac{1}{2}\ln 2 + \frac{3\pi}{4} i - \sum_{k=1}^{\infty} \frac{1}{k}\left(\frac{1+i}{2}\right)^k (z+1-i)^k$$

(c) Show that the radius of convergence for the power series in part (b) is $R = \sqrt{2}$. Explain why this does not contradict the result in part (a).

5. (a) Consider the function $f(z) = \mathrm{Log}(1 + z)$. What is the radius of the largest circle centered at the origin within which f is analytic?

(b) Expand f in a Maclaurin series. What is the radius of convergence of this series?

(c) Use the result in part (b) to find a Maclaurin series for $\mathrm{Log}(1 - z)$.

(d) Find a Maclaurin series for

$$\mathrm{Log}\left(\frac{1+z}{1-z}\right)$$

6. Find a Maclaurin series for $\operatorname{erf}(z)$

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, dt$$

7. Expand $f(z) = \dfrac{z}{(z+1)(z-2)}$ in a Laurent series valid for the indicated annular domain

$0 < |z+1| < 3$ $\qquad |z+1| > 3$ $\qquad 1 < |z| < 2$ $\qquad 0 < |z-2| < 3$

8. Determine the order of the poles for the given function

$$f(z) = \frac{\cot \pi z}{z^2} \qquad f(z) = \frac{1 - \cosh z}{z^4} \qquad f(z) = \frac{\sin z}{z^2 - z}$$