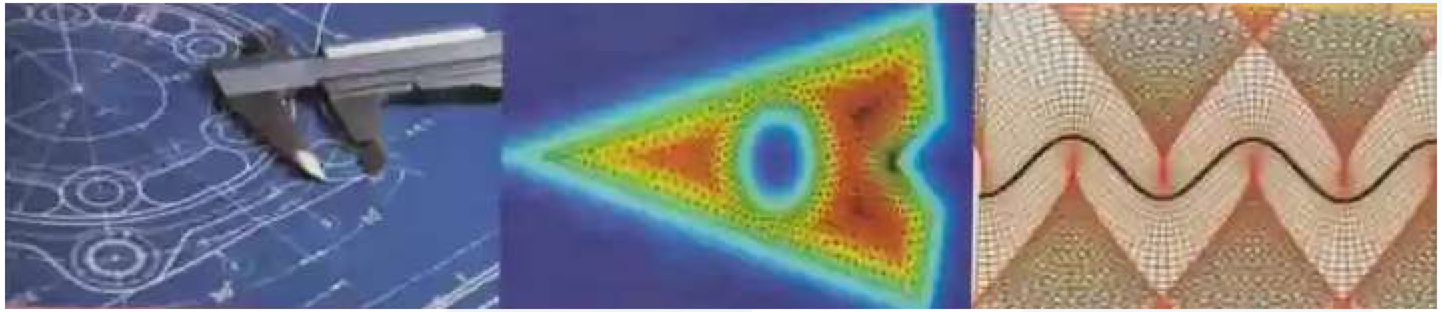
CEDC301: Mathematics Engineering

Exercises 5: Series and Residues: Part B



Ramez Koudsieh, Ph.D.

Faculty of Engineering

Department of Robotics and Intelligent Systems

Manara University

1. Use Cauchy's residue theorem to evaluate the given integral along the indicated contour

$$\oint_C \frac{ze^z}{z^2-1}\,dz,\ C: |z| = 2 \qquad \oint_C \frac{\cot \pi z}{z^2}\,dz,\ C: |z| = \frac{1}{2}$$

$$\oint_C \frac{e^{iz}+\sin z}{(z-\pi)^4}\,dz,\ C: |z-3| = 1 \qquad \oint_C \frac{z}{(z+1)(z^2+1)}\,dz,\ C: \text{ellipse } 16x^2+y^2 = 4$$

$$\oint_C \frac{2z-1}{z^2(z^3+1)}\,dz,\ C \text{ is the rectangle defined by } x=-2,\ y=-\frac{1}{2},\ y=1$$

2. Evaluate the Cauchy principal value of the given improper integral

$$\int_0^\infty \frac{\cos 3x}{(x^2+1)^2}\,dx \qquad \int_0^\infty \frac{x\sin x}{(x^2+1)(x^2+4)}\,dx$$

3. Evaluate the given trigonometric integral $\int_0^{2\pi} \frac{\cos^2\theta}{2+\sin\theta}\,d\theta$

4. Use an indented contour and residues to establish the given result

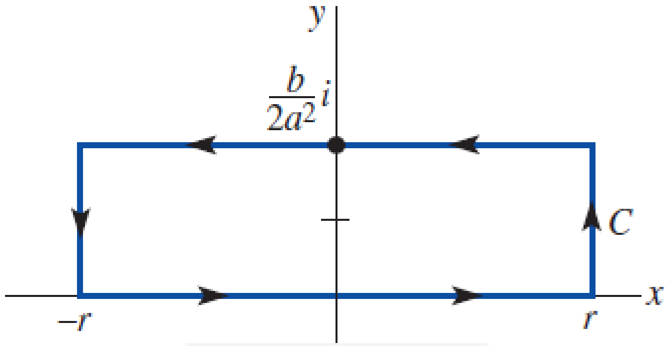
$$\text{P.V.}\int_{-\infty}^{\infty}\frac{\sin x}{x}\,dx=\pi \qquad \text{P.V.}\int_{-\infty}^{\infty}\frac{\sin x}{x(x^2+1)}\,dx=\pi(1-e^{-1})$$

5. Establish the general result $\int_0^{\pi}\frac{d\theta}{(a+\cos\theta)^2}=\frac{a\pi}{(\sqrt{a^2-1})^3},\ a>1$

6. Establish the general result

$$\int_0^{2\pi}\frac{\sin^2\theta}{a+b\cos\theta}\,d\theta=\frac{2\pi}{b^2}\left(a-\sqrt{a^2-b^2}\right),\ a>b>0$$

7. Show that $\int_0^\infty e^{-a^2x^2}\cos bx\,dx = e^{-b^2/4a^2}\sqrt{\pi}/2a$ by considering the complex integral $\oint_C e^{-a^2z^2}e^{ibz}\,dz$ along the contour C shown below. Use $\int_{-\infty}^\infty e^{-a^2x^2}\,dx = \sqrt{\pi}/a$



8. Use the contour shown below to show that

$$\text{P.V.}\int_{-\infty}^{\infty}\frac{e^{ax}}{1+e^x}\,dx = \frac{\pi}{\sin a\pi},\ 0 < a < 1$$