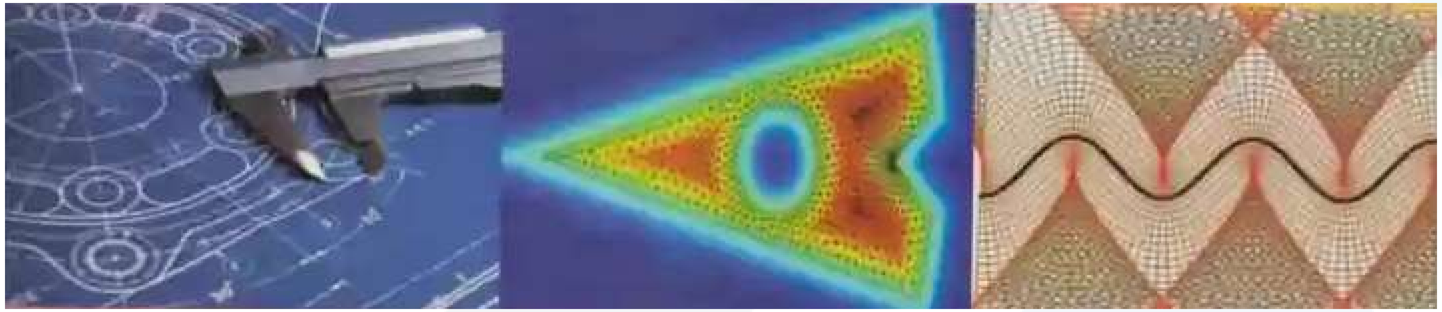
CEDC301: Mathematics Engineering

Exercises 6: Laplace Transform: Part A



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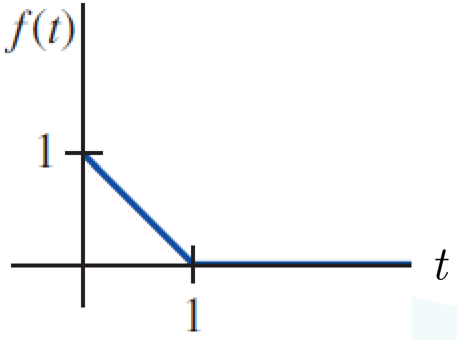
1. Use Definition find $\mathcal{L}\{f(t)\}$

$$f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$$



2. One definition of the gamma function $\Gamma(\alpha)$ is given by the improper integral

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt,\ \alpha > 0$$

Use this definition to show that $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$

3. Use exercise 2 to show that $\mathcal{L}(t^\alpha) = \dfrac{\Gamma(\alpha+1)}{s^{\alpha+1}},\ \alpha > -1$

4. Use the results in exercise 2 and exercise 3 and the fact that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ to find the Laplace transform of the given function

$$f(t) = t^{-1/2} \qquad\qquad f(t) = t^{1/2}$$

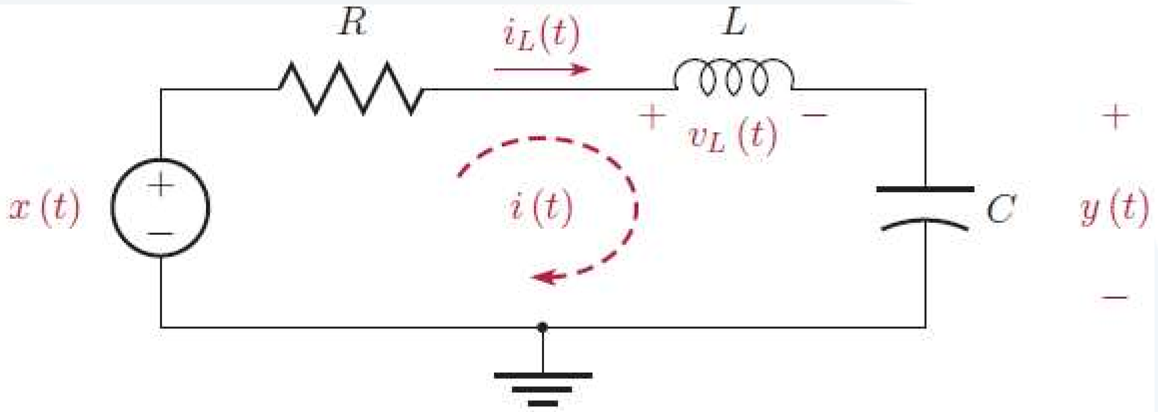
5. Find the given inverse transform

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)(s+2)}\right\} \qquad\qquad \mathcal{L}^{-1}\left\{\frac{6s+3}{(s^2+1)(s^2+4)}\right\}$$

6. Use the Laplace transform to solve the given initial-value problem

$$y' - y = 2\cos 5t,\ y(0) = 0 \qquad\qquad y'' + y = \sqrt{2}\sin\sqrt{2}t,\ y(0) = 10,\ y'(0) = 0$$

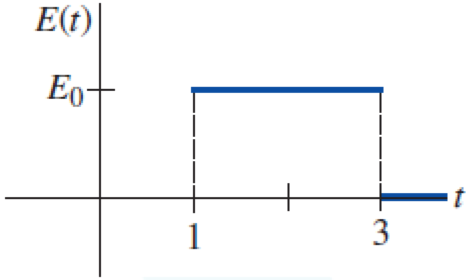
7. Consider the RLC circuit shown below, where the input is a voltage source $x(t)$ and the output the voltage $y(t)$ across the capacitor. Let $LC = 1$ and $R/L = 2$.



Determine the response of the system $y(t)$ to the unit-step function ($x(t) = u(t)$)

8. (a) Use the Laplace transform to find the charge $q(t)$ on the capacitor in an RC-series circuit when $q(0) = 0$, $R = 50\ \Omega, C = 0.01$ F, and $E(t)$ is as given in the figure below.

(b) Assume $E_0 = 100$ V. Graph $q(t)$ for $0 \le t \le 6$. Use the graph to estimate qmax, the maximum value of the charge.



9. Solve the given initial-value problem. Graph the solution

$$y'' + 16y = f(t),\ y(0) = 0,\ y'(0) = 1,\ f(t) = \begin{cases} \cos 4t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$$

10. Find the convolution $f * g$ of the given functions

$$f(t) = e^{-t},\ g(t) = e^{t} \qquad f(t) = e^{-t},\ g(t) = \sin t$$

11. Find

$$\mathcal{L}\left\{t\int_0^t \sin\tau\, d\tau\right\} \qquad \mathcal{L}\left\{t\int_0^t te^{-\tau}\, d\tau\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} \qquad \mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\}$$

12. Solve the given integral equation or integrodifferential equation

$$f(t) = te^{t} + \int_0^t \tau f(t-\tau)\, d\tau \qquad t - 2f(t) = \int_0^t (e^{\tau} - e^{-\tau}) f(t-\tau)\, d\tau$$

13. Find the Laplace transform of the given periodic function