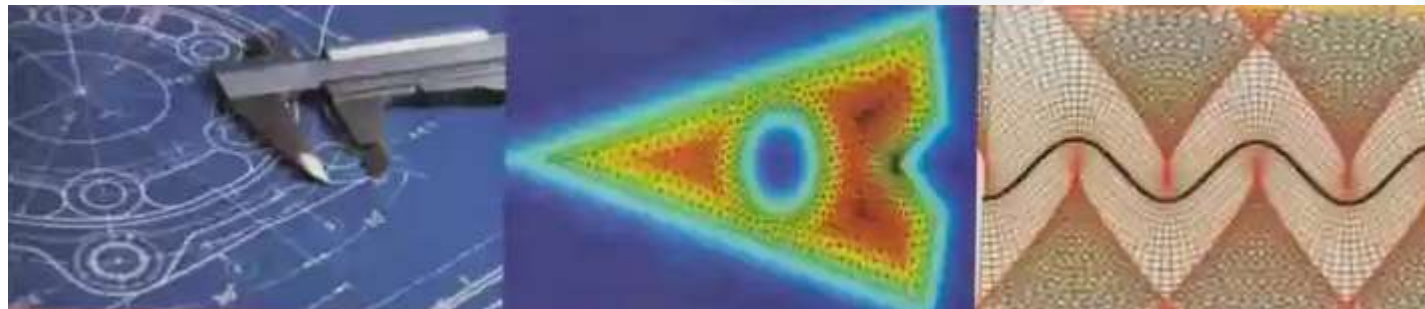


CEDC301: Mathematics Engineering

Exercises 8: Fourier Analysis: Part A



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1. Verify that the functions are orthogonal with respect to the indicated weight function on the given interval

$$H_0(x) = 1, H_1(x) = 2x, H_2(x) = 4x^2 - 2; w(x) = e^{-x^2}, (-\infty, \infty)$$

$$L_0(x) = 1, L_1(x) = -1 + x, L_2(x) = \frac{1}{2}x^2 - 2x + 1; w(x) = e^{-x}, [0, \infty)$$

2. Find the Fourier series of the function f on the given interval

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 \leq x < \pi \end{cases}$$

Use the result to show $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1 \times 3} - \frac{1}{3 \times 5} + \frac{1}{5 \times 7} - \frac{1}{7 \times 9} + \dots$

3. Find the Fourier series of the function f on the given interval

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \leq x < \pi \end{cases}$$

Use the result to show $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$

and $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

Find a series that gives the numerical value of $\pi^2/8$

4. Find the Fourier series of the function f on the given interval

$$f(x) = x + \pi, \quad -\pi < x < \pi$$

Use the result to show $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$