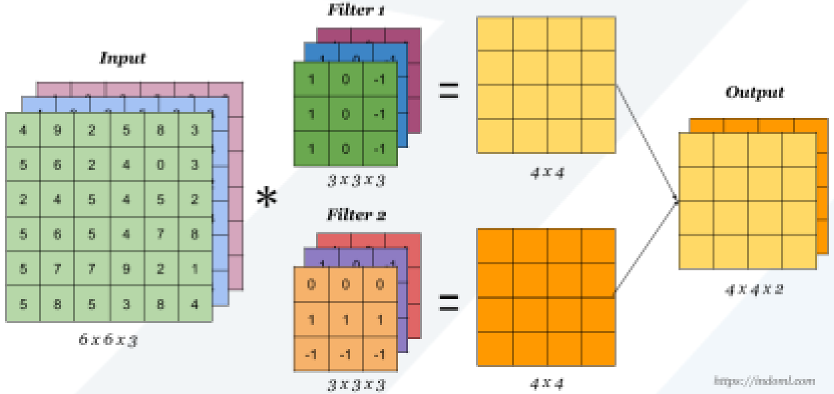
CECC122: Linear Algebra and Matrix Theory

Lecture Notes 5: Vector Spaces: Part A



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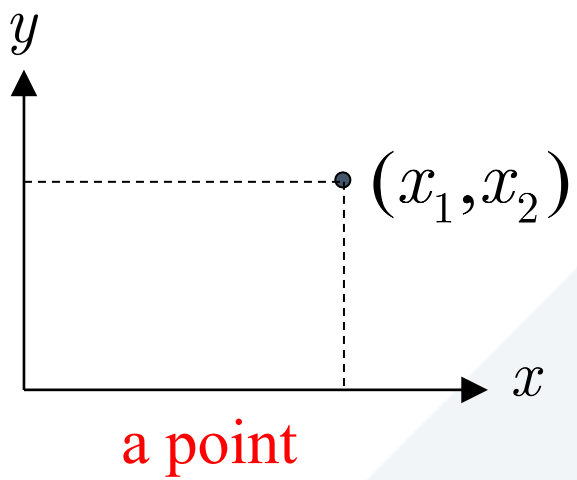
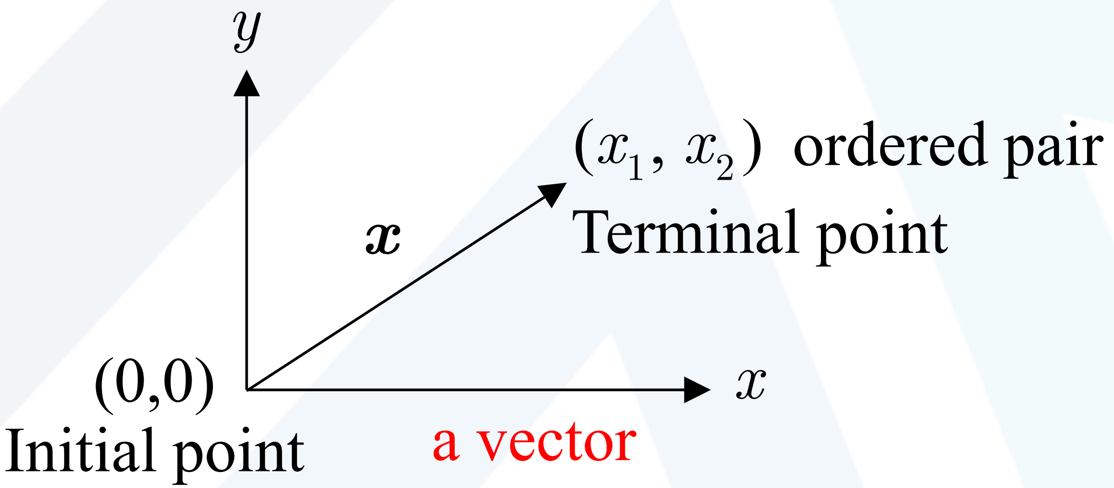
4.1 Vectors in R^n

- Vectors in the plane:

a vector $\boldsymbol{x}$ in the plane is represented by a directed line segment with its initial point at the origin and its terminal point at (x_1, x_2).



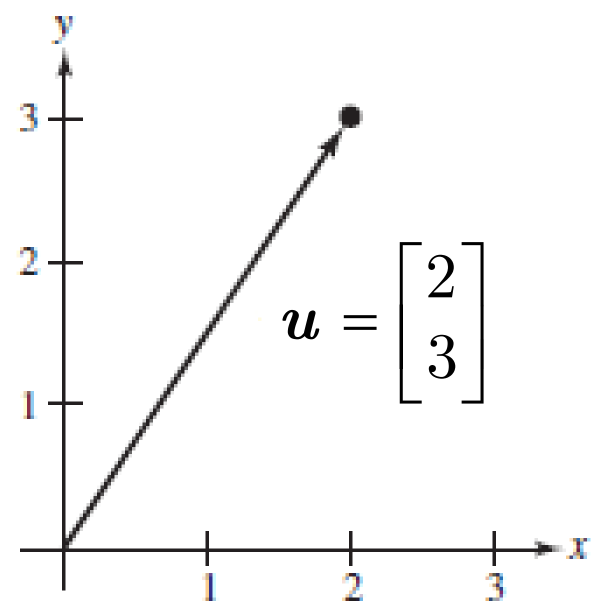


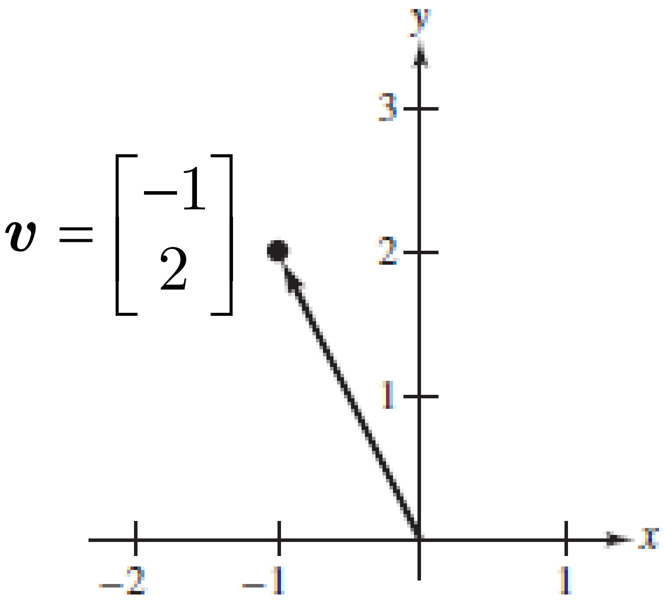
$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

x_1 = first component of $\boldsymbol{x}$

x_2 = second component of $\boldsymbol{x}$

- Ex 1:

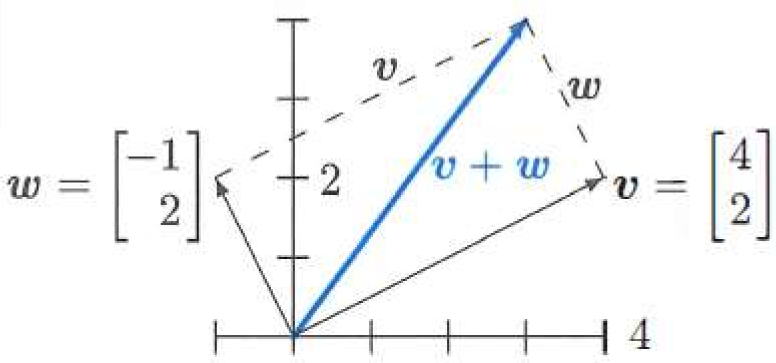


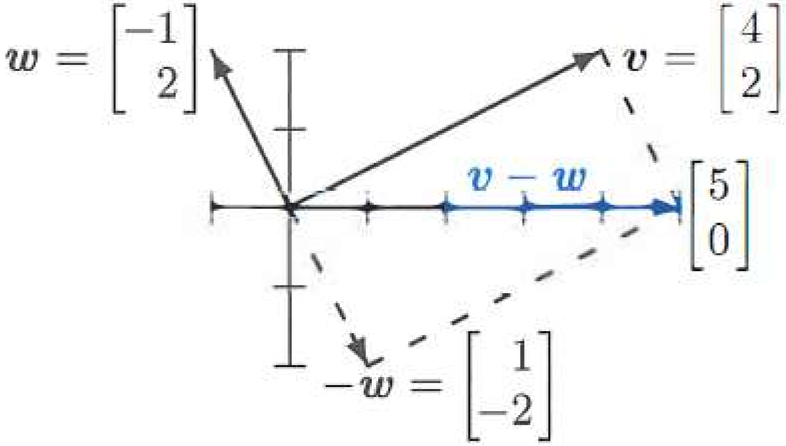


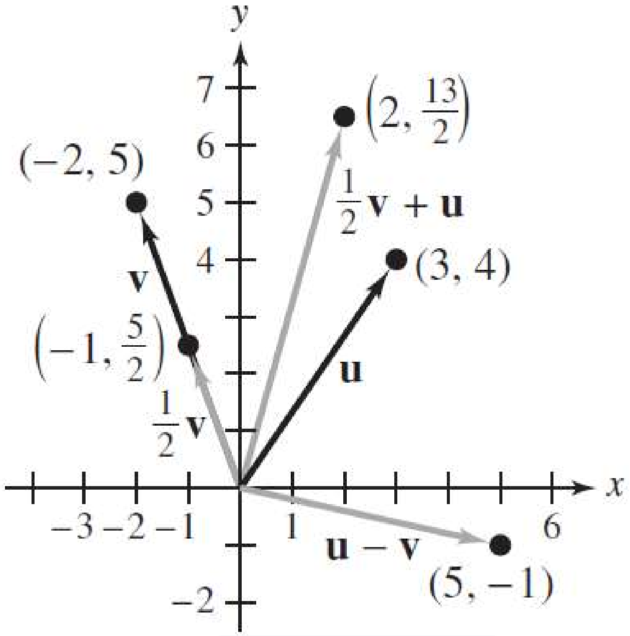
Vector Addition $\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad \boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad \boldsymbol{u} + \boldsymbol{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$

Scalar Multiplication $c\boldsymbol{v} = c\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} cv_1 \\ cv_2 \end{bmatrix}$ $\quad -\boldsymbol{v} = (-1)\boldsymbol{v} \Rightarrow \boldsymbol{u} - \boldsymbol{v} = \boldsymbol{u} + (-\boldsymbol{v})$

- Ex 2:


v
w
2
v + w
w = [-1 2]
v = [4 2]
4


w = [-1 2]
v = [4 2]
v − w
[5 0]
−w = [1 −2]


y
x
(2, 13/2)
(−2, 5)
1/2 v + u
(3, 4)
v
u
(−1, 5/2)
1/2 v
u − v
(5, −1)
7
6
5
4
−2
−3 −2 −1
1
6

- n-space: R^n

R^1 = 1-space = set of all real number (x_1, x_2)

R^2 = 2-space = set of all ordered pair of real numbers (x_1, x_2, x_3)

R^3 = 3-space = set of all ordered triple of real numbers $(x_1, x_2, \cdots, x_n)$

$\vdots$

R^n = n-space = set of all ordered n-tuple of real numbers

- Notes: An n-tuple $(x_1, x_2, \ldots, x_n)$ can be viewed as

(1) a point in R^n with the x_i's as its coordinates.

(2) a vector $\boldsymbol{x}$ in R^n with the x_i's as its components.

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

a vector $\boldsymbol{x}$ in R^n will be represented also as $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$

$$\boldsymbol{u} = (u_1, u_2, \cdots, u_n), \quad \boldsymbol{v} = (v_1, v_2, \cdots, v_n) \quad \text{(two vectors in } R^n\text{)}$$

- Equal:

  $\boldsymbol{u} = \boldsymbol{v}$ if and only if $u_1 = v_1,\ u_2 = v_2,\ \cdots,\ u_n = v_n$

- Vector addition (the sum of $\boldsymbol{u}$ and $\boldsymbol{v}$):

  $$\boldsymbol{u} + \boldsymbol{v} = (u_1 + v_1,\ u_2 + v_2,\ \cdots,\ u_n + v_n)$$

- Scalar multiplication (the scalar multiple of $\boldsymbol{u}$ by c):

  $$c\boldsymbol{u} = (cu_1, cu_2, \cdots, cu_n)$$

- Notes:

  The sum of two vectors and the scalar multiple of a vector in R^n are called the standard operations in R^n.

- Negative:

$$-\boldsymbol{u} = (-u_1, -u_2, \cdots, -u_n)$$

- Difference:

$$\boldsymbol{u} - \boldsymbol{v} = (u_1 - v_1,\ u_2 - v_2,\ \cdots,\ u_n - v_n)$$

- Zero vector:

$$\mathbf{0} = (0,\ 0,\ \cdots,\ 0)$$

- Notes:

(1) The zero vector $\mathbf{0}$ in R^n is called the additive identity in R^n.

(2) The vector $-\boldsymbol{v}$ is called the additive inverse of $\boldsymbol{v}$.

- Ex 3:

Let $\boldsymbol{u} = (-1, 0, 1)$ and $\boldsymbol{v} = (2, -1, 5)$ in R^3.

Perform each vector operation:

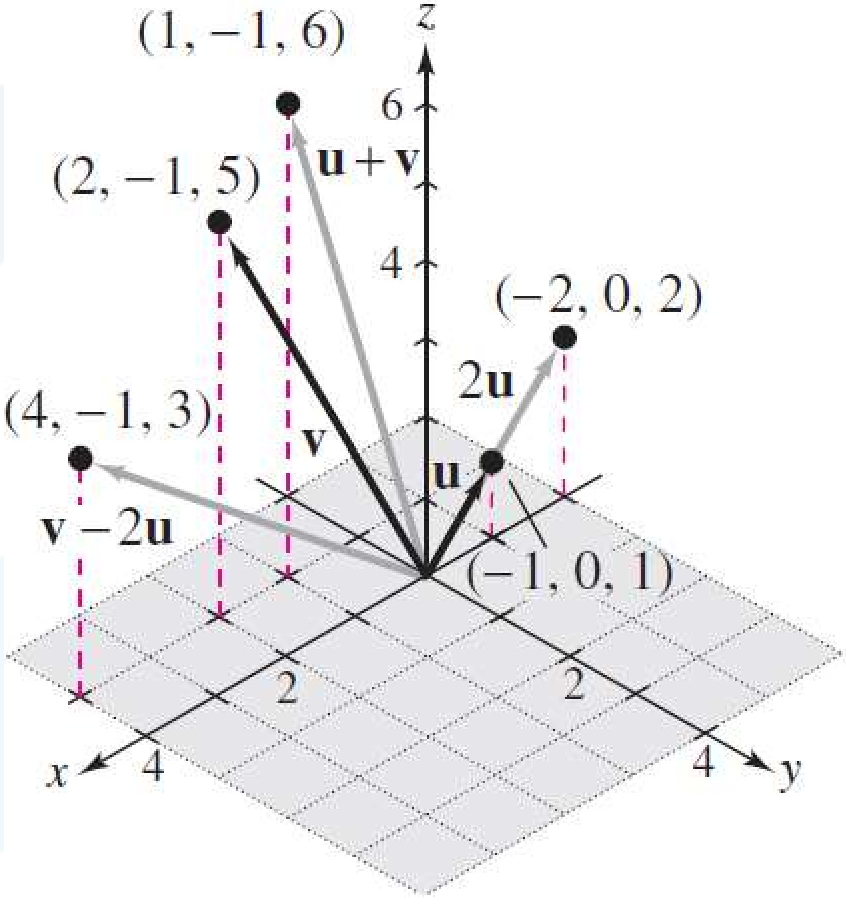
$(a)\ \boldsymbol{u}+\boldsymbol{v}$ $\quad (b)\ 2\boldsymbol{u}$ $\quad (c)\ \boldsymbol{v}-2\boldsymbol{u}$

Sol:

$(a)\ \boldsymbol{u}+\boldsymbol{v} = (-1, 0, 1)+(2, -1, 5) = (1, -1, 6)$

$(b)\ 2\boldsymbol{u} = 2\,(-1, 0, 1) = (-2, 0, 2)$

$(c)\ \boldsymbol{v}-2\boldsymbol{u} = (2, -1, 5)-(-2, 0, 2) = (4, -1, 3)$



- Theorem 4.1: (Properties of vector addition and scalar multiplication)

Let $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$ be vectors in R^n, and let c and d be scalars

(1) $\boldsymbol{u}+\boldsymbol{v}$ is a vector in R^n	Closure under addition
(2) $\boldsymbol{u}+\boldsymbol{v}=\boldsymbol{v}+\boldsymbol{u}$	Commutative property of addition
(3) $(\boldsymbol{u}+\boldsymbol{v})+\boldsymbol{w}=\boldsymbol{u}+(\boldsymbol{v}+\boldsymbol{w})$	Associative property of addition
(4) $\boldsymbol{u}+\boldsymbol{0}=\boldsymbol{u}$	Additive identity property
(5) $\boldsymbol{u}+(-\boldsymbol{u})=\boldsymbol{0}$	Additive inverse property
(6) $c\boldsymbol{u}$ is a vector in R^n	Closure under scalar multiplication
(7) $c(\boldsymbol{u}+\boldsymbol{v})=c\boldsymbol{u}+c\boldsymbol{v}$	Distributive property
(8) $(c+d)\boldsymbol{u}=c\boldsymbol{u}+d\boldsymbol{u}$	Distributive property
(9) $c(d\boldsymbol{u})=(cd)\boldsymbol{u}$	Associative property of multiplication
(10) $1(\boldsymbol{u})=\boldsymbol{u}$	Multiplicative identity property

- Ex 4: (Vector operations in R^4)

Let $\boldsymbol{u} = (2, -1, 5, 0)$, $\boldsymbol{v} = (4, 3, 1, -1)$ and $\boldsymbol{w} = (-6, 2, 0, 3)$ be vectors in R^4. Solve $\boldsymbol{x}$ for each of the following:

$(a)\ \boldsymbol{x} = 2\boldsymbol{u} - (\boldsymbol{v} + 3\boldsymbol{w})$

$(b)\ 3(\boldsymbol{x} + \boldsymbol{w}) = 2\boldsymbol{u} - \boldsymbol{v} + \boldsymbol{x}$

Sol: (a)
$$\begin{aligned}\boldsymbol{x} &= 2\boldsymbol{u} - (\boldsymbol{v} + 3\boldsymbol{w}) \\ &= 2\boldsymbol{u} - \boldsymbol{v} - 3\boldsymbol{w} \\ &= (4, -2, 10, 0) - (4, 3, 1, -1) - (-18, 6, 0, 9) \\ &= (4 - 4 + 18, -2 - 3 - 6, 10 - 1 - 0, 0 + 1 - 9) \\ &= (18, -11, 9, -8)\end{aligned}$$

$$
\begin{aligned}
(b)\ 3(\boldsymbol{x}+\boldsymbol{w}) &= 2\boldsymbol{u}-\boldsymbol{v}+\boldsymbol{x} \\
3\boldsymbol{x}+3\boldsymbol{w} &= 2\boldsymbol{u}-\boldsymbol{v}+\boldsymbol{x} \\
3\boldsymbol{x}-\boldsymbol{x} &= 2\boldsymbol{u}-\boldsymbol{v}-3\boldsymbol{w} \\
2\boldsymbol{x} &= 2\boldsymbol{u}-\boldsymbol{v}-3\boldsymbol{w} \\
\boldsymbol{x} &= \boldsymbol{u}-\frac{1}{2}\boldsymbol{v}-\frac{3}{2}\boldsymbol{w} \\
&= (2, 1, 5, 0) + (-2, \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}) + (9, -3, 0, \frac{-9}{2}) \\
&= (9, \frac{-11}{2}, \frac{9}{2}, -4)
\end{aligned}
$$

Theorem 4.2: (Properties of additive identity and additive inverse)

Let $\boldsymbol{v}$ be a vector in R^n, and c be a scalars. Then the properties below are true:

(1) The additive identity is unique. That is, if $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v}$, then $\boldsymbol{u} = \mathbf{0}$

(2) The additive inverse of $\boldsymbol{v}$ is unique. That is, if $\boldsymbol{v} + \boldsymbol{u} = \mathbf{0}$, then $\boldsymbol{u} = -\boldsymbol{v}$

(3) $0\boldsymbol{v} = \mathbf{0}$

(4) $c\,\mathbf{0} = \mathbf{0}$

(5) If $c\boldsymbol{v} = \mathbf{0}$, then $c = 0$ or $\boldsymbol{v} = \mathbf{0}$

(6) $-(-\boldsymbol{v}) = \boldsymbol{v}$

- Linear combination:

The vector $\boldsymbol{x}$ is called a linear combination of $\boldsymbol{v_1}, \boldsymbol{v_2}, \ldots, \boldsymbol{v_n}$ if it can be expressed in the form $\boldsymbol{x} = c_1\boldsymbol{v_1} + c_2\boldsymbol{v_2} + \cdots + c_n\boldsymbol{v_n}$ $\quad c_1, c_2, \ldots, c_n$: scalars

- Ex 5: Given $\boldsymbol{x} = (-1, -2, -2)$, $\boldsymbol{u} = (0, 1, 4)$, $\boldsymbol{v} = (-1, 1, 2)$, and $\boldsymbol{w} = (3, 1, 2)$ in R^3, find a, b, and c such that $\boldsymbol{x} = a\boldsymbol{u} + b\boldsymbol{v} + c\boldsymbol{w}$.

Sol:

$$\begin{array}{rcrcrcr} & & -b & + & 3c & = & -1 \\ a & + & b & + & c & = & -2 \\ 4a & + & 2b & + & 2c & = & -2 \end{array}$$

$\Rightarrow a = 1,\ b = -2,\ c = -1$

Thus $\boldsymbol{x} = \boldsymbol{u} - 2\boldsymbol{v} - \boldsymbol{w}$

4.2 Vector Spaces

- Vector spaces:

Let V be a set on which two operations (vector addition and scalar multiplication) are defined. If the following axioms are satisfied for every $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$ in V and every scalar c and d, then V is called a vector space.

Addition:

(1) $\boldsymbol{u} + \boldsymbol{v}$ is in V — Closure under addition

(2) $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$ — Commutative property

(3) $\boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w}) = (\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w}$ — Associative property

(4) V has a zero vector $\mathbf{0}$: for every $\boldsymbol{u}$ in V, $\boldsymbol{u} + \mathbf{0} = \boldsymbol{u}$ — Additive identity

(5) For every $\boldsymbol{u}$ in V, there is a vector in V denoted by $-\boldsymbol{u}$: $\boldsymbol{u} + (-\boldsymbol{u}) = \mathbf{0}$ — Scalar identity

Scalar multiplication:

(6) $c\boldsymbol{u}$ is a vector in V — Closure under scalar multiplication

(7) $c(\boldsymbol{u}+\boldsymbol{v}) = c\boldsymbol{u} + c\boldsymbol{v}$ — Distributive property

(8) $(c+d)\boldsymbol{u} = c\boldsymbol{u} + d\boldsymbol{u}$ — Distributive property

(9) $c(d\boldsymbol{u}) = (cd)\boldsymbol{u}$ — Associative property

(10) $1(\boldsymbol{u}) = \boldsymbol{u}$ — Scalar identity

- Notes:

(1) A vector space $(V, +, .)$ consists of four entities:

a nonempty set V of vectors, a set of scalars, and two operations $(+, .)$

(2) $V = \{\mathbf{0}\}$ zero vector space

- Examples of vector spaces:

(1) n-tuple space: $V = R^n$

$$(u_1, u_2, \cdots, u_n) + (v_1, v_2, \cdots, v_n) = (u_1 + v_1, u_2 + v_2, \cdots, u_n + v_n) \quad \text{vector addition}$$

$$k(u_1, u_2, \cdots, u_n) = (ku_1, ku_2, \cdots, ku_n) \quad \text{scalar multiplication}$$

(2) Matrix space: $V = M_{m\text{x}n}$ (the set of all $m \times n$ matrices with real values)

Ex: $(m = n = 2)$

$$\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \begin{bmatrix} u_{11} + v_{11} & u_{12} + v_{12} \\ u_{21} + v_{21} & u_{22} + v_{22} \end{bmatrix} \quad \text{vector addition}$$

$$k\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix} \quad \text{scalar multiplication}$$

(3) n-th degree polynomial space: $V = P_n(x)$

(the set of all real polynomials of degree n or less)

$$p(x) + q(x) = (a_0 + b_0) + (a_1 + b_1)x + \cdots + (a_n + b_n)x^n$$

$$kp(x) = ka_0 + ka_1x + \cdots + ka_nx^n$$

(4) Function space: $V = c(-\infty, \infty)$

(the set of all real functions)

$$(f + g)(x) = f(x) + g(x)$$

$$(kf)(x) = kf(x)$$

- Theorem 4.3: (Properties of scalar multiplication)

  Let $\boldsymbol{v}$ any element of a vector space V, and let c be any scalars. Then the following properties are true:

  (1) $0\boldsymbol{v} = \mathbf{0}$

  (2) $c\mathbf{0} = \mathbf{0}$

  (3) If $c\boldsymbol{v} = \mathbf{0}$, then $c = 0$ or $\boldsymbol{v} = \mathbf{0}$

  (4) $(-1)\boldsymbol{v} = -\boldsymbol{v}$

4.3 Subspaces of Vector Spaces

- Subspace:

$(V, +, .)$: a vector space

$$\left.\begin{array}{l} W \neq \emptyset \\ W \subseteq V \end{array}\right\}$$: a nonempty subset

$(W, +, .)$: a vector space (under the operations of addition and scalar multiplication defined in V)

$\Rightarrow W$ is a subspace of V

- Trivial subspace: Every vector space V has at least two subspaces

(1) Zero vector space $\{\mathbf{0}\}$ is a subspace of V.

(2) V is a subspace of V.

- Theorem 4.4: (Test for a subspace)

  If W is a nonempty subset of a vector space V, then W is a subspace of V if and only if the following conditions hold:

  (1) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are in W, then $\boldsymbol{u}+\boldsymbol{v}$ is in W.

  (2) If $\boldsymbol{u}$ is in W and c is any scalar, then $c\boldsymbol{u}$ is in W.

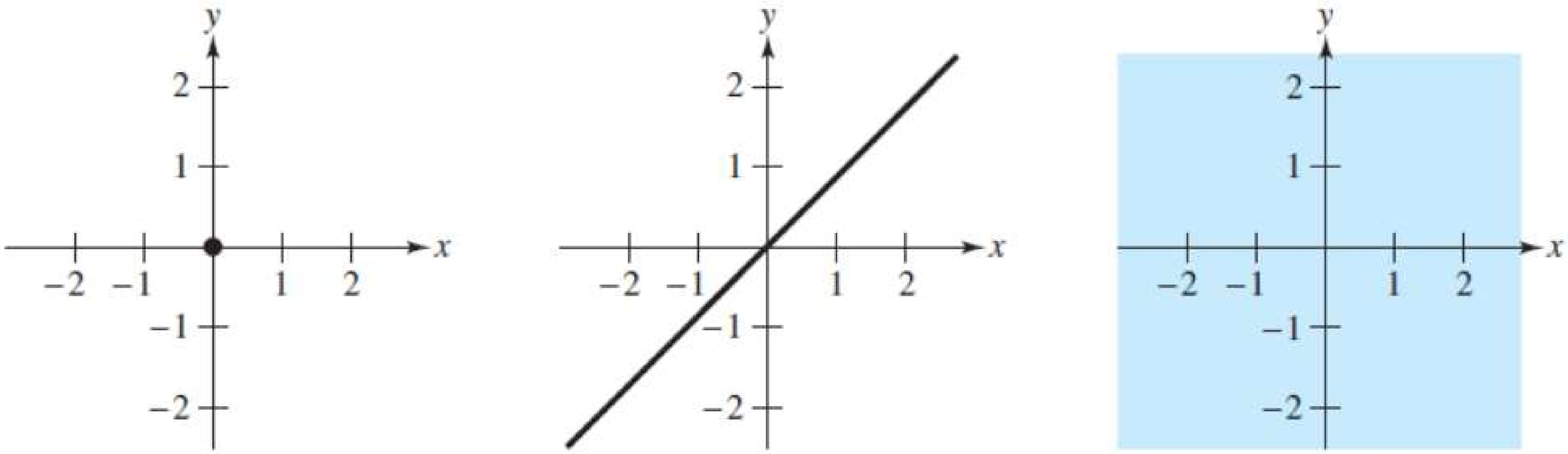
- Notes:

  (1) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are in W, c and d are any scalars, then $c\boldsymbol{u}+d\boldsymbol{v}$ is in W.

  $\Rightarrow W$ is a subspace of V

  (2) If W is a subspace of a vector space V, then W contains the zero vector $\mathbf{0}$ of V

■ Ex 1: Subspace of R^2



$W = \{(0, 0)\}$

W = all points on a line passing through the origin

$W = R^2$

(1) $\{\mathbf{0}\}$ $\qquad \mathbf{0} = (0, 0)$

(2) Lines through the origin

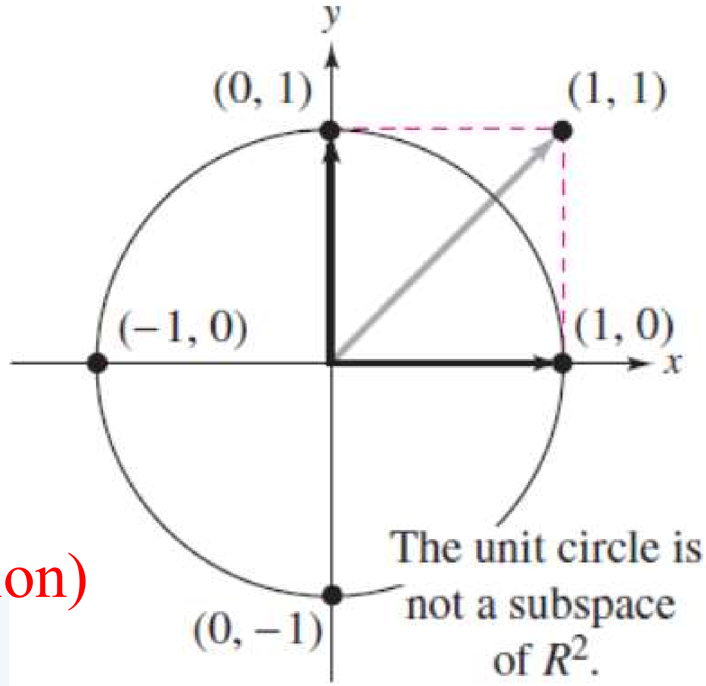
(3) R^2

- Ex 2: (A Subset of R^2 That Is Not a Subspace)

  Show that the subset of R^2 consisting of all points on $x^2 + y^2 = 1$ is not a subspace

  Sol:

  points (1, 0) and (0, 1) are in the subset, but their sum $(1, 0) + (0, 1) = (1, 1)$ is not. (not closed under addition)



- Ex 3: Subspace of R^3

  (1) $\{\mathbf{0}\}$ $\qquad \mathbf{0} = (0, 0, 0)$

  (2) Lines through the origin

  (3) Planes through the origin

  (4) R^3

- Ex 4: (Determining subspaces of R^2)

Which of the following two subsets is a subspace of R^2?

(a) The set of points on the line given by $x + 2y = 0$. Yes

(b) The set of points on the line given by $x + 2y = 1$. No

- Theorem 4.5: (The intersection of two subspaces is a subspace)

If V and W are both subspaces of a vector space U, then the intersection of V and W (denoted by $V \cap W$) is also a subspace of U.

4.4 Spanning Sets and Linear Independence

- **Linear combination:**

A vector $\boldsymbol{v}$ in a vector space V is called a linear combination of the vectors $\boldsymbol{u_1}$, $\boldsymbol{u_2}, \ldots, \boldsymbol{u_k}$ in V if $\boldsymbol{v}$ can be written in the form

$$\boldsymbol{v} = c_1\boldsymbol{u_1} + c_2\boldsymbol{u_2} + \cdots + c_k\boldsymbol{u_k} \qquad c_1, c_2, \ldots, c_k\text{: scalars}$$

- **Ex 1: (Finding a linear combination)**

$$\boldsymbol{v_1} = (1,\ 2,\ 3), \qquad \boldsymbol{v_2} = (0,\ 1,\ 2), \qquad \boldsymbol{v_3} = (-1,\ 0,\ 1)$$

Prove (a) $\boldsymbol{w} = (1, 1, 1)$ is a linear combination of $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}$

(b) $\boldsymbol{w} = (1, -2, 2)$ is not a linear combination of $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}$

Sol: $(a)\ \boldsymbol{w} = c_1\boldsymbol{v_1} + c_2\boldsymbol{v_2} + c_3\boldsymbol{v_3}$

$$(1,1,1) = c_1(1,2,3) + c_2(0,1,2) + c_3(-1,0,1)$$
$$= (c_1 - c_3,\ 2c_1 + c_2,\ 2c_2 + c_3)$$

$$\Rightarrow \begin{array}{rcrcrl} c_1 & & & - & c_3 & = 1 \\ 2c_1 & + & c_2 & & & = 1 \\ 3c_1 & + & 2c_2 & + & c_3 & = 1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 1 \end{array}\right] \xrightarrow{\text{Gauss-Jordan Elimination}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow c_1 = 1 + t,\ c_2 = -1 - 2t,\ c_3 = t$ (this system has infinitely many solutions)

$t = 1 \Rightarrow \boldsymbol{w} = 2\boldsymbol{v_1} - 3\boldsymbol{v_2} + \boldsymbol{v_3}$

(b) $\boldsymbol{w} = c_1\boldsymbol{v_1} + c_2\boldsymbol{v_2} + c_3\boldsymbol{v_3}$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & -2 \\ 3 & 2 & 1 & 2 \end{array}\right] \xrightarrow{\text{Gauss-Jordan Elimination}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & 7 \end{array}\right]$$

$\Rightarrow$ this system has no solution $(0 \neq 7)$

$\Rightarrow \boldsymbol{w} \neq c_1\boldsymbol{v_1} + c_2\boldsymbol{v_2} + c_3\boldsymbol{v_3}$

- A spanning set of a vector space:

  If every vector in a given vector space can be written as a linear combination of vectors in a given set S, then S is called a spanning set of the vector space.

- Ex 2: (A spanning set for R^3)

  The set $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ spans R^3 because any vector $\boldsymbol{u} = (u_1, u_2, u_3)$ in R^3 can be written as

$$\boldsymbol{u} = u_1(1, 0, 0) + u_2(0, 1, 0) + u_3(0, 0, 1) = (u_1, u_2, u_3)$$

- The span of a set: span (S)

  If $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ is a set of vectors in a vector space V, then the span of S is the set of all linear combinations of the vectors in S,

$$\text{span}(S) = \left\{c_1\boldsymbol{v_1} + c_2\boldsymbol{v_2} + \cdots + c_k\boldsymbol{v_k} \mid \forall c_i \in R\right\}$$

(the set of all linear combinations of the vectors in S)

- Linear Independent (L.I.) and Linear Dependent (L.D.):

$S = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \ldots, \boldsymbol{v_k}\}$ is a set of vectors in a vector space V,

$$c_1\boldsymbol{v_1} + c_2\boldsymbol{v_2} + \cdots + c_k\boldsymbol{v_k} = 0$$

(1) If the equation has only the trivial solution ($c_1 = c_2 = \ldots\ c_k = 0$), then S is called linearly independent.

(2) If the equation has a non trivial solution (i.e. not all zeros), then S is called linearly dependent.

- Notes

(1) $\emptyset$ is linearly independent.

(2) $\mathbf{0} \in S \Rightarrow S$ is linearly dependent.

(3) $\boldsymbol{v} \neq \mathbf{0} \Rightarrow \{\boldsymbol{v}\}$ is linearly independent.

(4) $S_1 \subseteq S_2$

S_1 is linearly dependent $\Rightarrow S_2$ is linearly dependent

S_2 is linearly independent $\Rightarrow S_1$ is linearly independent

- Ex 3: (Testing for linearly independent)

Determine whether the following set of vectors in R^3 is L.I. or L.D.

$$S = \{\underset{\boldsymbol{v}_1}{(1, 2, 3)}, \underset{\boldsymbol{v}_2}{(0, 1, 2)}, \underset{\boldsymbol{v}_3}{(-2, 0, 1)}\}$$

Sol:

$$c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + c_3\boldsymbol{v}_3 = 0 \Rightarrow \begin{array}{rcrcrl} c_1 & & & - & 2c_3 & = 0 \\ 2c_1 & + & c_2 & & & = 0 \\ 3c_1 & + & 2c_2 & + & c_3 & = 0 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right] \xrightarrow{\text{Gauss-Jordan Elimination}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\Rightarrow c_1 = c_2 = c_3 = 0$ (only the trivial solution) $\Rightarrow S$ is linearly independent

- Independence of two vectors:

Two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in a vector space V are linearly dependent if and only if one is a scalar multiple of the other.

- Ex 4: (Testing for linear dependent of 2 Vectors)

(1) $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2\} = \{(1, 2, 0), (-2, 2, 1)\}$ is L.I. because $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are not scalar multiples of each other.

(2) $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2\} = \{(4, -4, -2), (-2, 2, 1)\}$ is L.D. because $\boldsymbol{v}_1 = -2\boldsymbol{v}_2$