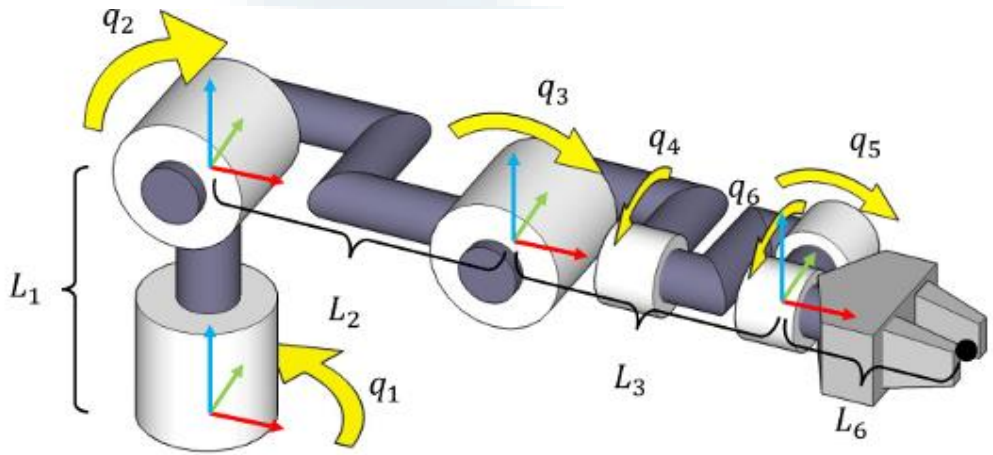
Inverse kinematic model

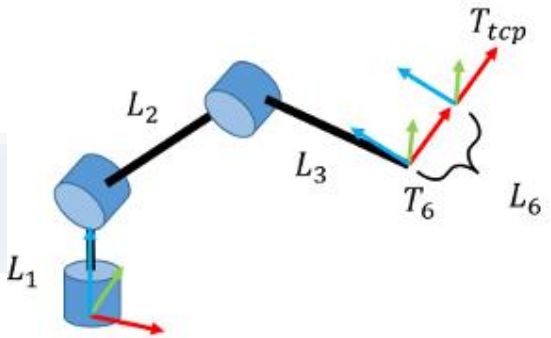
Paul Method

Arm+Wrist+ End Effector

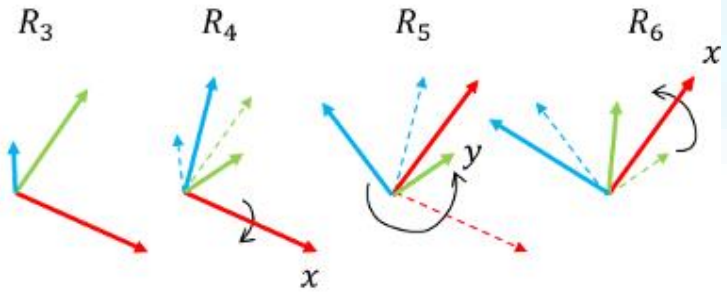
Paul Method



(a)

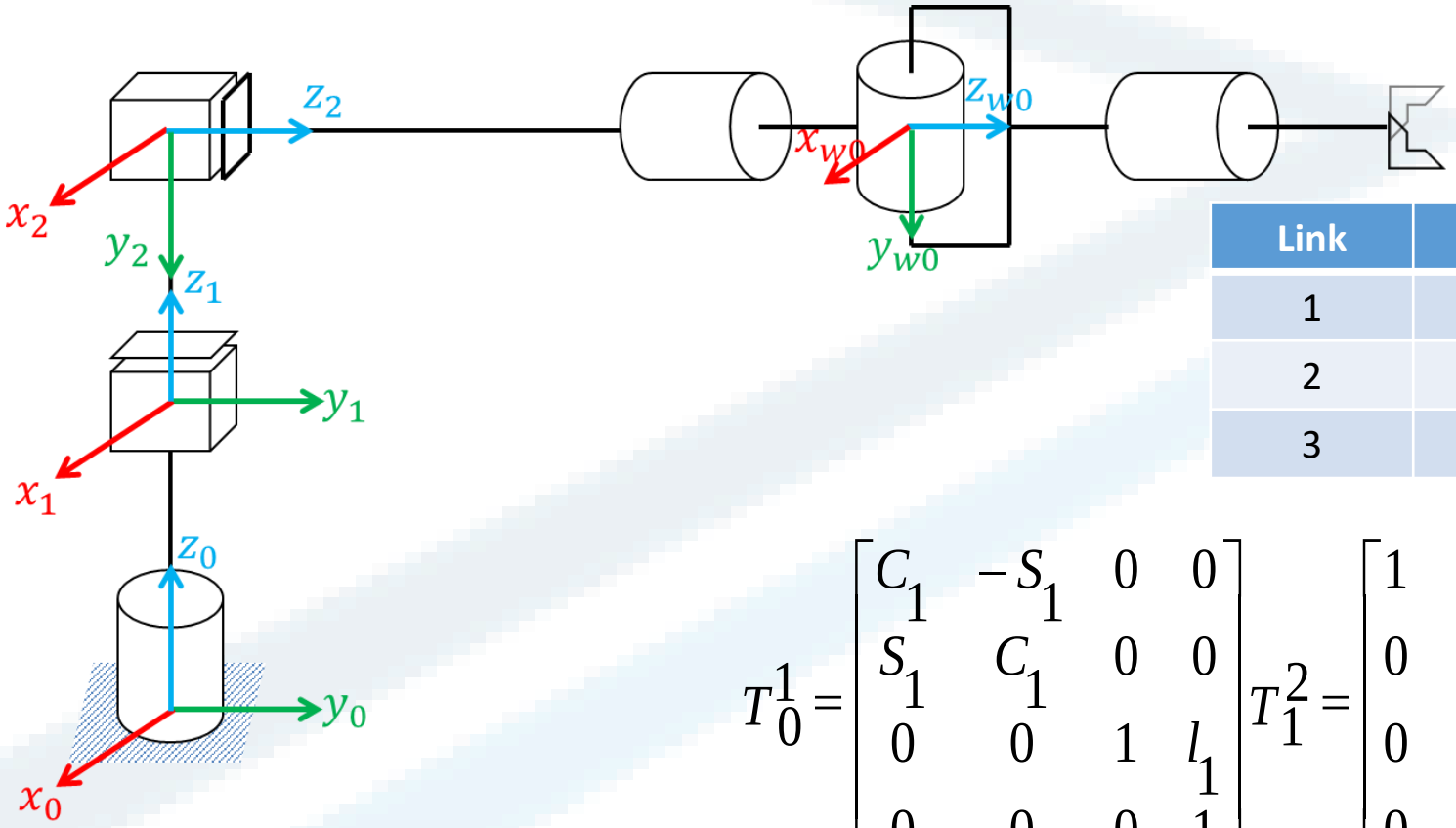


(b)



(c)

Arm to wrist

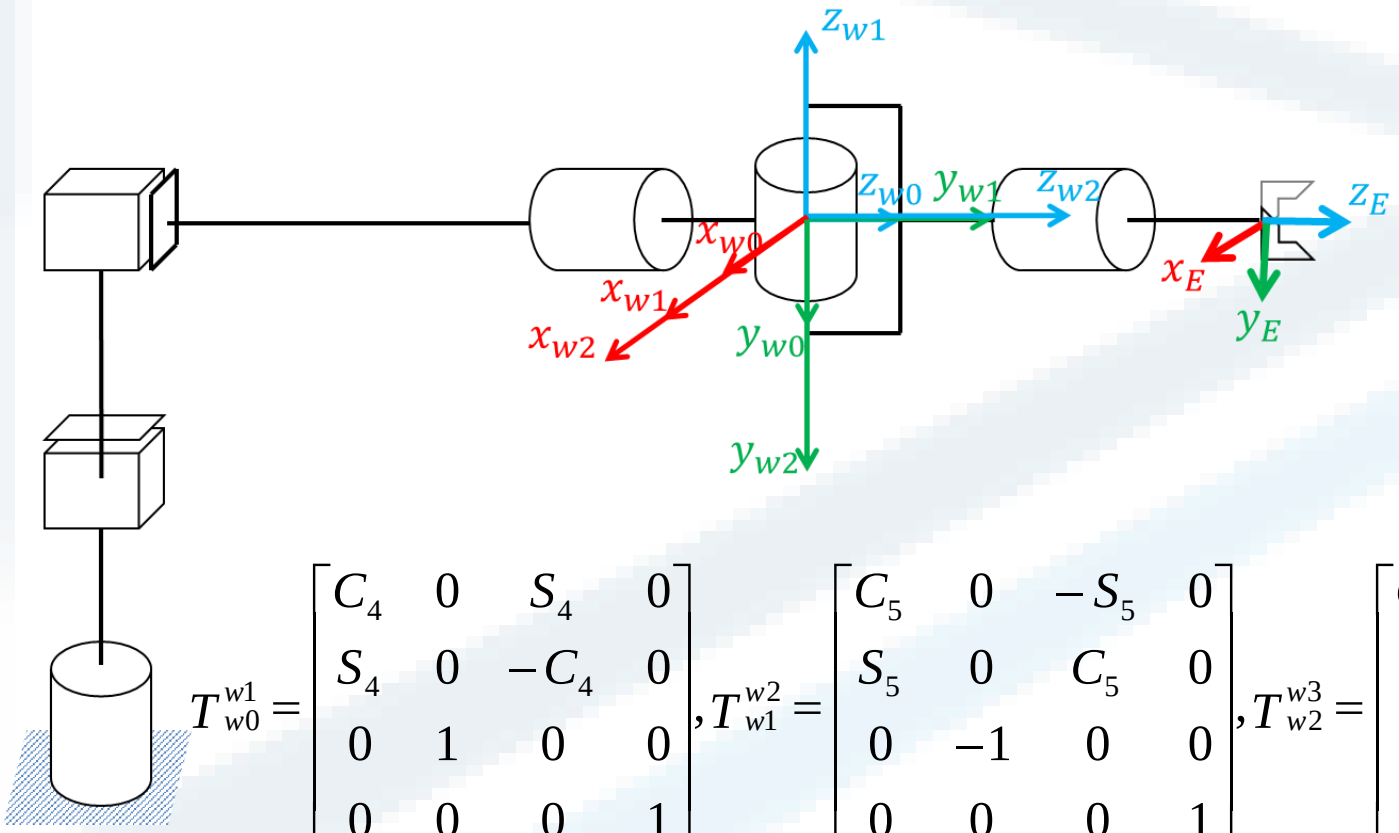


Link	a	α	d	θ
1	0	0	L1	θ1
2	0	-π/2	d2	0
3	0	0	d3	0

$$T_0^1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_2^{w0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical wrist

Wrist to End-effector



Link	a	α	d	θ
1w	0	π/2	0	θ4
2w	0	-π/2	0	θ5
3w	0	0	0	θ6
	0	0	L6	0

$$T_{w0}^{w1}=\begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{w1}^{w2}=\begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{w2}^{w3}=\begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{w3}^{E}=\begin{pmatrix} I & P_{w3}^{E} \\ 0 & 1 \end{pmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^E = T_0^{w0} T_{w0}^{w3} T_{w3}^E$$

$$T_0^{w0} = \begin{pmatrix} R_0^{w0} & P_0^{w0} \\ 0 & 1 \end{pmatrix}$$

$$T_{w0}^{w3} = \begin{pmatrix} R_{w0}^{w3} & P_{w0}^{w3} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} R_{w0}^{w1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R_{w1}^{w2} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R_{w2}^{w3} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} R_{w0}^{w1} R_{w1}^{w2} R_{w2}^{w3} & 0 \\ 0 & 1 \end{pmatrix}$$

$$T_0^{w3} = \begin{pmatrix} R_0^{w0} & P_0^{w0} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R_{w0}^{w1} R_{w1}^{w2} R_{w2}^{w3} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} R_0^{w0} R_{w0}^{w1} R_{w1}^{w2} R_{w2}^{w3} & P_0^{w0} \\ 0 & 1 \end{pmatrix}$$

$$\left.\begin{array}{l} T_0^E T_E^{w3} = T_0^{w0} T_{w0}^{w3} \\ T_0^E T_E^{w3} = T_0^{w3} \\ T_0^E T_E^{w3} = T_0^{w0} T_{w0}^{w3} \\ T_{w0}^0 T_0^E T_E^{w3} = T_{w0}^{w3} \end{array}\right\} \Rightarrow$$

$$T_0^E = T_0^{w0} T_{w0}^{w3} T_{w3}^E$$

$$T_0^{w0} = \begin{pmatrix} R_0^{w0} & P_0^{w0} \\ 0 & 1 \end{pmatrix} = \begin{bmatrix} C_1 & 0 & -S_1 & -d_3 S_1 \\ S_1 & 0 & C_1 & d_3 C_1 \\ 0 & -1 & 0 & l_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{w0}^{w3} = \begin{bmatrix} C_4 C_5 C_6 - S_4 S_6 & -C_4 C_5 S_6 - S_4 C_6 & -C_4 S_5 & 0 \\ S_4 C_5 C_6 + C_4 S_6 & -S_4 C_5 S_6 + C_4 C_6 & -S_4 S_5 & 0 \\ S_5 C_6 & -S_5 S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^{w3} = \begin{pmatrix} R_0^{w3} & P_0^{w0} \\ 0 & 1 \end{pmatrix}$$

$$T_{w3}^E = \begin{pmatrix} I & P_{w3}^E \\ 0 & 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow T_E^{w3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} T_0^E T_E^{w3} &= T_0^{w0} T_{w0}^{w3} \\ T_0^E T_E^{w3} &= T_0^{w3} \\ T_0^E T_E^{w3} &= T_0^{w0} T_{w0}^{w3} \\ T_{w0}^0 T_0^E T_E^{w3} &= T_{w0}^{w3} \end{aligned}$$

Numerical example

$$\left.\begin{array}{lllll} & & L_1 = 100 & L_6 = 250 \\ (E)Position: & d_x = 0 & d_y = 200 & d_z = 500 \\ (E)Orientation: & \psi = \pi/2 & \theta = 0 & \varphi = 0 \end{array}\right\} \Rightarrow IKM = ?$$

EULER

$$T_0^E = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 200 \\ 0 & 0 & 1 & 500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Arm2Wrist

$$T_0^{w3} = \begin{pmatrix} R_0^{w0} R_{w0}^{w1} R_{w1}^{w2} R_{w2}^{w3} & P_0^{w0} \\ 0 & 1 \end{pmatrix}$$

$$T_0^{w0} = \begin{pmatrix} R_0^{w0} & P_0^{w0} \\ 0 & 1 \end{pmatrix}$$

Theta1, D2,D3

$$T_0^E T_E^{w3} = T_0^{w3} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 200 \\ 0 & 0 & 1 & 500 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 200 \\ 0 & 0 & 1 & 250 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^{w0} = \begin{pmatrix} R_0^{w0} & P_0^{w0} \\ 0 & 1 \end{pmatrix} = T_0^1 * T_1^2 * T_2^{w0} = \begin{bmatrix} C_1 & 0 & -S_1 & -d_3 S_1 \\ S_1 & 0 & C_1 & d_3 C_1 \\ 0 & -1 & 0 & l_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_2 = 150$$

$$d_3 = 200$$

$$\theta_1 = 0$$

$$\left.\begin{array}{l} d_2 = 150 \\ d_3 = 200 \\ \theta_1 = 0 \end{array}\right\} \Rightarrow T_0^{w0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 200 \\ 0 & -1 & 0 & 250 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow T_{w0}^{0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 250 \\ 0 & 1 & 0 & -200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{w0}^{0} T_0^{E} T_E^{w3} = T_{w0}^{w3}$$

$$T_{w0}^{w3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 250 \\ 0 & 1 & 0 & -200 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 200 \\ 0 & 0 & 1 & 500 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Wrist

$$T_{w0}^{w3} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{w0}^{w3} = T_{w0}^{w1} * T_{w1}^{w2} * T_{w2}^{w3} = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & -C_4S_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & -S_4S_5 & 0 \\ C_6S_5 & -S_6S_5 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

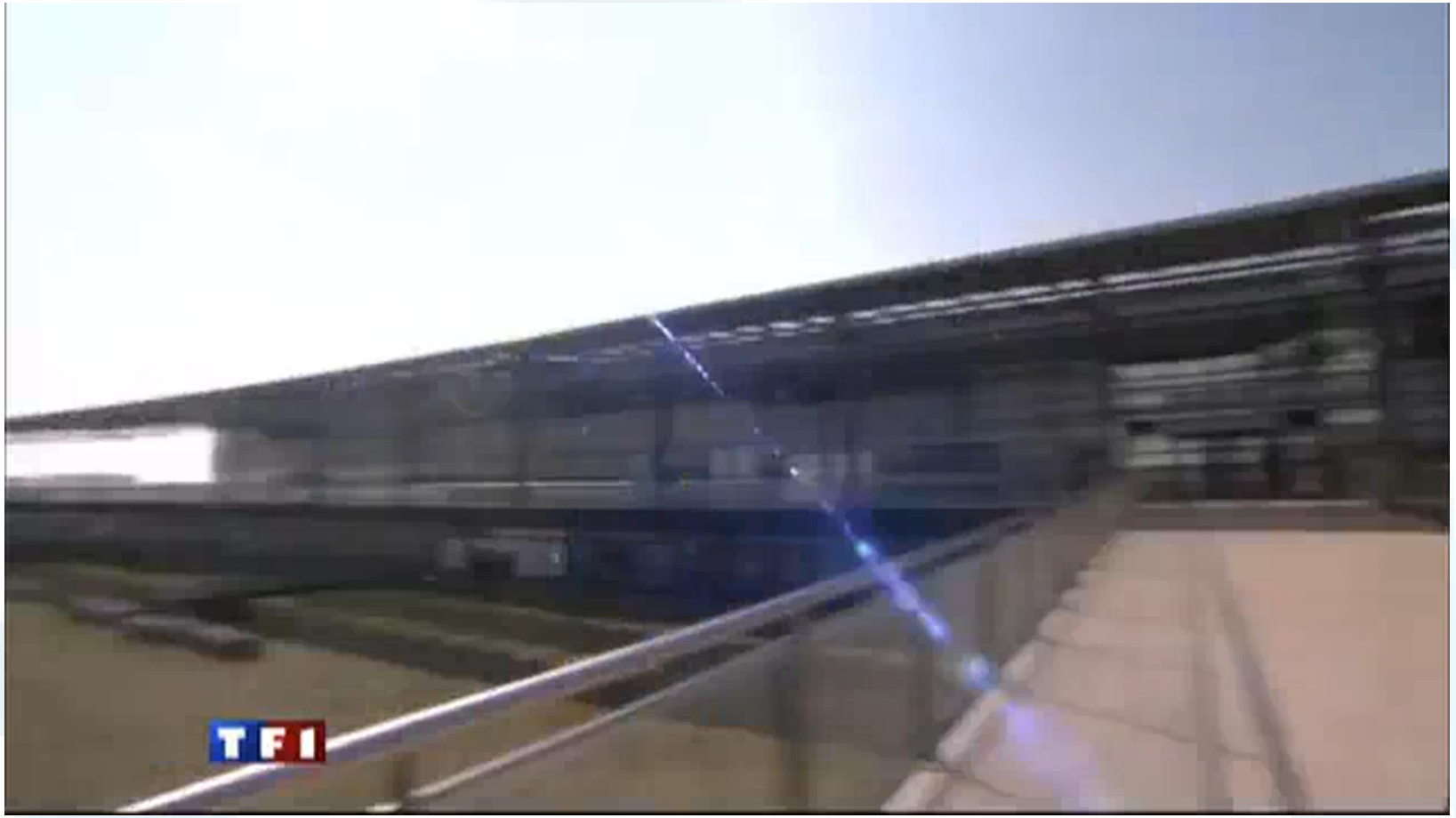
Theta 4,5,6

$$\theta_5 = \pm\frac{\pi}{2}$$

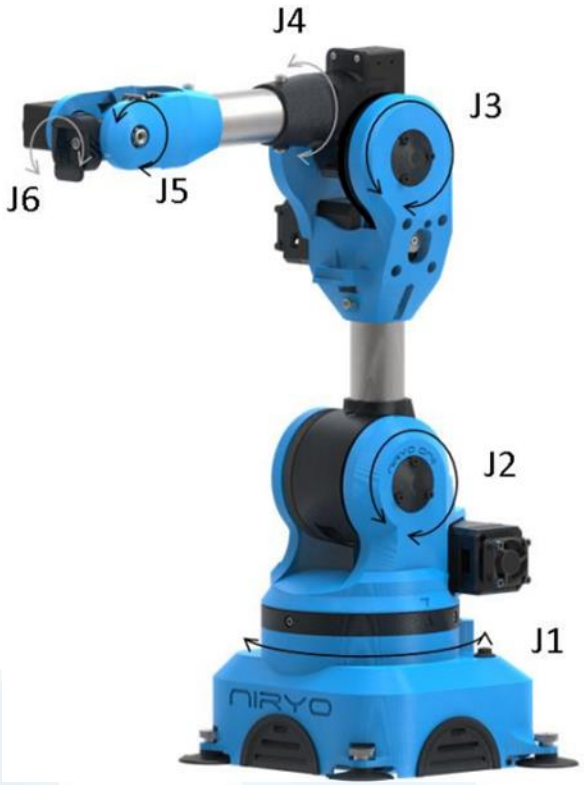
$$\theta_4 = a\tan 2(\pm 1,0) = \pm\frac{\pi}{2}$$

$$\theta_6 = a\tan 2(0,1) = 0$$

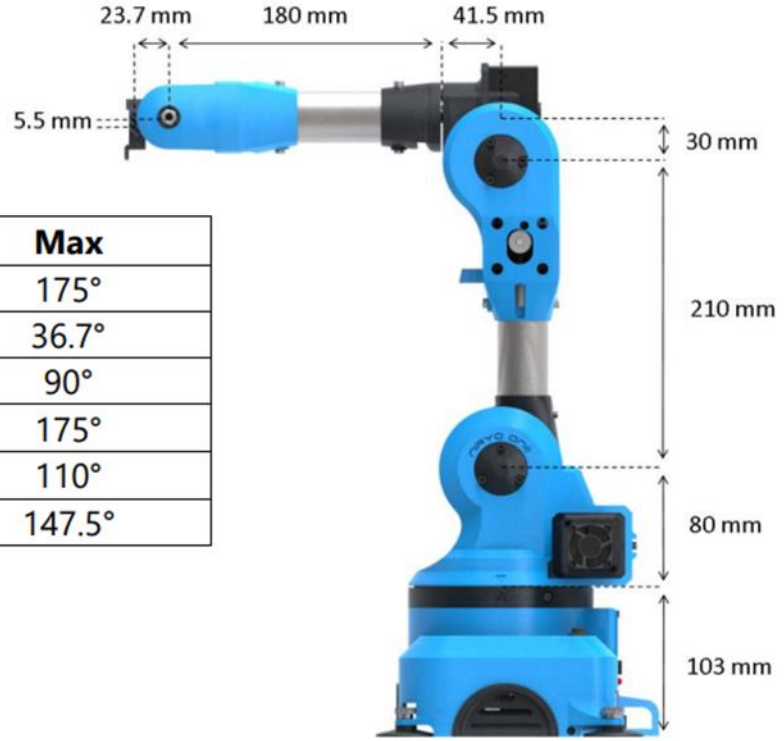
6 Rotations Robot



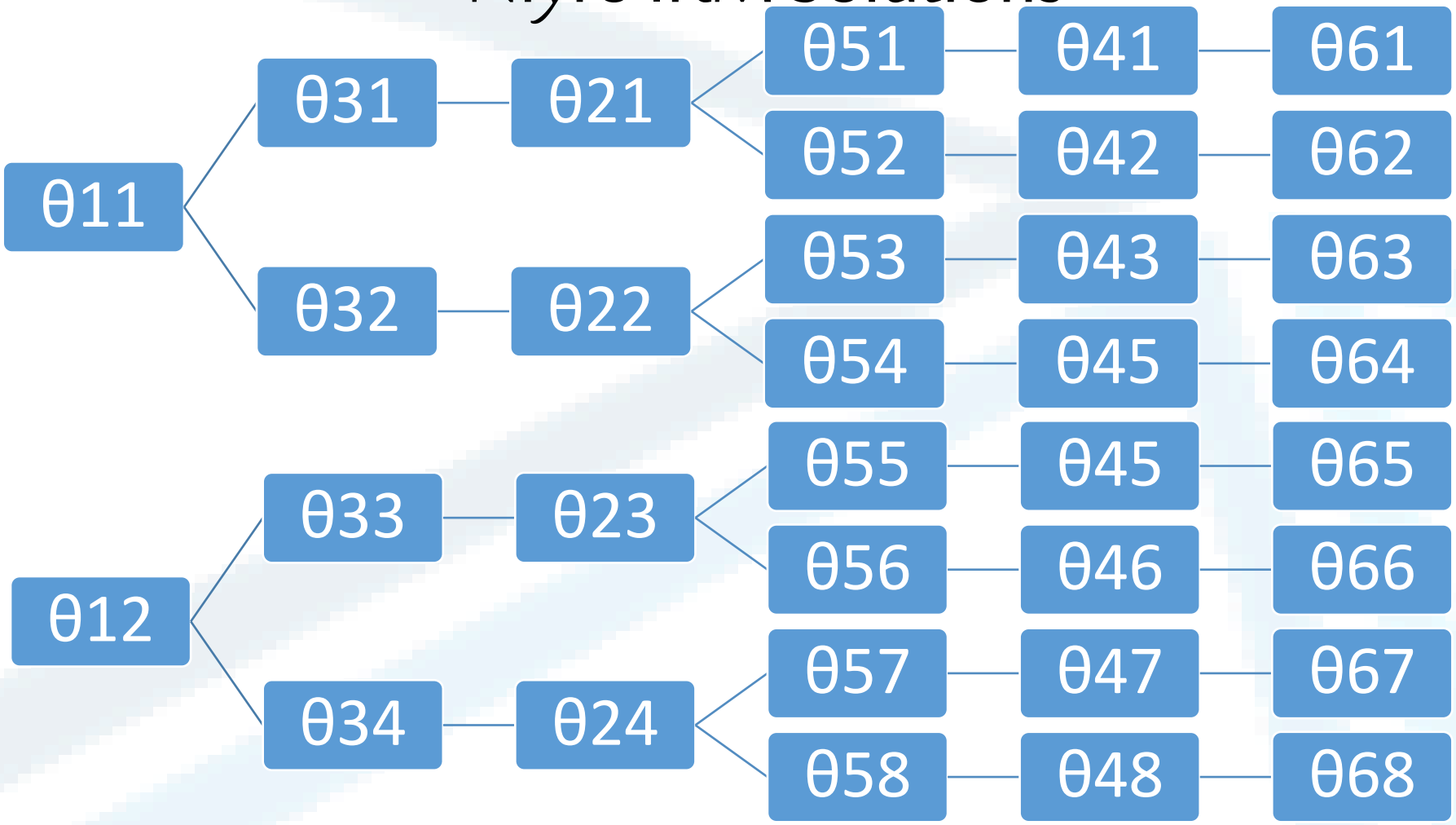
Niryo IKM



	Min	Max
J1	-175°	175°
J2	-90°	36.7°
J3	-80°	90°
J4	-175°	175°
J5	-100°	110°
J6	-147.5°	147.5°



Niyro IKM Solutions



Example

```
>> Generalized_Parameters=[pi/2,pi/6,pi/4,pi/2,pi/3,pi/12]

Generalized_Parameters =

    1.5708    0.5236    0.7854    1.5708    1.0472    0.2618

>> [ Actual_Parameters ] = Niryo_DKM(Generalized_Parameters);

T1 =
```

	Position	Euler_Angels	Euler_Orientation
Ex	61.17	Psi	1.64
Ey	-67.296	Theta	2.6362
Ez	626.99	Phi	1.5016

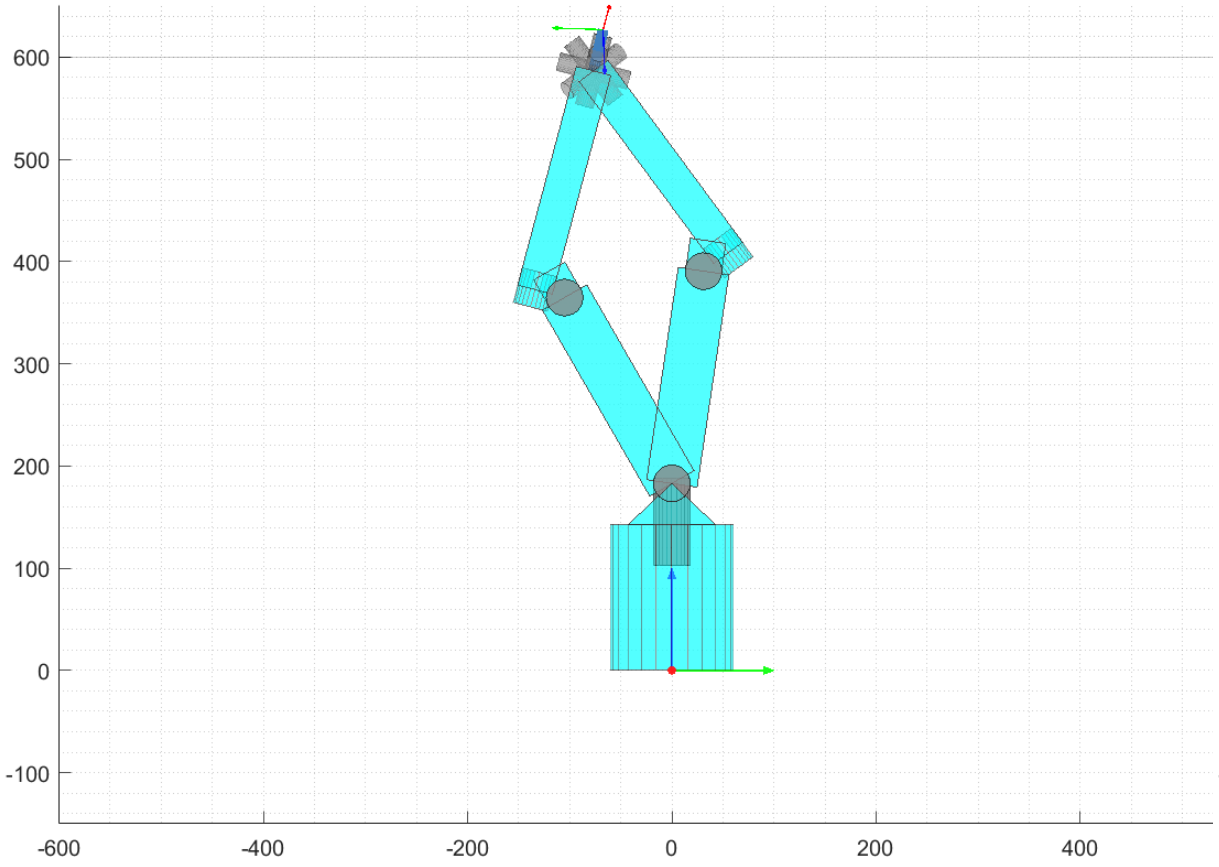
```
>> [ Solutions] = Niryo IKM( Actual Parameters );
```

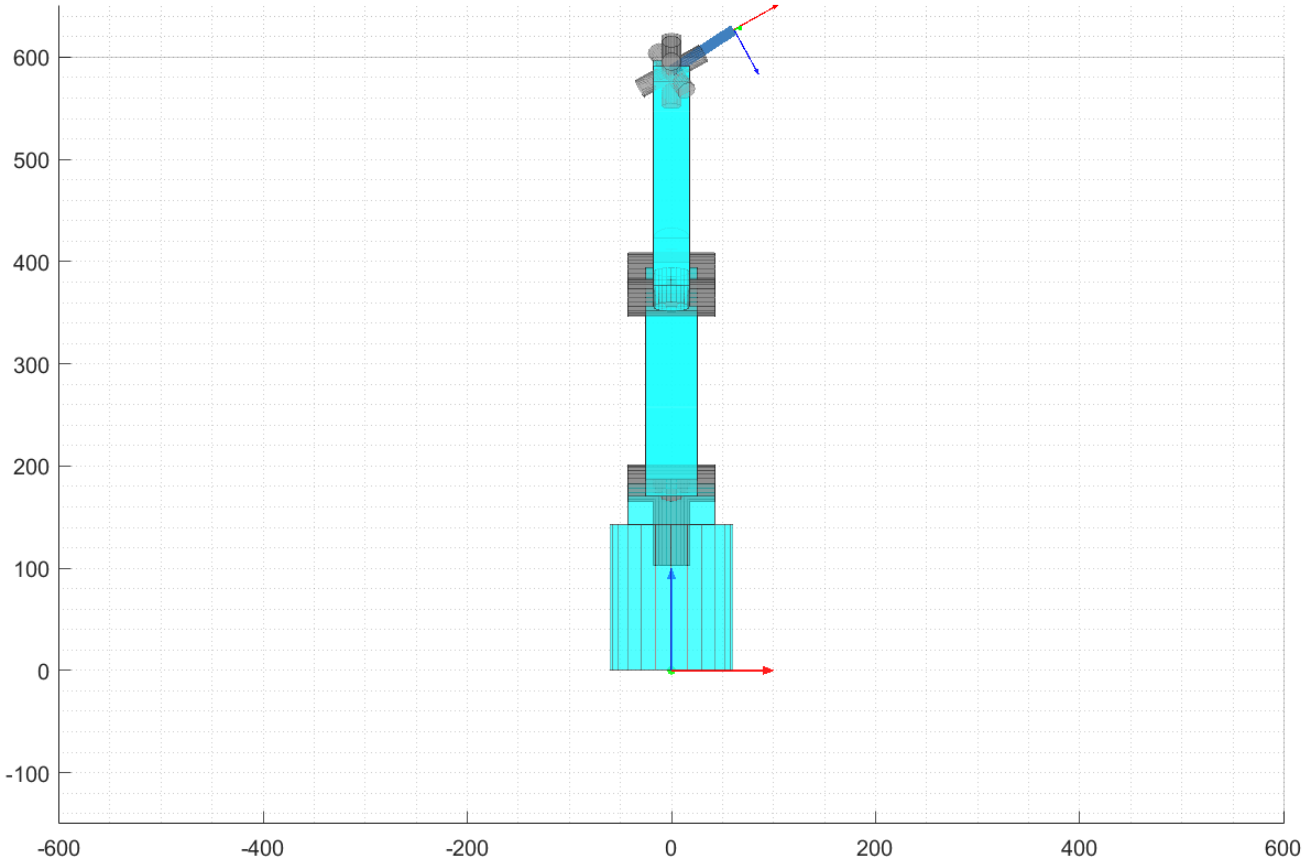
Solutions

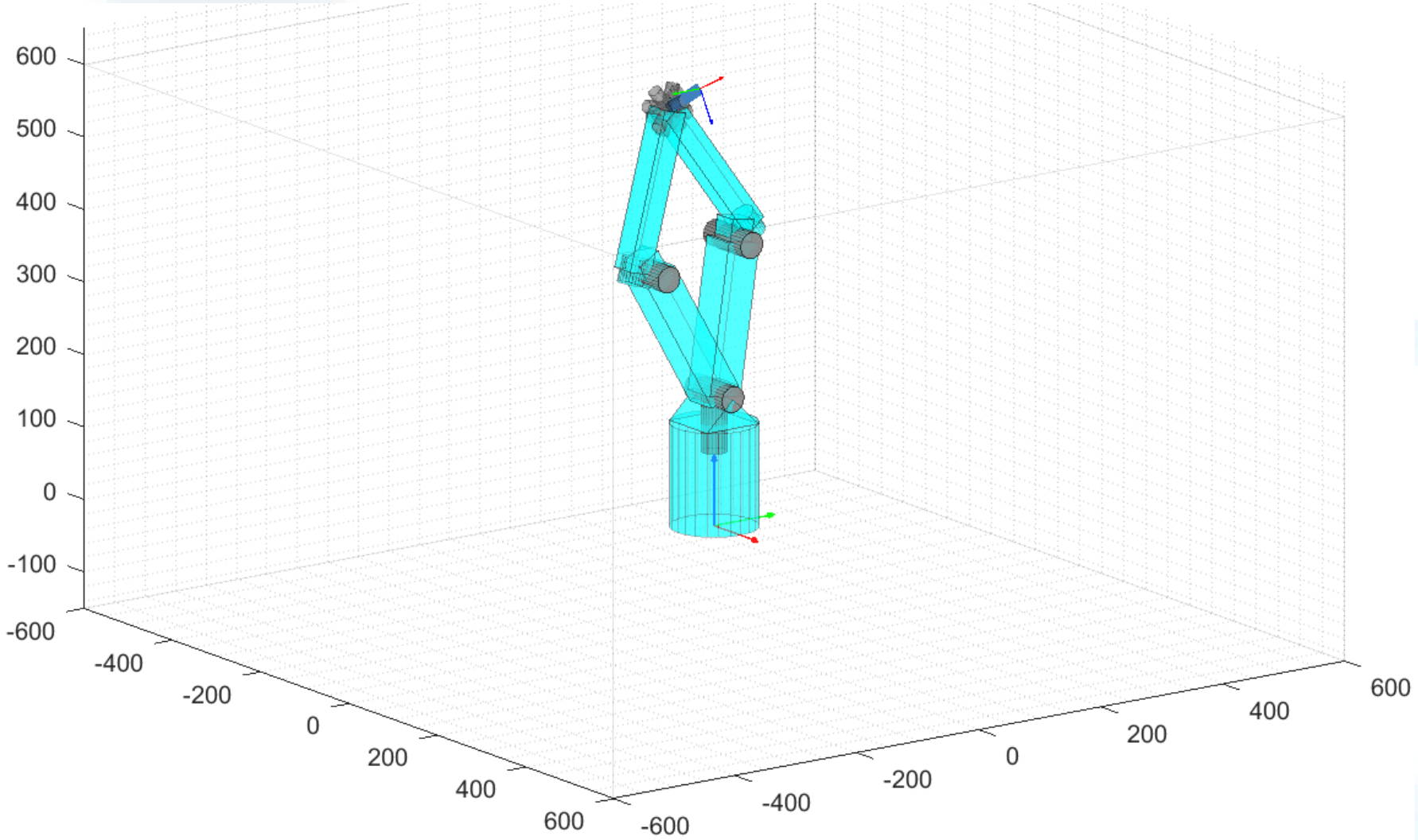
```
>> [ Solutions] = Niryo_IKM( Actual_Parameters );

T1 =
```

	Solution1	Solution2	Solution3	Solution4	Solution5	Solution6	Solution7	Solution8
Theta1	-90	-90	-90	-90	90	90	90	90
Theta2	-30	-30	8.49	8.49	-8.49	-8.49	30	30
Theta3	119.57	119.57	45	45	119.57	119.57	45	45
Theta4	98.73	-81.27	114.32	-65.68	-71.22	108.78	-90	90
Theta5	-61.18	61.18	-71.87	71.87	-66.17	66.17	-60	60
Theta6	177.33	-2.67	139.55	-40.45	154.92	-25.08	-165	15
IsOk	0	0	1	1	0	0	0	1
JL	0	0	0	0	0	0	0	0


600
500
400
300
200
100
0
-100
-600
-400
-200
0
200
400


600
500
400
300
200
100
0
-100
-600
-400
-200
0
200
400
600


600
500
400
300
200
100
0
-100
-600
-400
-200
0
200
400
600
-600
-400
-200
0
200
400
600

Homework

Thanks

ملاحظة: هذه المحاضرة تتيح فهم الموديل الهندسي العكسي لروبوت تسلسلي ذو ٦ درجات حرية و توضح طريقة بول المستخدمة في إيجاد البارامترات

يجب العمل على فهم هذه الطريقة بشكل جيد حتى نتمكن من استخدامها في حساب موديل السرعة العكسي