Robot Control

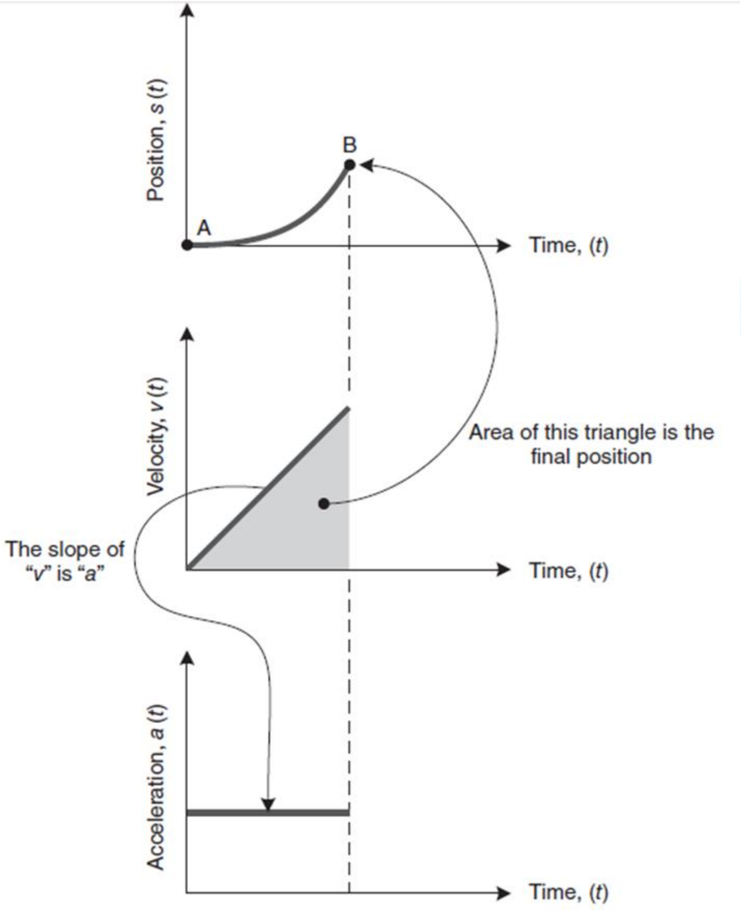
Velocity of an axis

Industrial Motion Control Book

Basic concepts

$$v_{(t)} = \frac{ds}{dt} \rightarrow s = \int v_{(t)}.dt$$

$$a_{(t)} = \frac{dv}{dt} \rightarrow v = \int a_{(t)}.dt$$



Geometric Rules for Motion Profile

- Position at time *t* is equal to the area under the velocity curve up to time *t*, and the acceleration is the slope of the velocity curve.

$$v = v_0 + a(t - t_0)$$

$$s = s_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

- Where:

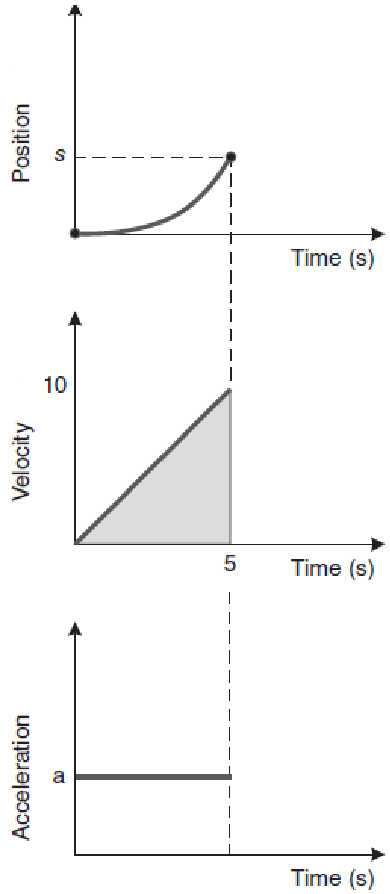
t_0 is the initial time

v_0 is the initial velocity

s_0 is the initial position. The acceleration "a" is constant.

Example 1

- Given the velocity profile, find the position and acceleration at $t = 5$ sec.



Solution

- Slope of the velocity profile is the acceleration

$$a = \frac{10}{5} = 2\ cm/sec^2$$

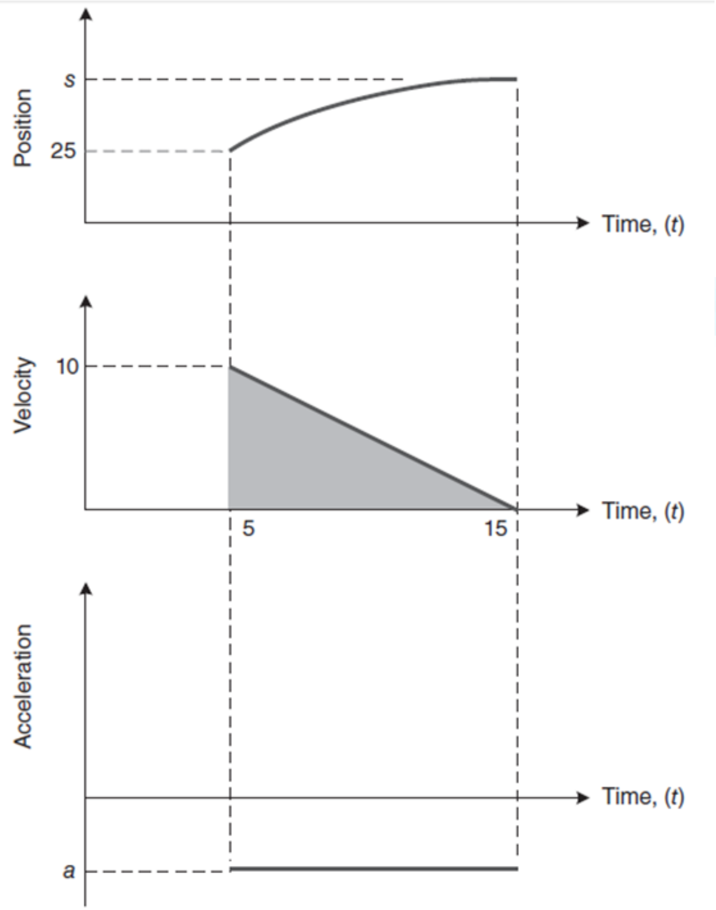
- The triangular area under the velocity curve up to $t = 5$ sec is the position reached at $t = 5$ sec.

$$s = \frac{1}{2}(10 \times 5) = 25\ cm$$

Example 2

An axis is **traveling** at a speed of 10 cm/sec. At $t = 5$ sec it starts to **slow down** as given by the velocity profile in the figure beside.

- What is the axis position when it stops? Assume that the axis starts decelerating at 25 cm.



Solution

Slope of the velocity profile is the acceleration.

In this case, the slope is negative since the axis is decelerating. Therefore,

$$a = \frac{-10}{10} = -1\ cm/sec^2$$

The triangular area under the velocity curve is the position reached at $t = 15$ sec.

$$s = 25 + 10(15 - 5) - \frac{1}{2} \times (15 - 5)^2 = 75\ cm$$

Common Motion Profiles

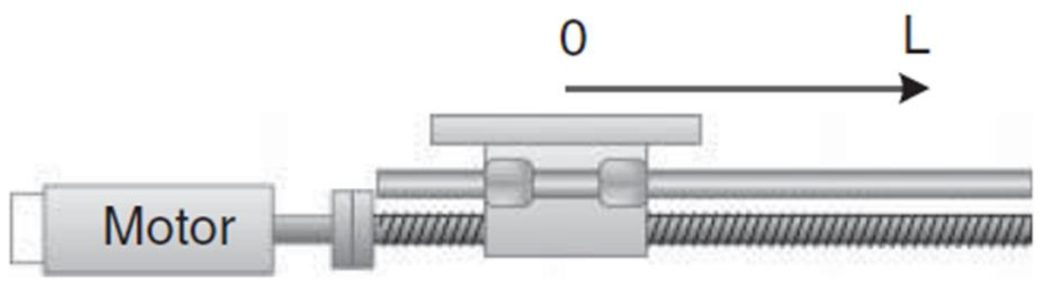
Trapezoidal Velocity Profile

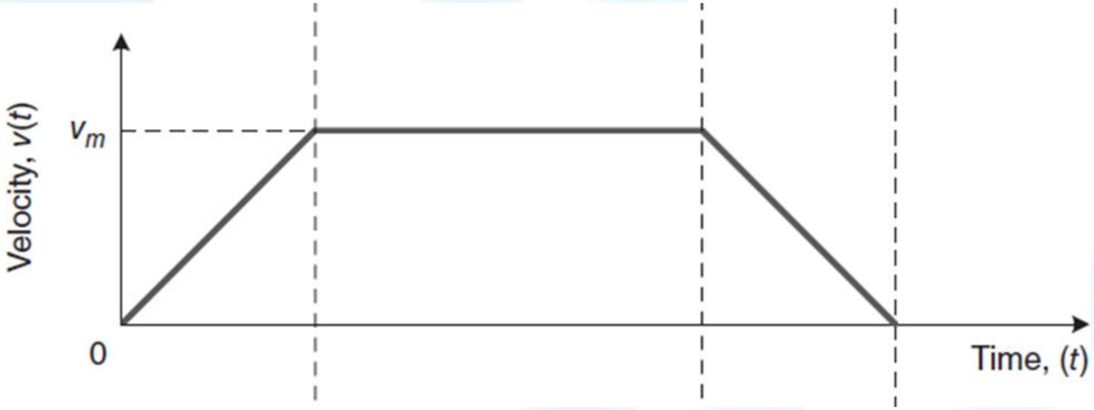
S-curve velocity profile

Trapezoidal Velocity Profile

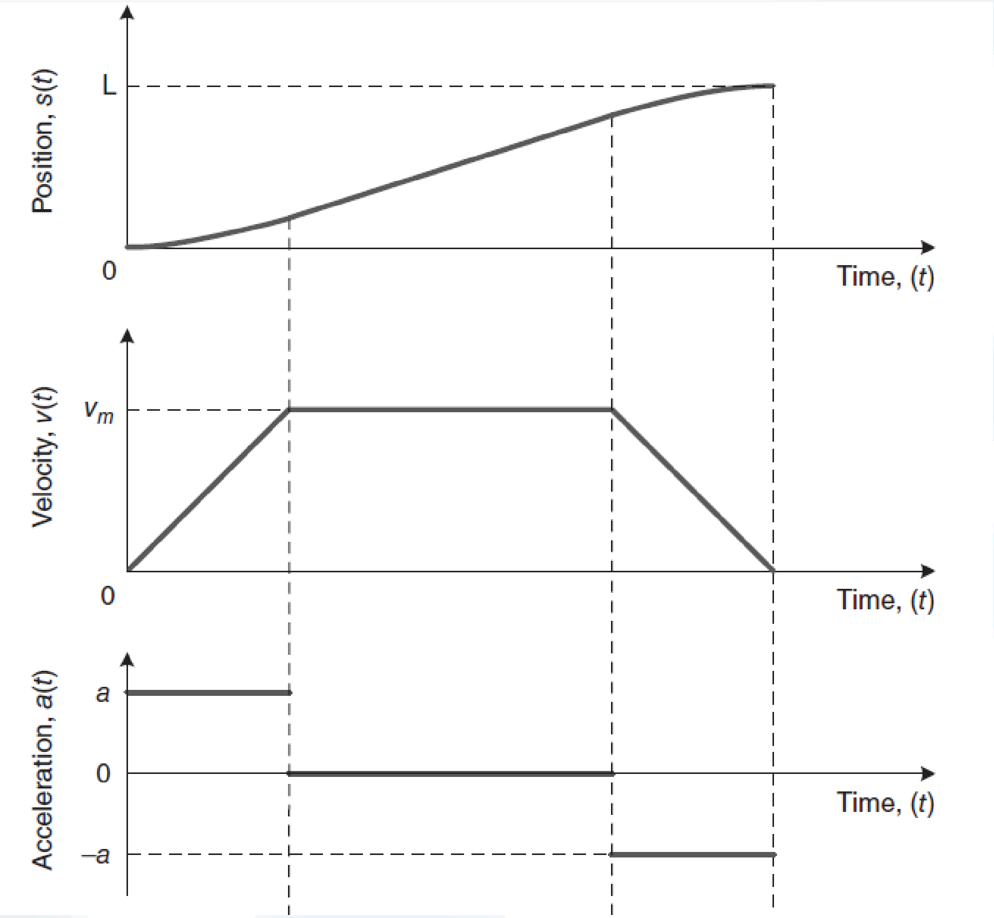
To move an axis of a machine, usually the following desired motion parameters are known:

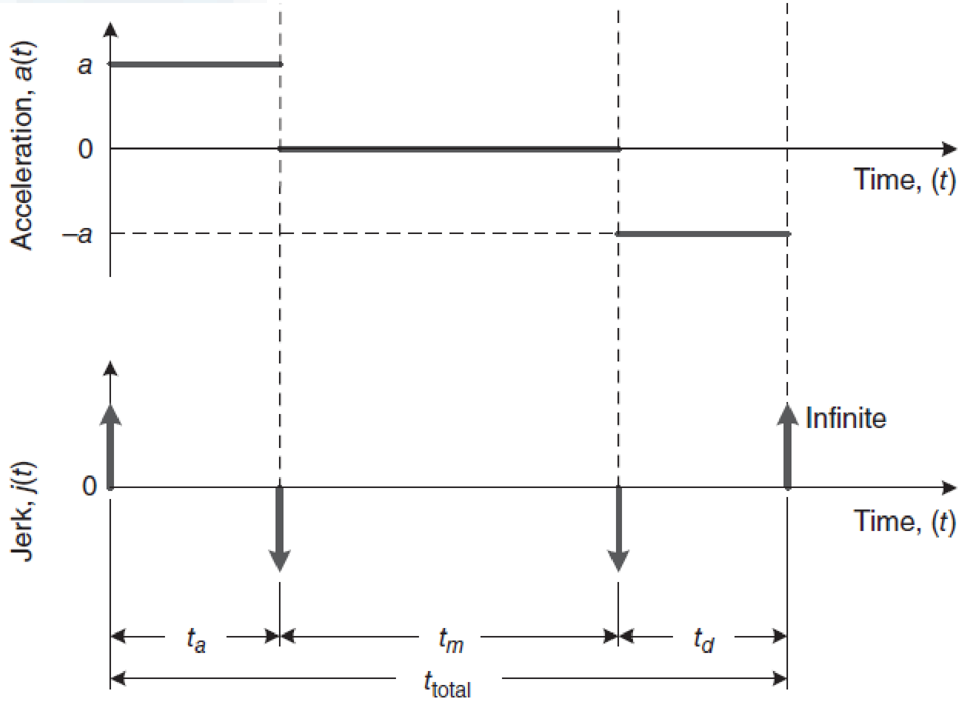
- Move velocity v_m
- Acceleration a
- Distance s to be traveled by the axis





Geometric Approach





Geometric Approach

$$t_a = t_d = \frac{v_m}{a}$$

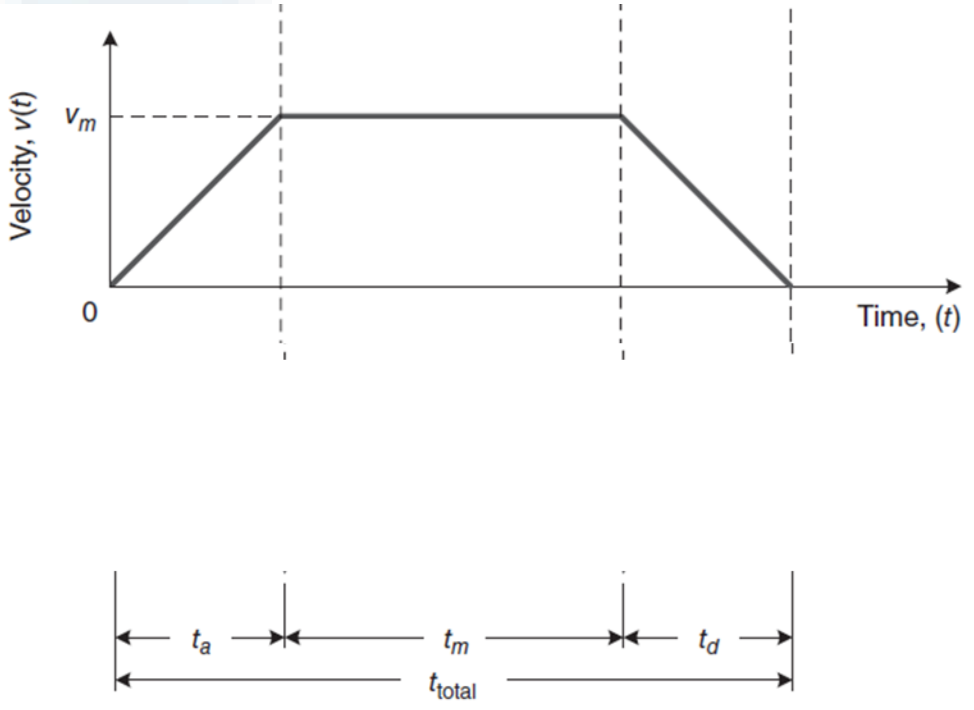
$$t_{total} = t_a + t_m + t_d$$

$$L = \frac{t_a \times v_m}{2} + t_m \times v_m + \frac{t_d \times v_m}{2}$$

$$L = v_m \times (t_a + t_m)$$

The move time can then be found as:

$$t_m = \frac{L}{v_m} - t_a$$



Analytical Approach

$$for\ 0 \le t < t_a, \qquad t_0 = 0; v_0 = 0; s_0 = 0;$$

$$s_{(t)} = \frac{1}{2}at^2$$

$$for\ t_a \le t < (t_a + t_m), \qquad t_0 = t_a; v_0 = v_m; s_0 = s_{(t_a)}; a = 0;$$

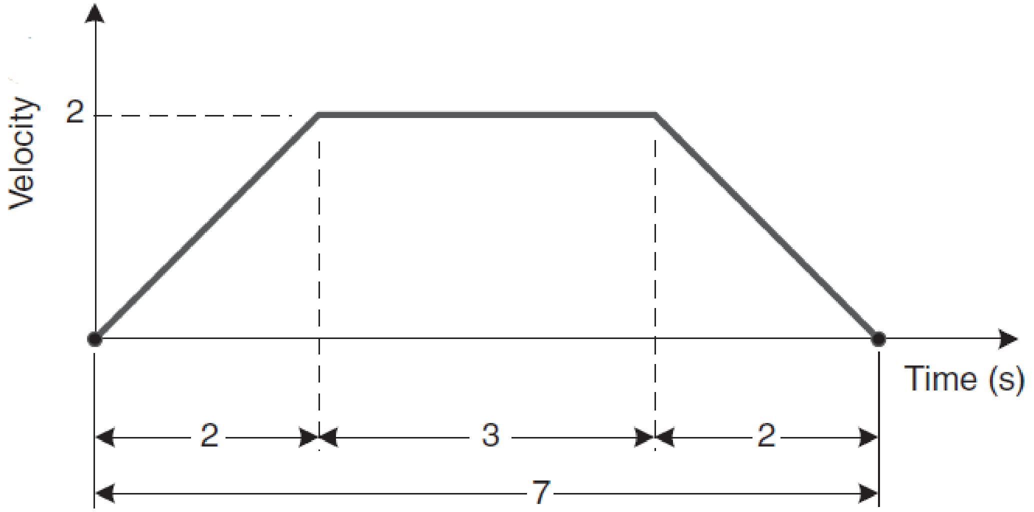
$$s_{(t)} = s_{(t_a)} + v_m \times (t - t_a)$$

$$for\ (t_a + t_m) \le t \le t_{total}, t_0 = (t_a + t_m); v_0 = v_m; s_0 = s_{(t_a+t_m)}$$

$$s_{(t)} = s_{(t_a+t_m)} + v_m \times \left(t - (t_a + t_m)\right) + \frac{1}{2}a \times \left(t - (t_a + t_m)\right)^2$$

Example 3

- The *X*-axis of robot moves for 10 cm.
- The maximum acceleration allowed for this axis is 1 cm/s2.
- If the axis needs to move at a desired maximum velocity of 2 cm/s, how long will it take to complete this motion?



Solution

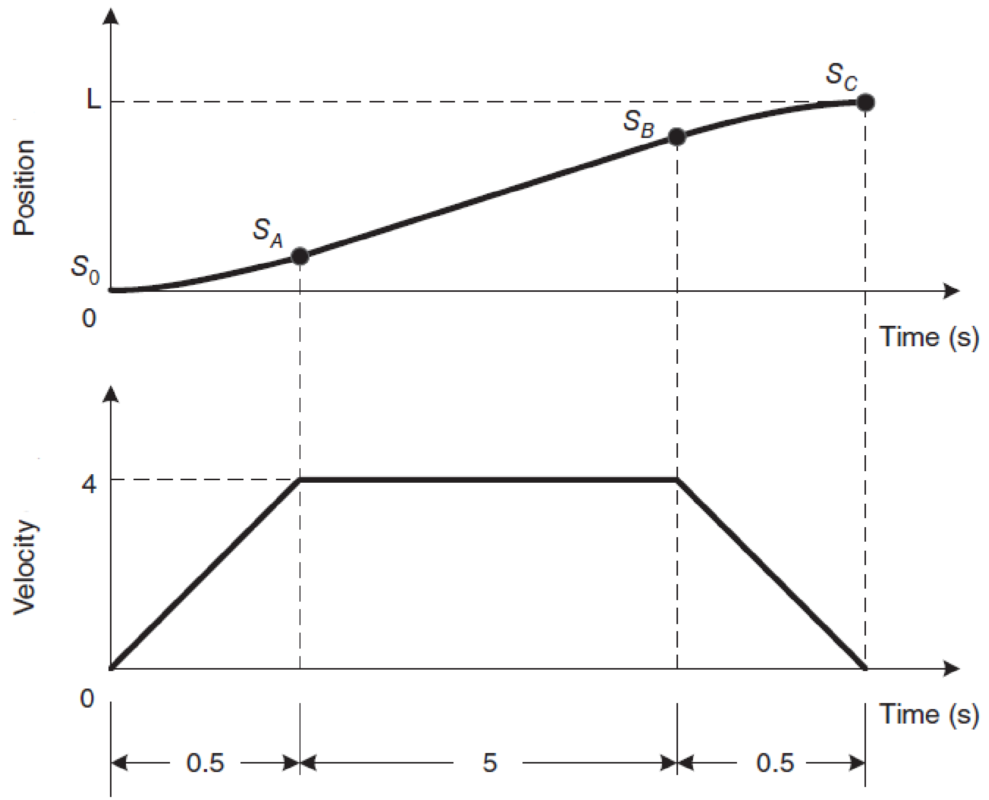
$$t_a = t_d = \frac{v_m}{a} = \frac{2}{1} = 2\ sec$$

$$t_m = \frac{L}{v_m} - t_a \Rightarrow t_m = \frac{10}{2} - 2 = 3\ sec$$

$$t_{total} = t_a + t_m + t_d \Rightarrow t_{total} = 2 + 3 + 2 = 7\ sec$$

Example 4

- Given the velocity profile in Figure beside, calculate S_A, S_B, S_C using the geometric rules for motion profile



Solution

- S_A can be found from the area of the triangle under the velocity curve as:

$$S_A = \frac{4 \times 0.5}{2} = 1\ cm$$

- S_B is the total area under the velocity curve up to point B.

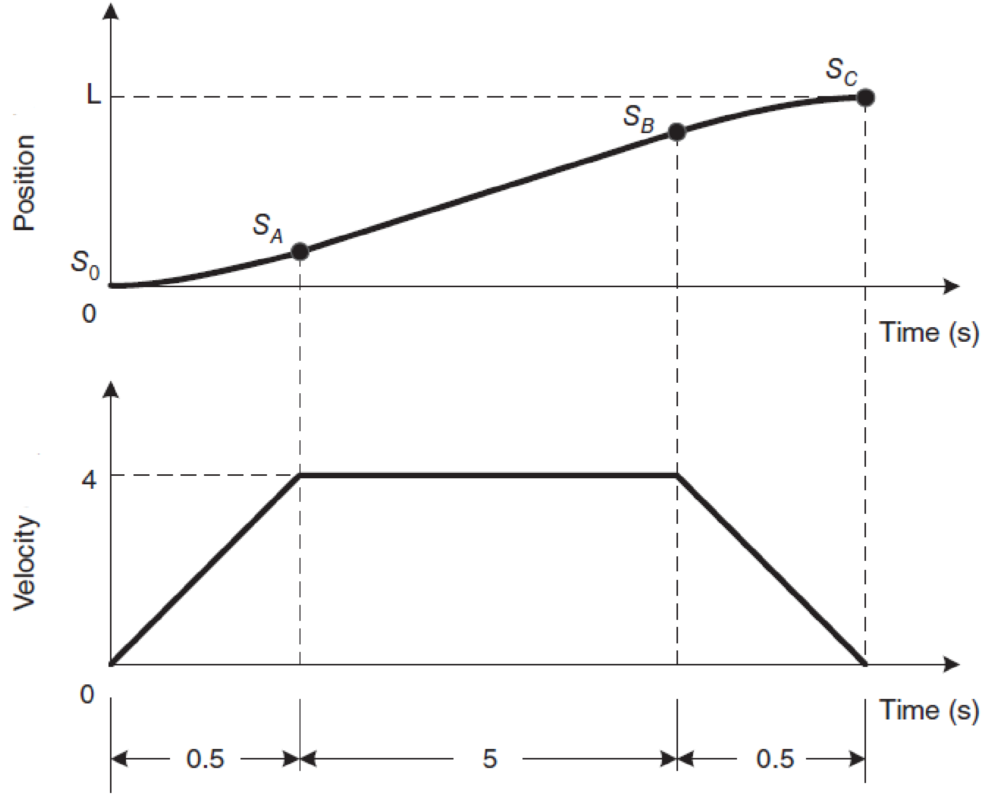
$$S_B = S_A + 4 \times 5 = 21\ cm$$

- S_C is the total area under the velocity curve up to point C.

$$S_C = S_A + S_B + \frac{4 \times 0.5}{2} = 22\ cm$$

Example 5

- Given the velocity profile in Figure beside, calculate S_A, S_B, S_C using the analytical method



Solution

- $a = 4/0.5 = 8$ cm/s2.
- *Acceleration phase* $(0 \le t \le 0.5)$

$$t_0 = 0; v_0 = 0; s_0 = 0; a = 8$$
$$s(t) = \frac{1}{2}a(t - t_0)^2 = 1\ cm$$

- *Constant velocity phase* $(0.5 < t \le 5.5)$

$$t_0 = 0.5; v_0 = v_A; s_0 = s_A; a = 0$$
$$s(t) = S_A + v_A(t - t_0) + \frac{1}{2}a(t - t_0)^2 \Rightarrow s(t) = 21\ cm$$

- *Deceleration phase* $(5.5 < t \le 6)$

$$t_0 = 5.5; v_0 = v_B; s_0 = s_B; a = -8$$
$$s(t) = S_B + v_B(t - t_0) + \frac{1}{2}a(t - t_0)^2 \Rightarrow s(t) = 22\ cm$$

Thanks

This is an introduction to the S-Curve motion profile.