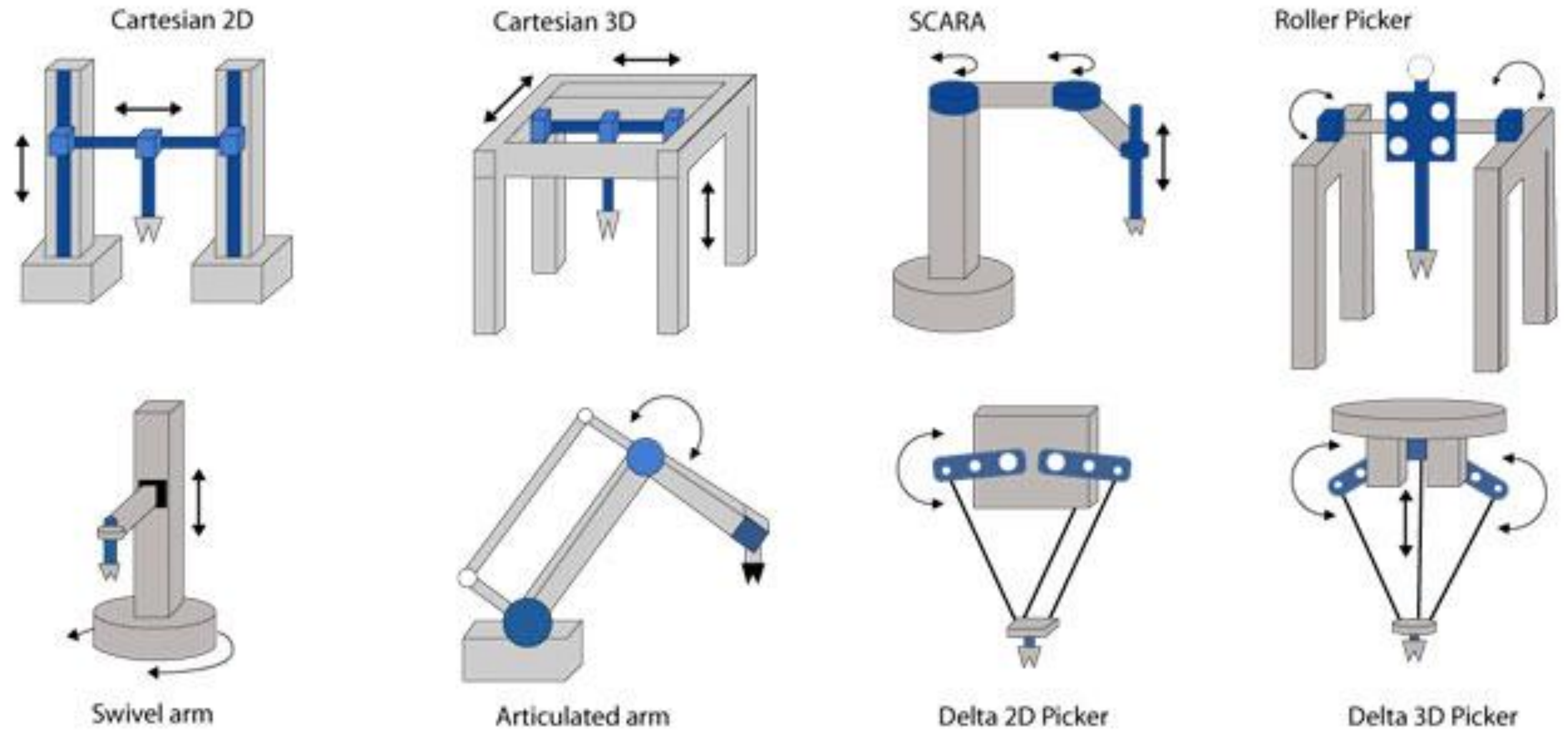
Robotics

Introduction and tools

History

- Karel Chapek (1890-1938)……. 1917.
- Issac Asimov (1920-1992), Run-around 1941, IROBOT 1951.
- ROBOT: (Robotics Institute of America RIA): A reprogrammable multifunctional manipulator designed to move material, parts, tools or specialized devices through variable programmed motions for the performance of a variety of tasks.
- ROBOTICS: A term refers to the study and use of robots.

Serial & parallel robots



Robotics Science

Mechanical Part

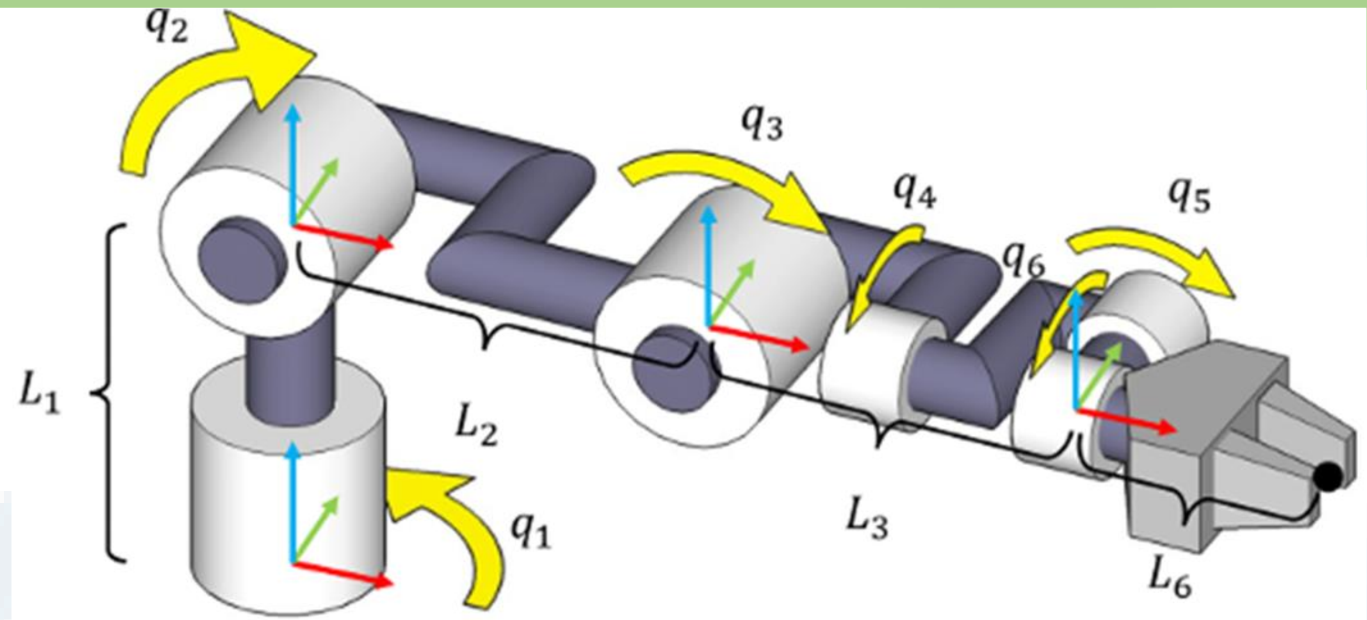
- Coordinates & Frames
- Position & Orientation
- Joints & Movements
- Transformations
- Kinematic model (direct & inverse)
- Velocity model (direct & inverse)
- Dynamic model (direct & inverse)

Control Part

- Sensors
- Motors
- Controllers
- Programming
- Communications
- ……

Serial Robot Kinematic Analysis

Robot Presentation → Transformation Matrices → Kinematic, velocity and acceleration models



Matrices

Homogeneous transformation, rotation matrix , partitioned matrix, orientation matrix.

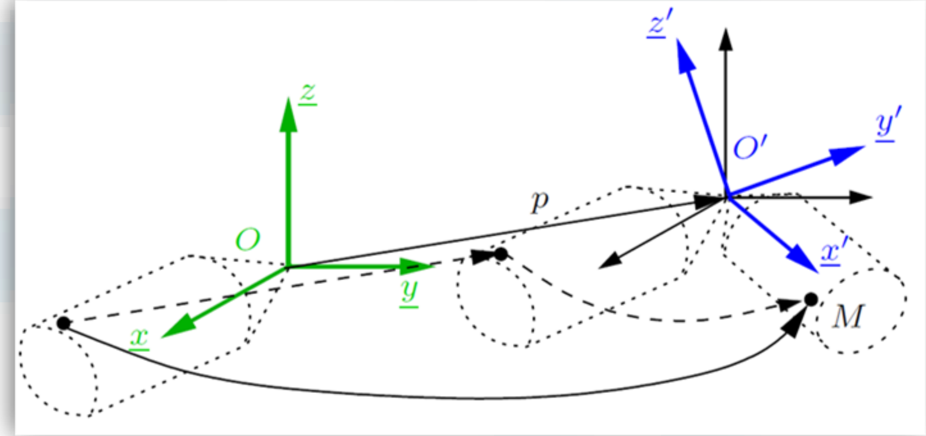
Geometric (Homogeneous) transformation

$$Transformation = Rotation + Translation$$

$$T = R + P$$

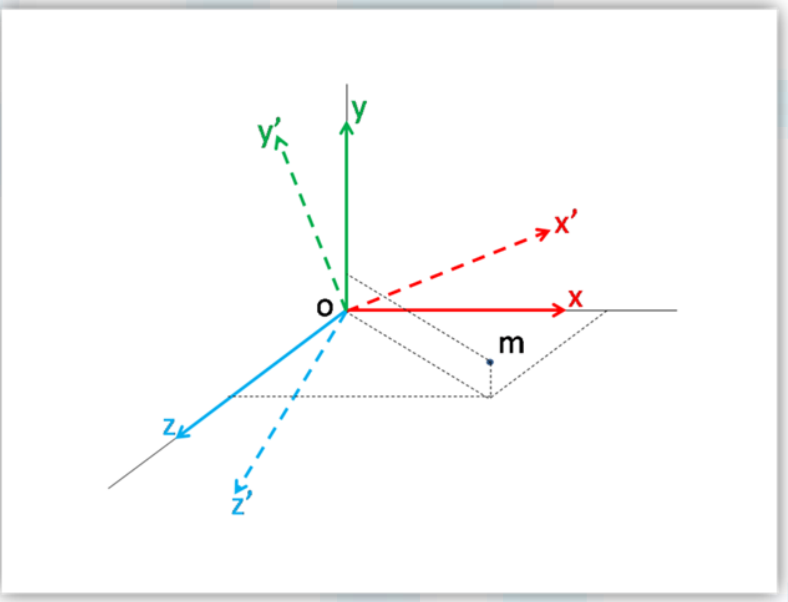
$$\begin{pmatrix} m \\ 1 \end{pmatrix} = \begin{pmatrix} R & P \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m' \\ 1 \end{pmatrix} \Leftrightarrow \overline{m} = T.\overline{m}'$$

$$\left.\begin{array}{l} T_1 = \begin{pmatrix} R_1 & P_1 \\ 0 & 1 \end{pmatrix} \\ T_2 = \begin{pmatrix} R_2 & P_2 \\ 0 & 1 \end{pmatrix} \end{array}\right\} \Rightarrow T_1 \times T_2 = \begin{pmatrix} R_1 R_2 & R_1 P_2 + P_1 \\ 0 & 1 \end{pmatrix}$$



Rotation matrix

The rotation matrix from R *to* $\acute{R}$ *is* the matrix formed by the coordinates of $\acute{R}$ *unit vectors in* R



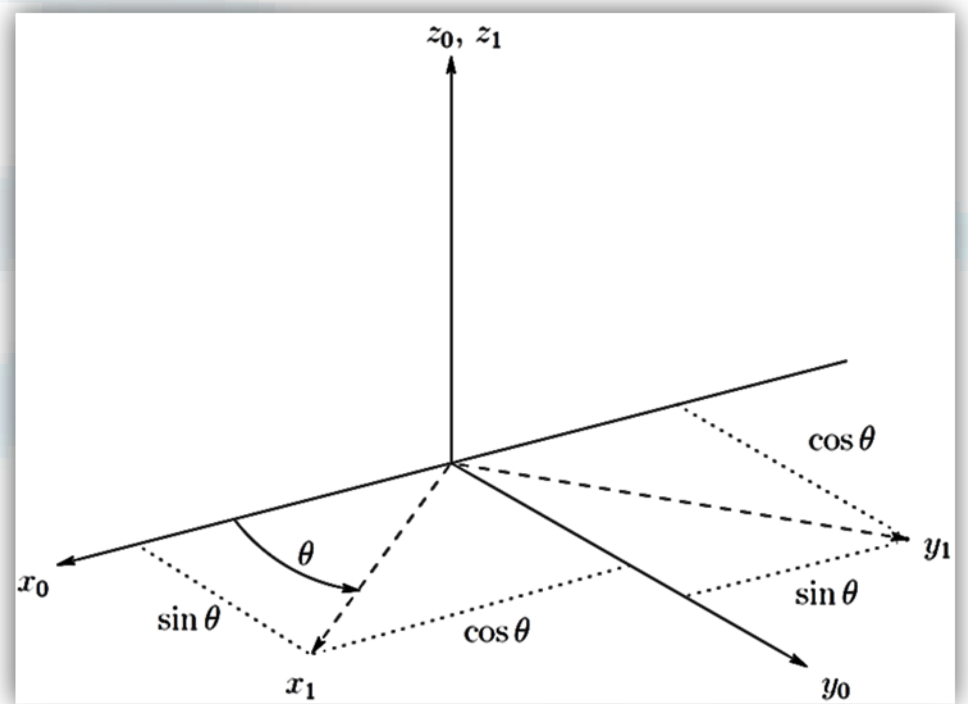
$$Rotation_{R\to\acute{R}} = \begin{bmatrix} \vec{\acute{\imath}}.\vec{\imath} & \vec{\acute{\jmath}}.\vec{\imath} & \vec{\acute{k}}.\vec{\imath} \\ \vec{\acute{\imath}}.\vec{\jmath} & \vec{\acute{\jmath}}.\vec{\jmath} & \vec{\acute{k}}.\vec{\jmath} \\ \vec{\acute{\imath}}.\vec{k} & \vec{\acute{\jmath}}.\vec{k} & \vec{\acute{k}}.\vec{k} \end{bmatrix}$$

Rotation about Cartesian axes

Rotation about Z

$$0 \xrightarrow{R_0^1} 1 : R_0^1 = \begin{bmatrix} \vec{x_1}.\vec{x_0} & \vec{y_1}.\vec{x_0} & \vec{z_1}.\vec{x_0} \\ \vec{x_1}.\vec{y_0} & \vec{y_1}.\vec{y_0} & \vec{z_1}.\vec{y_0} \\ \vec{x_1}.\vec{z_0} & \vec{y_1}.\vec{z_0} & \vec{z_1}.\vec{z_0} \end{bmatrix}$$

$$\left.\begin{aligned} \vec{x_1}.\vec{x_0} &= \cos(\theta) \\ \vec{x_1}.\vec{y_0} &= \sin(\theta) \\ \vec{y_1}.\vec{x_0} &= -\sin(\theta) \\ \vec{y_1}.\vec{y_0} &= \cos(\theta) \\ \vec{z_1}.\vec{z_0} &= 1 \end{aligned}\right\} \Rightarrow R_0^1 = R_{z,\theta} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Rotation about y and x

$$R_{y,\theta}=\begin{bmatrix}\cos(\theta) & 0 & \sin(\theta)\\ 0 & 1 & 0\\ -\sin(\theta) & 0 & \cos(\theta)\end{bmatrix},R_{x,\theta}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(\theta) & -\sin(\theta)\\ 0 & \sin(\theta) & \cos(\theta)\end{bmatrix}$$

Example:

$$R_0^1=R_{y,\varphi},R_1^2=R_{z,\theta}\Rightarrow R_0^2=R_{y,\varphi}\times R_{z,\theta}$$

$$R_0^2=\begin{bmatrix}\cos(\varphi) & 0 & \sin(\varphi)\\ 0 & 1 & 0\\ -\sin(\varphi) & 0 & \cos(\varphi)\end{bmatrix}\times\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0\\ \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$R_0^2=\begin{bmatrix}\cos(\varphi)\cos(\theta) & -\cos(\varphi)\sin(\theta) & \sin(\varphi)\\ \sin(\theta) & \cos(\theta) & 0\\ -\sin(\varphi)\cos(\theta) & \sin(\varphi)\sin(\theta) & \cos(\varphi)\end{bmatrix}=\begin{bmatrix}c_\varphi c_\theta & -c_\varphi s_\theta & s_\varphi\\ s_\theta & c_\theta & 0\\ -s_\varphi c_\theta & s_\varphi s_\theta & c_\varphi\end{bmatrix}$$

NOTE
${R_0^2}^T = {R_0^2}^{-1} = R_2^0$
Det(R)=1

Partitioned Matrix

$$B \in R^{m_1 \times m_1}, C \in R^{m_1 \times m_2}, D \in R^{m_2 \times m_1}, E \in R^{m_2 \times m_2}$$

$$\left.\begin{array}{c} A \in R^{m \times m} \\ m = m_1 + m_2 \end{array}\right\} \Rightarrow A = \begin{bmatrix} B & C \\ D & E \end{bmatrix} \textit{ is partitioned matrix}$$

$$A^{-1} = \begin{bmatrix} (B - C.E^{-1}.D)^{-1} & -B^{-1}.C(E - D.B^{-1}C)^{-1} \\ -E^{-1}.D(B - C.E^{-1}.D)^{-1} & (E - D.B^{-1}C)^{-1} \end{bmatrix}$$

Geometric transformation is a partitioned matrix

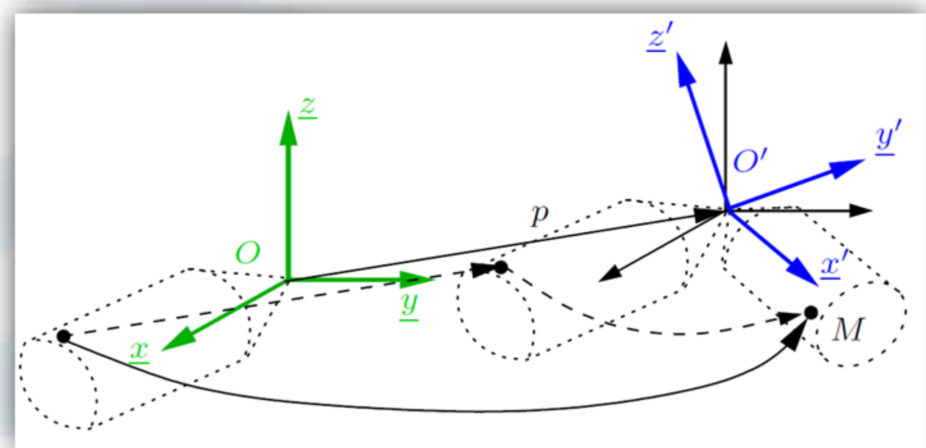
$$B \in R^{3\times3}, C \in R^{3\times1}, D \in R^{1\times3}, E \in R^{1\times1}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, E = 1 \Rightarrow A = \begin{bmatrix} B & C \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} B^{-1} & -B^{-1}.C \\ 0 & 1 \end{bmatrix}$$

$$B : Orthogonal$$

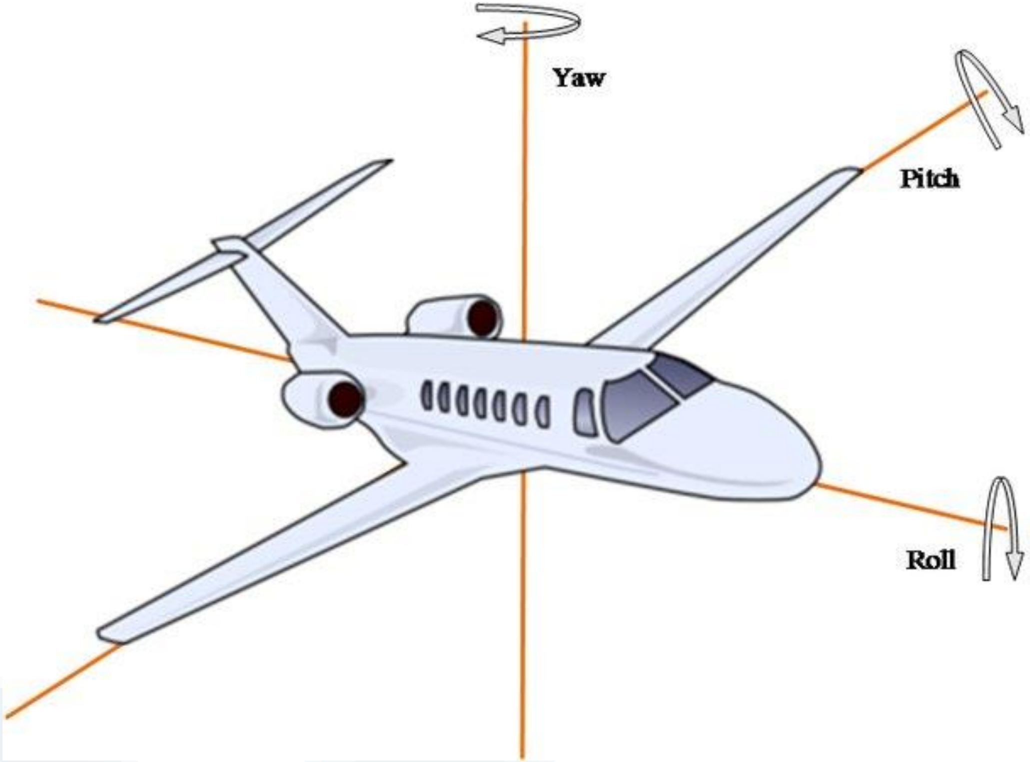
$$A^{-1} = \begin{bmatrix} B^{T} & -B^{T}.C \\ 0 & 1 \end{bmatrix}$$



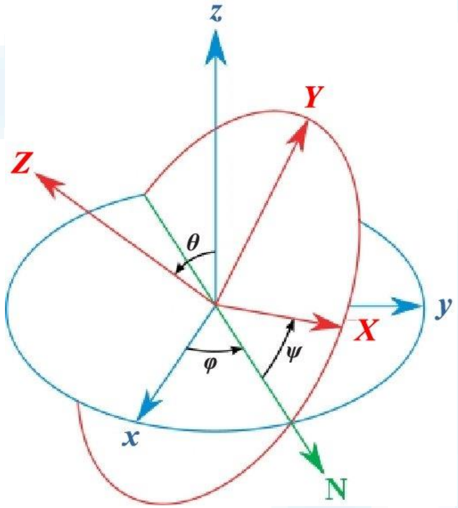
$$T = \begin{pmatrix} R & P \\ 0 & 1 \end{pmatrix} \Rightarrow T^{-1} = \begin{pmatrix} R^{T} & -R^{T}P \\ 0 & 1 \end{pmatrix}$$

3D Orientation

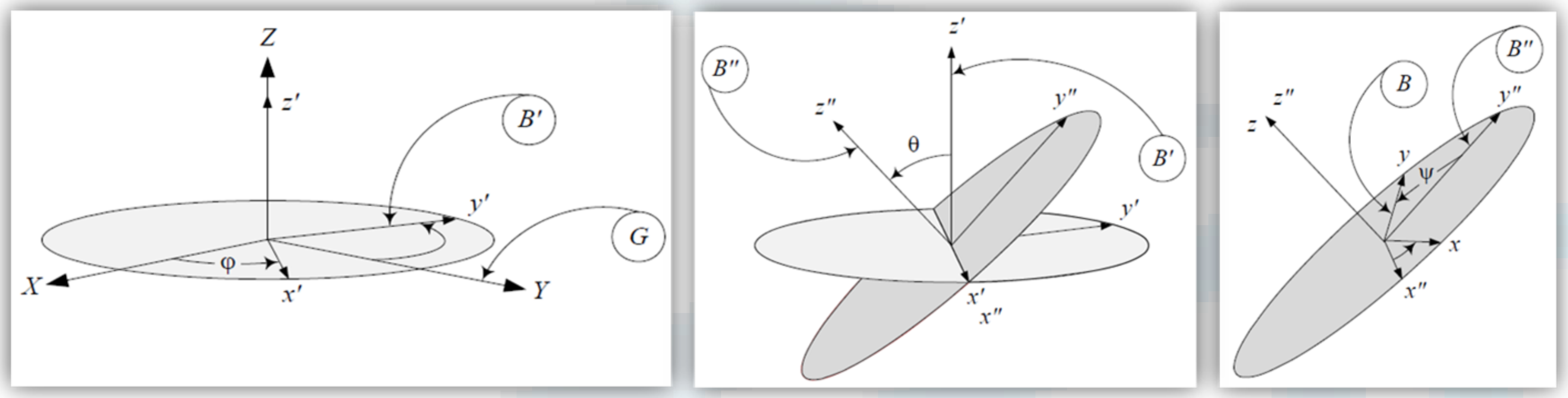
Roll-Pitch-Yaw



Euler



Euler angles



$$R = R_{z,\psi}.R_{x_\psi,\theta}.R_{z_\theta,\varphi}$$

$$R = \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{bmatrix} . \begin{bmatrix} c_\varphi & -s_\varphi & 0 \\ s_\varphi & c_\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_\psi c_\varphi - s_\psi c_\theta s_\varphi & -c_\psi s_\varphi - s_\psi c_\theta c_\varphi & s_\psi s_\theta \\ s_\psi c_\varphi + c_\psi c_\theta s_\varphi & -s_\psi s_\varphi + c_\psi c_\theta c_\varphi & -c_\psi s_\theta \\ s_\theta s_\varphi & s_\theta c_\varphi & c_\theta \end{bmatrix} = \begin{bmatrix} x'_x & y'_x & z'_x \\ x'_y & y'_y & z'_y \\ x'_z & y'_z & z'_z \end{bmatrix}$$

Euler Condition Example

Rotation Matrix 1

-0.3536	-0.3536	0.8660
0.7071	-0.7071	-0.0000
0.6124	0.6124	0.5000

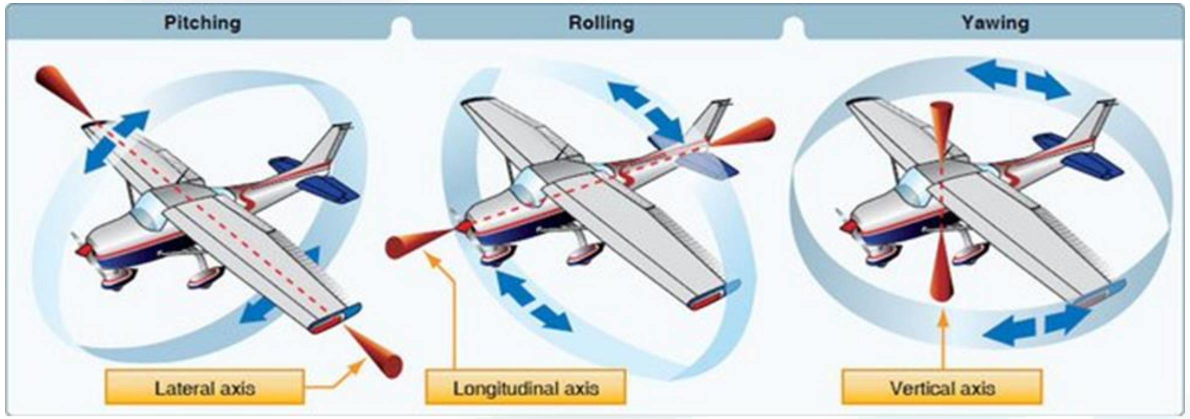
Rotation Matrix 2

-0.7071	-0.7071	0
0.7071	-0.7071	0
0	0	1.0000

$$\begin{bmatrix} c_\psi c_\varphi - s_\psi c_\theta s_\varphi & -c_\psi s_\varphi - s_\psi c_\theta c_\varphi & s_\psi s_\theta \\ s_\psi c_\varphi + c_\psi c_\theta s_\varphi & -s_\psi s_\varphi + c_\psi c_\theta c_\varphi & -c_\psi s_\theta \\ s_\theta s_\varphi & s_\theta c_\varphi & c_\theta \end{bmatrix}$$

$$z_z^{'} \neq \pm 1 \Rightarrow \theta = a\cos(z_z^{'}) \Rightarrow \begin{cases} \psi = a\tan 2(z_x^{'}, -z_y^{'}) \\ \varphi = a\tan 2(x_z^{'}, y_z^{'}) \end{cases}$$

Roll-Pitch-Yaw



$$R = R_{z,\alpha}.R_{y_\alpha,\beta}.R_{x_\beta,\gamma}$$

$$R = \begin{bmatrix} c_\alpha & -s_\alpha & 0 \\ s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} c_\beta & 0 & s_\beta \\ 0 & 1 & 0 \\ -s_\beta & 0 & c_\beta \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & -s_\gamma \\ 0 & s_\gamma & c_\gamma \end{bmatrix}$$

$$R = \begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\gamma + c_\alpha s_\beta s_\gamma & s_\alpha s_\gamma + c_\alpha s_\beta c_\gamma \\ s_\alpha c_\beta & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & -c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix} = \begin{bmatrix} x'_x & y'_x & z'_x \\ x'_y & y'_y & z'_y \\ x'_z & y'_z & z'_z \end{bmatrix}$$

RPY Condition

Rotation Matrix 1

0.0000	-0.7071	0.7071
0.5000	0.6124	0.6124
-0.8660	0.3536	0.3536

Rotation Matrix 2

0.0000	-0.2588	0.9659
0.0000	0.9659	0.2588
-1.0000	0.0000	0.0000

$$\begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\gamma + c_\alpha s_\beta s_\gamma & s_\alpha s_\gamma + c_\alpha s_\beta c_\gamma \\ s_\alpha c_\beta & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & -c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix}$$

$$\boxed{\beta \neq \pm\frac{\pi}{2} \Rightarrow \beta = a\tan 2(-x'_z, \sqrt{x'^{2}_x + x'^{2}_y}) \Rightarrow \begin{cases} \alpha = a\tan 2(x'_y, x'_x) \\ \gamma = a\tan 2(y'_z, z'_z) \end{cases}}$$

Trigonometric

Trigonometric Formula

- $\cos(\theta) = a \xRightarrow{2\ solutions\ in\ 2\ methods} \theta = \pm acos(a) \equiv \begin{cases} \theta = atan2(\sqrt{1-a^2}, a) \\ \theta = atan2(-\sqrt{1-a^2}, a) \end{cases}$
- $\sin(\theta) = a \xRightarrow{2\ solutions\ in\ 2\ methods} \begin{cases} \theta = asin(a) \\ \theta = \pi - asin(a) \end{cases} \equiv \begin{cases} \theta = atan2(a, \sqrt{1-a^2}) \\ \theta = atan2(a, -\sqrt{1-a^2}) \end{cases}$
- $a \times \cos(\theta) - b \times \sin(\theta) = 0 \xRightarrow{two\ solutions} \begin{cases} \theta = atan2(a, b) \\ \theta = atan2(-a, -b) \end{cases}$
- $a \times \cos(\theta) + b \times \sin(\theta) = c \xRightarrow{2\ solutions} \theta = atan2\left(c, \pm\sqrt{a^2 + b^2 - c^2}\right) - atan2(a, b)$

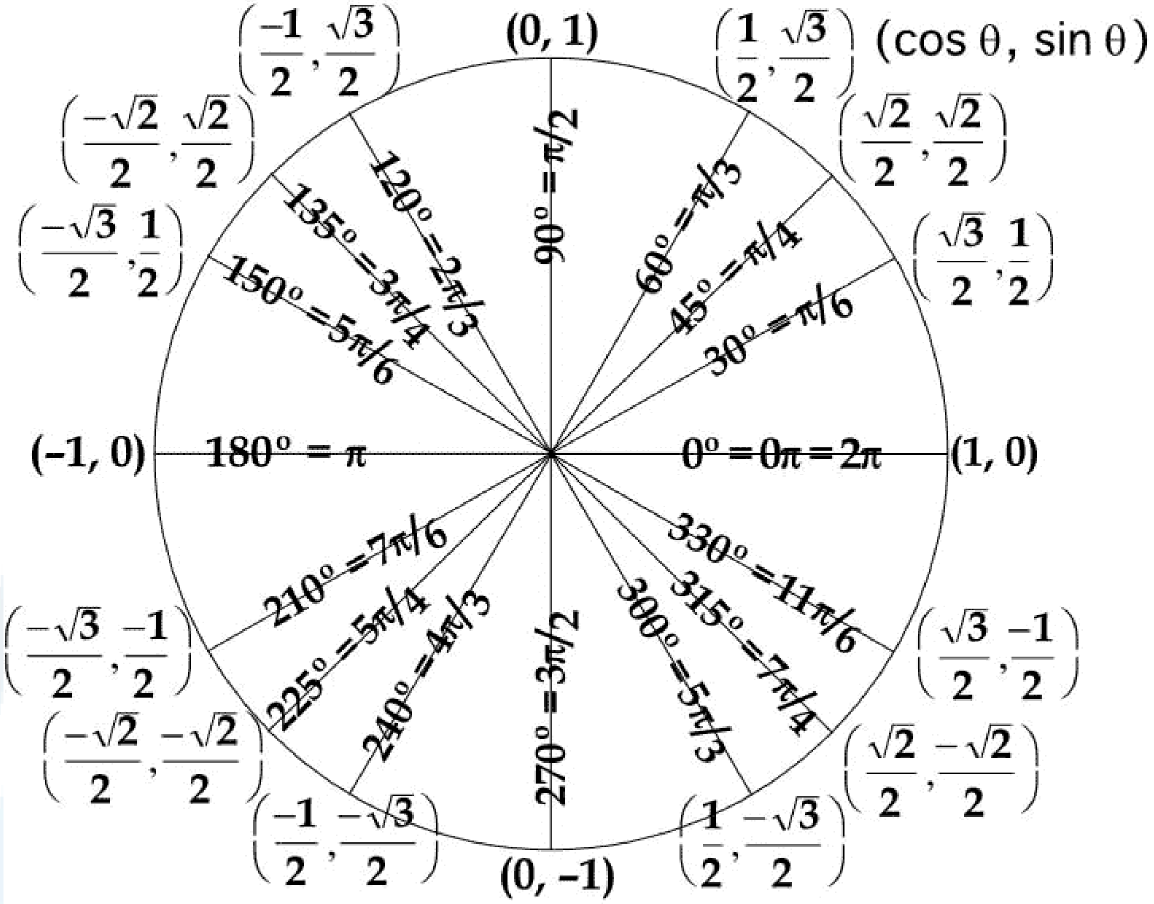
Important case

$$\left.\begin{aligned} a \times \cos(\theta) + b \times \sin(\theta) = m \\ c \times \cos(\theta) + d \times \sin(\theta) = n \end{aligned}\right\} \xrightarrow{solution} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \frac{-1}{ad - bc} \begin{bmatrix} -d & b \\ c & -a \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} \Rightarrow \theta = atan2(B, A)$$

Remember……… Unit Circle



Thanks