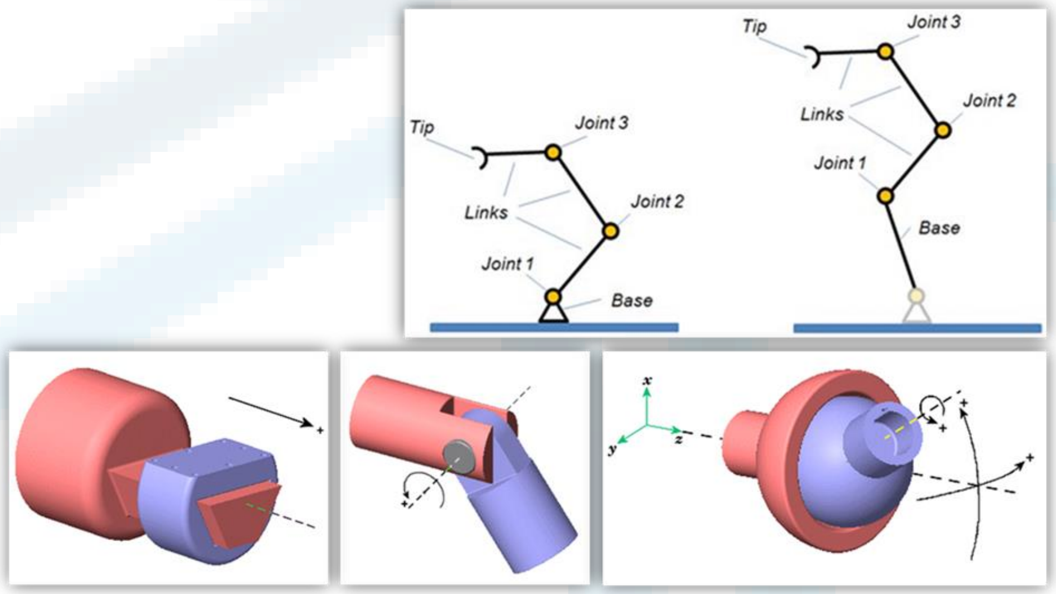
Direct kinematic model

Denavit-Hartenberg

Kinematic chains

- Kinematic chain is a set of segments and joints.
- Serial robot is a set of kinematic chains
- Joints
  - Simple joint (1 DOF)
  - Composed joint (2 or 3 DOF)

$$
Robot\begin{cases} n\ \text{joint}\left(\text{link i - 1}\xrightarrow{\text{joint i}}\textit{link}\ \text{i}\right) \\ n+1\ \text{link}\left(\text{joint i actuates link i}\right) \end{cases}
$$



Serial Robot Studying

Denavit-Hartenberg

Frame definition → Distance and Rotation Between Frames → Transformations → Kinematic Models

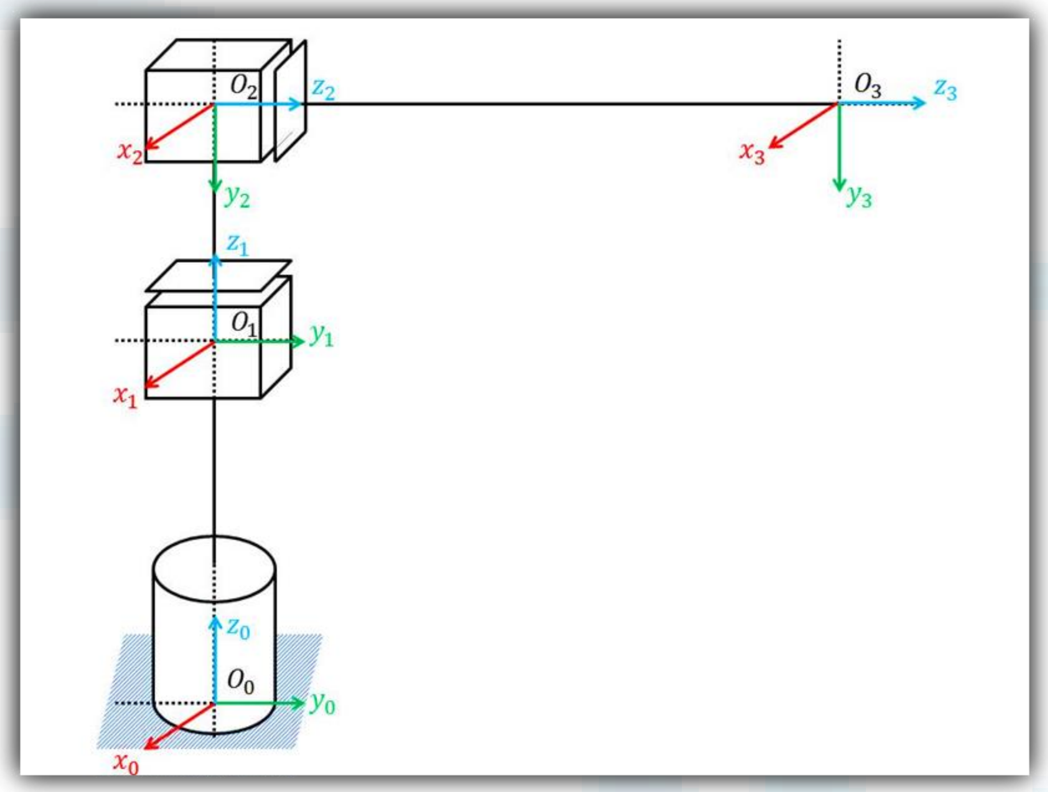
Denavit-hartenberg method

For every joint, we have to determine:

1. DH Frames
2. DH Parameters
3. DH Transformation

DH Frames (Theorems)

- $x_i \perp z_{i-1}$
- x_i, z_{i-1} in intersection
- $\mathrm{z_i}$ is the action axis of joint $\mathrm{i+1}$



Three cases

1 - $\mathrm{z_i, z_{i-1}}$ not in the same plan

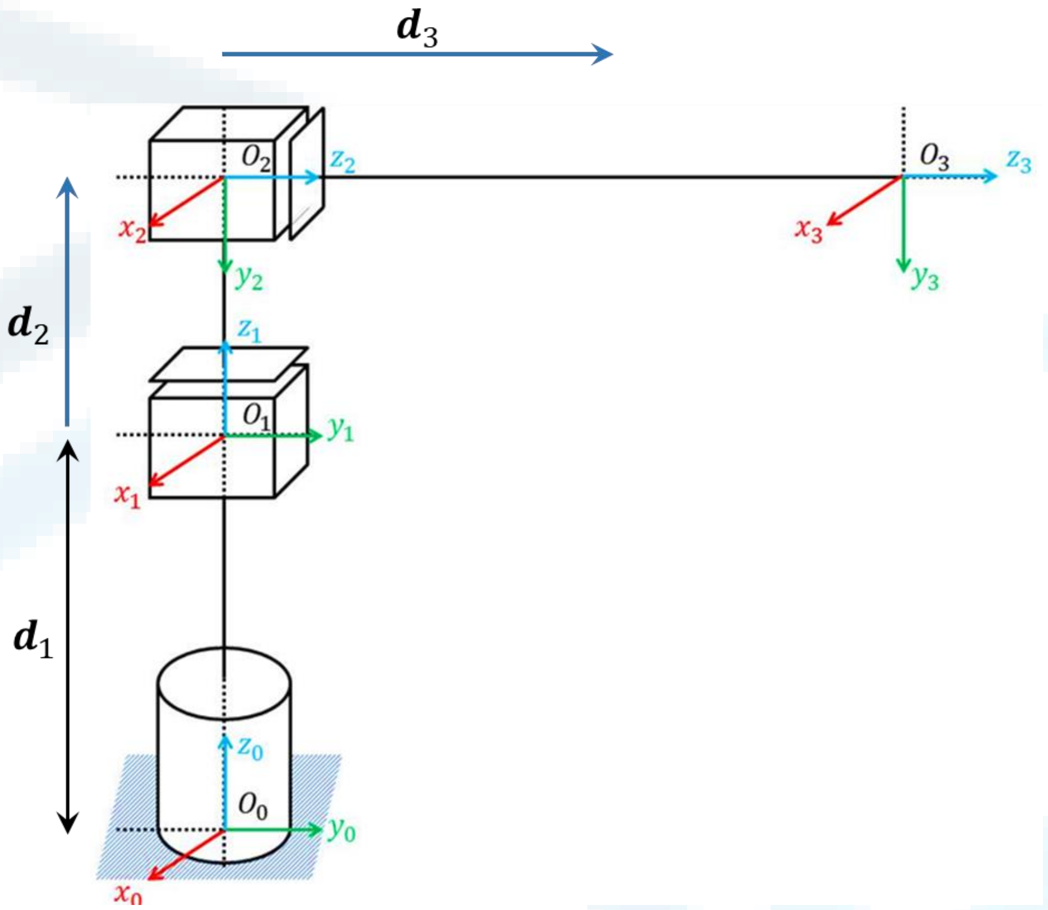
2 - $\mathrm{z_i, z_{i-1}}$ in intersection

3 - $\mathrm{z_i, z_{i-1}}$ parallel

DH Parameters

- a_i : the distance between z_{i-1} and z_i axes along the x_i axis
- α_i : the angle between z_{i-1} and z_i axis about x_i axis
- d_i : the distance between x_{i-1} and x_i axes along the z_{i-1} axis
- θ_i : the angle between x_{i-1} axis and x_i axis about the z_{i-1} axis

Joint	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	0	-90	d_2	0
3	0	0	d_3	0
....				



DH Transformation

$$T_{i-1}^{i} = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$T_{i-1}^{i} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

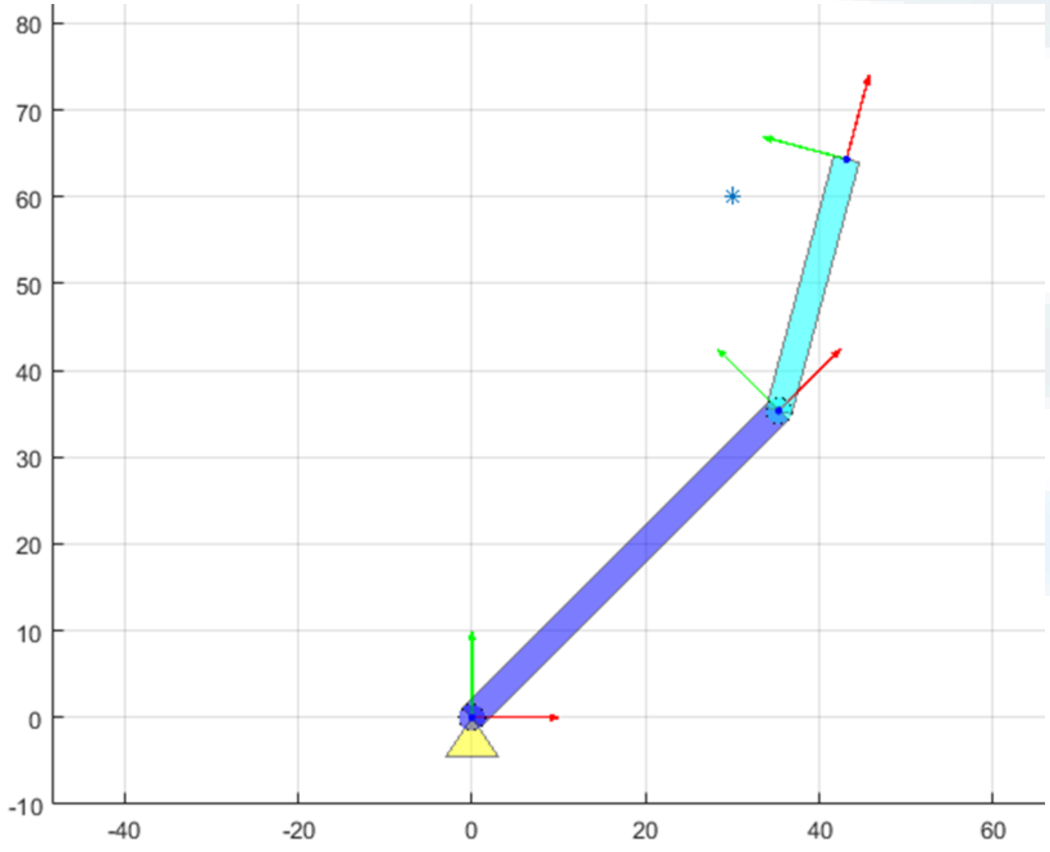
$$T_{i-1}^{i} = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH table & matrices

link	a_i	$\propto_i$	d_i	θ_i
1	0	0	d_1	$\theta_1{}^*$
2	0	-90	$d_2{}^*$	0
3	0	0	$d_3{}^*$	0

$$T_{i-1}^{i}=\begin{bmatrix} C_{\theta_i} & -S_{\theta_i}C_{\alpha_i} & S_{\theta_i}S_{\alpha_i} & a_iC_{\theta_i} \\ S_{\theta_i} & C_{\theta_i}C_{\alpha_i} & -C_{\theta_i}S_{\alpha_i} & a_iS_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \left\{ \begin{array}{l} T_0^1=\begin{bmatrix} C_{\theta_1} & -S_{\theta_1} & 0 & 0 \\ S_{\theta_1} & C_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_1^2=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_2^3=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \right.$$

Simple case : 2R planar robot



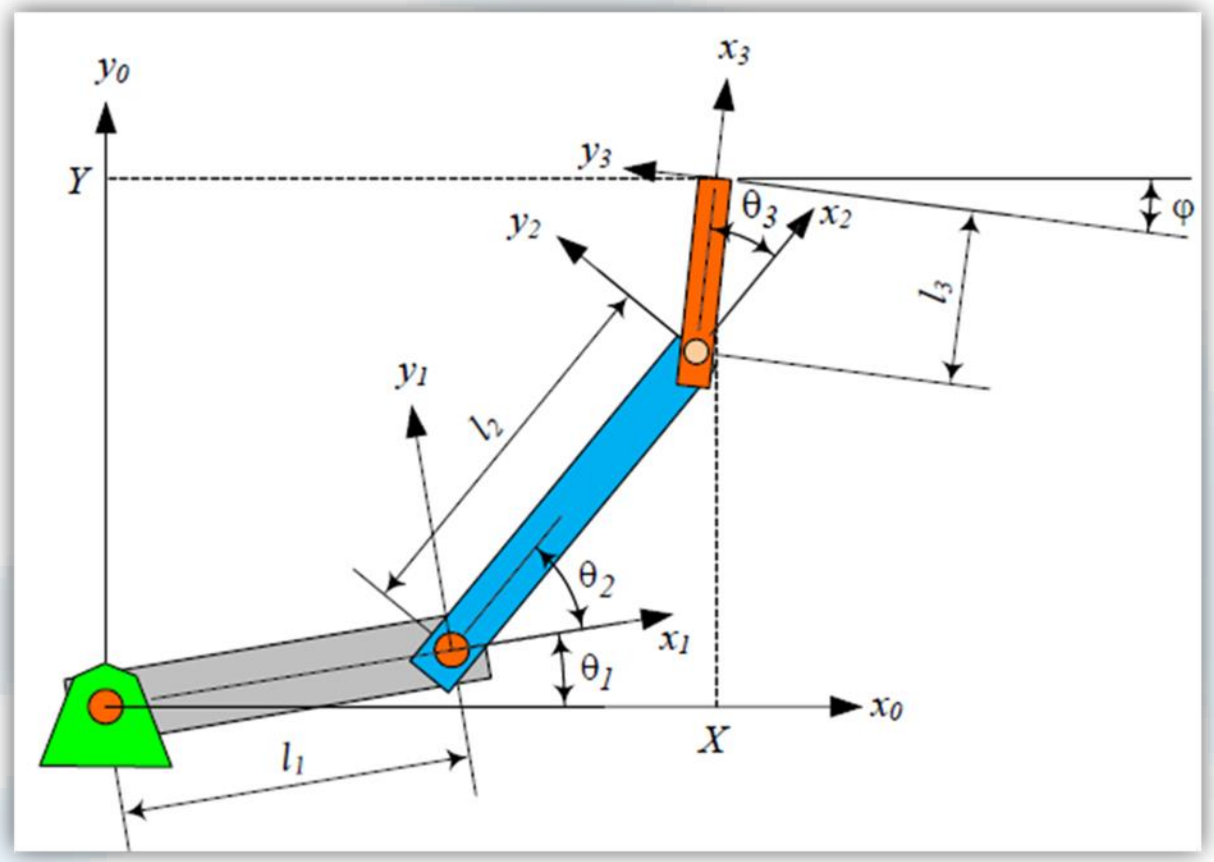
Link	a_i	α_i	d_i	θ_i
1	50	0	0	$\boldsymbol{\theta_1}$
2	30	0	0	$\boldsymbol{\theta_2}$

```
T01 =

[ cos(theta1), -sin(theta1), 0, 50*cos(theta1)]
[ sin(theta1),  cos(theta1), 0, 50*sin(theta1)]
[           0,            0, 1,              0]
[           0,            0, 0,              1]
T12 =

[ cos(theta2), -sin(theta2), 0, 30*cos(theta2)]
[ sin(theta2),  cos(theta2), 0, 30*sin(theta2)]
[           0,            0, 1,              0]
[           0,            0, 0,              1]
```

3R planer robot



DH table & matrices

link	a_i	α_i	d_i	θ_i
1	l1	0	0	θ1
2	l2	0	0	θ2
3	l3	0	0	θ3

$$A_i = T_{i-1}^{i} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

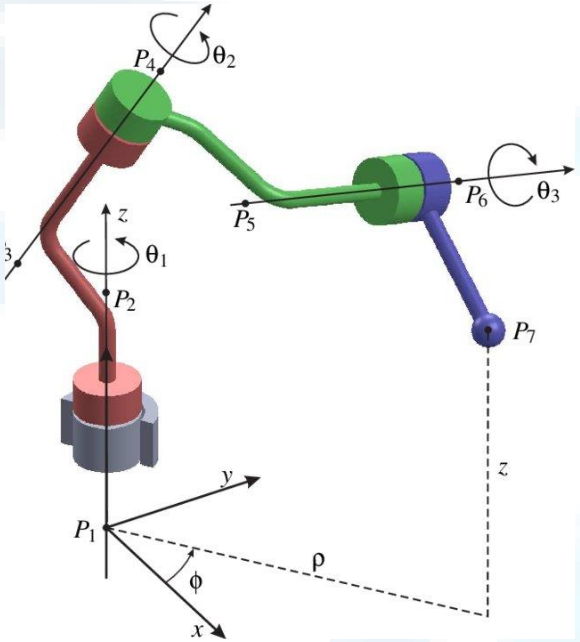
$$T_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1c_1 \\ s_1 & c_1 & 0 & l_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2c_2 \\ s_2 & c_2 & 0 & l_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_2^3 = \begin{bmatrix} c_3 & -s_3 & 0 & l_3c_3 \\ s_3 & c_3 & 0 & l_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Direct kinematic model

Actual & Generalized parameters

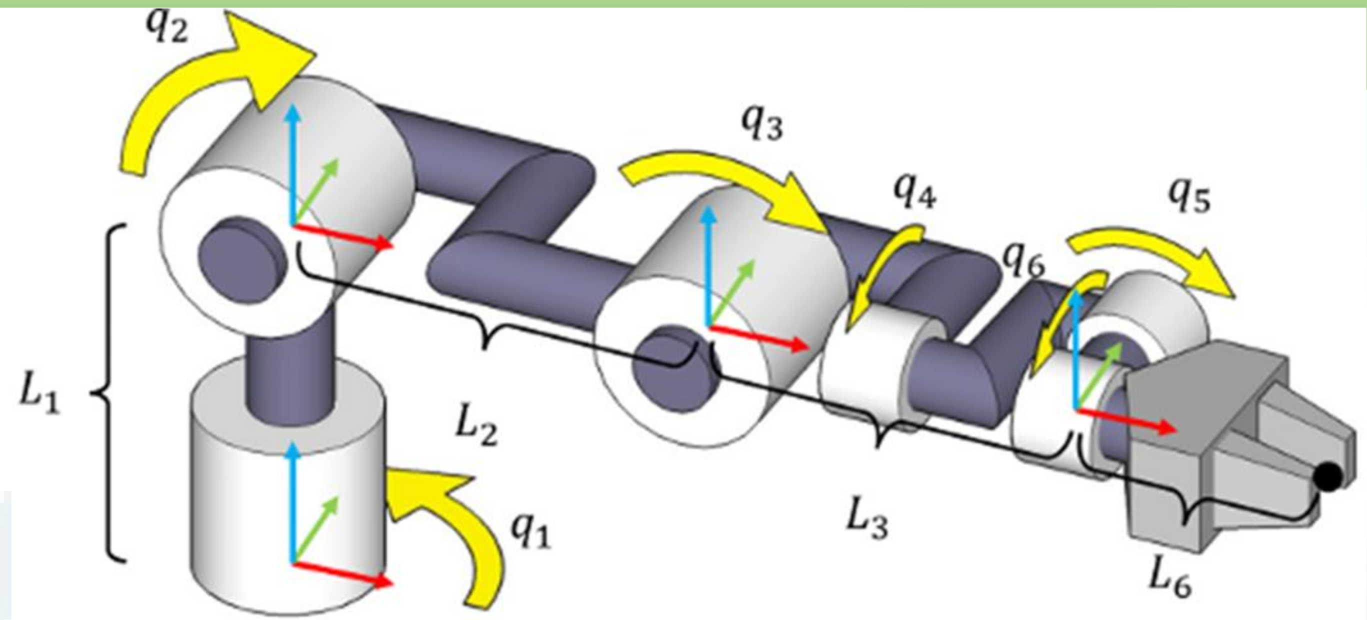
Actual & Generalized Parameters

- Actual parameters of a serial robot are the coordinates of its end-effector
- Generalized parameters are the DH variables



Serial Robot Kinematic Analysis

Robot Presentation → DH Matrices → DKM & IKM



Direct and inverse kinematic models

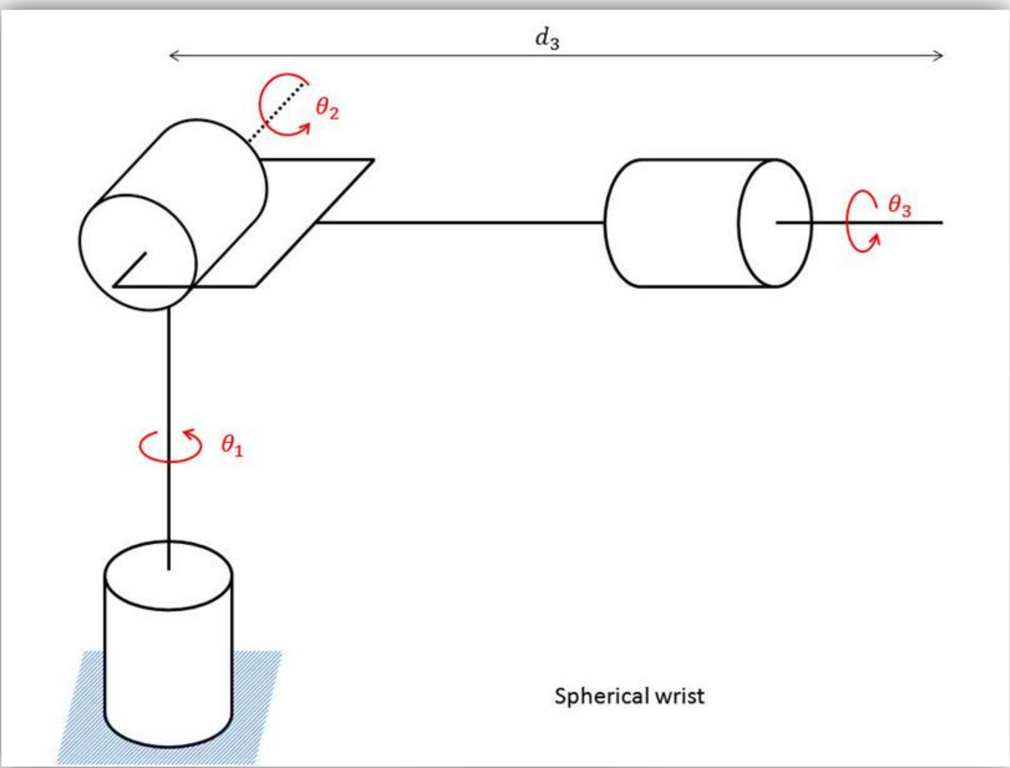
DKM

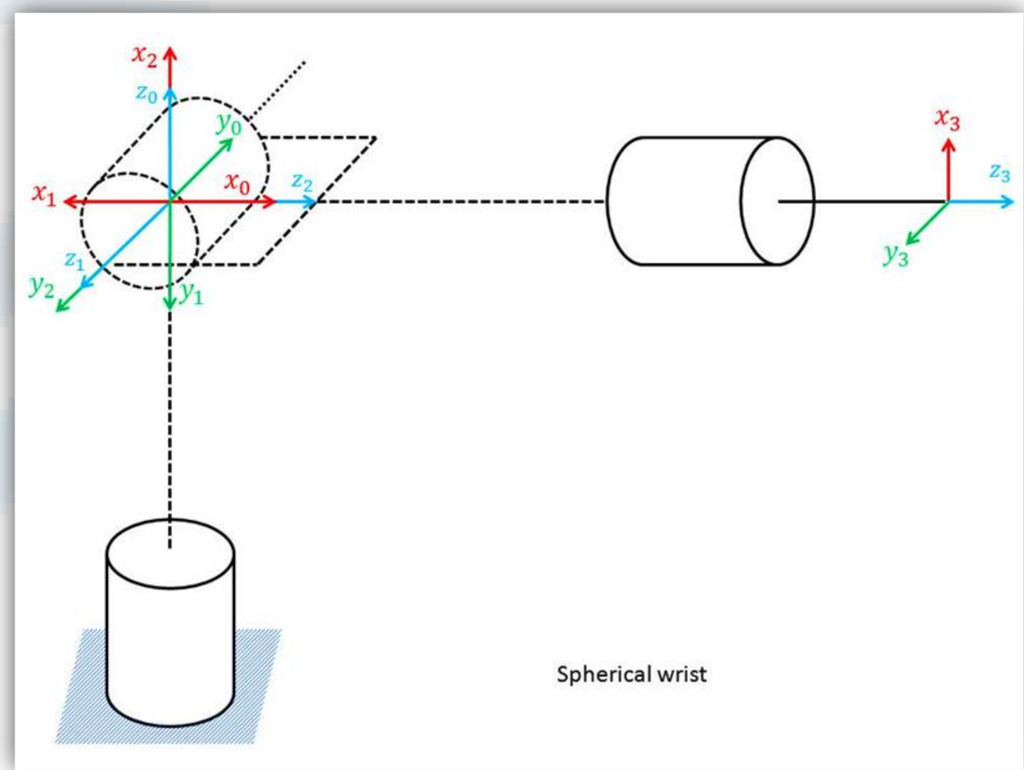
$$\{x, y, z, r_x, r_y, r_z\} = f(\theta_i, d_i)$$
$$i = 1,2..,n$$

IKM

$$\{\theta_i, d_i\} = f(x, y, z, r_x, r_y, r_z)$$
$$i = 1,2,..,n$$

Example : Spherical wrist





DH table & matrices

link	a_i	$\propto_i$	d_i	θ_i
1	0	-90	0	$\theta_1{}^*$
2	0	90	0	$\theta_2{}^*$
3	0	0	d_3	$\theta_3{}^*$

$$T_{i-1}^{i} = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} T_0^1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_1^2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_2^3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{cases}$$

```
T_0_2 =

[ C1*C2, -S1, C1*S2, 0]
[ C2*S1,  C1, S1*S2, 0]
[   -S2,   0,    C2, 0]
[     0,   0,     0, 1]
```

```
T_0_3 =

[ C1*C2*C3 - S1*S3, - C3*S1 - C1*C2*S3, C1*S2, 200*C1*S2]
[ C1*S3 + C2*C3*S1,   C1*C3 - C2*S1*S3, S1*S2, 200*S1*S2]
[          -C3*S2,               S2*S3,    C2,    200*C2]
[                0,                  0,     0,         1]
```

Numerical example

Find (the actual parameters) the position and the orientation of the end effector when:

$$\theta_1 = \frac{\pi}{3}, \theta_2 = \frac{\pi}{4}, \theta_3 = \frac{\pi}{6}$$

Suppose $d_3 = 200\ mm$

Remember

Euler Matrix

$$\begin{bmatrix} c_\psi c_\varphi - s_\psi c_\theta s_\varphi & -c_\psi s_\varphi - s_\psi c_\theta c_\varphi & s_\psi s_\theta \\ s_\psi c_\varphi + c_\psi c_\theta s_\varphi & -s_\psi s_\varphi + c_\psi c_\theta c_\varphi & -c_\psi s_\theta \\ s_\theta s_\varphi & s_\theta c_\varphi & c_\theta \end{bmatrix}$$

RPY Matrix

$$\begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\gamma + c_\alpha s_\beta s_\gamma & s_\alpha s_\gamma + c_\alpha s_\beta c_\gamma \\ s_\alpha c_\beta & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & -c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix}$$

DKM =

-0.1268	-0.9268	0.3536	70.7107
0.7803	0.1268	0.6124	122.4745
-0.6124	0.3536	0.7071	141.4214
0	0	0	1.0000

	Position	RPY_Angels	RPY_Orientation	Euler_Angels	Euler_Or:
Ex	70.711	Alfa	1.7319	Psi	2.618
Ey	122.47	Beta	0.65906	Theta	0.7854
Ez	141.42	Gamma	0.46365	Phi	-1.0472
IsOk	1	IsOk	1	IsOk	1

Thanks