جامعة المنارة

قسم هندسة الروبوت والأنظمة الذكية

الدارات الكهربائية

الدكتور المهندس

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4

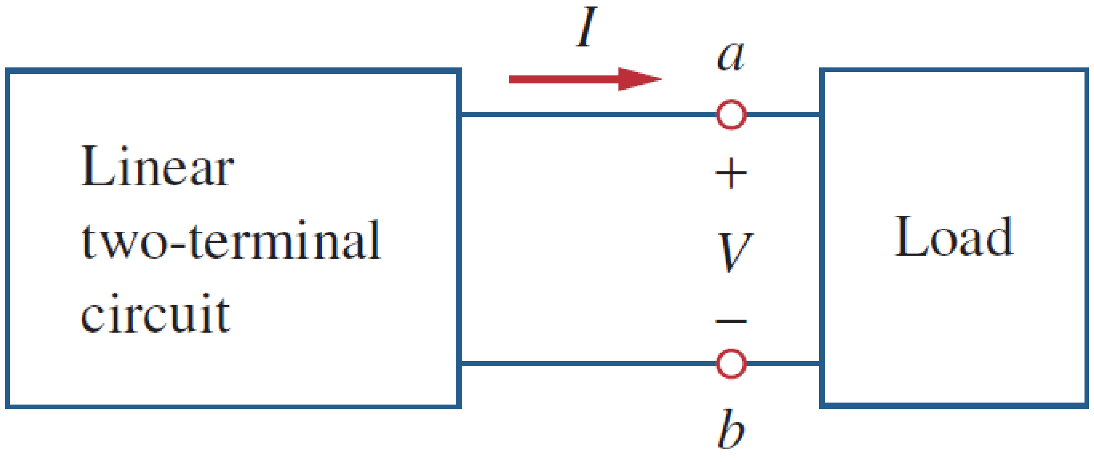
طرق تحليل الدارات الكهربائية

METHODS ANALYSIS OF ELECTRICAL CIRCUITS

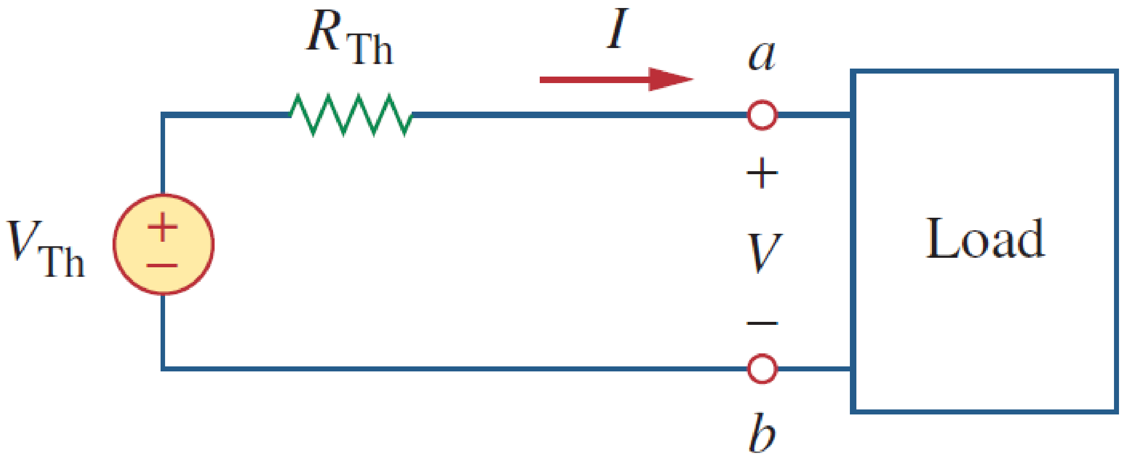
نظرية ثيفينين (منبع الجهد المكافئ) :Thevenin's Theorem

يمكن استبدال أي دارة كهربائية خطية لها نهايتان خارجيتان (a)، (b) بمنبع جهد مكافئ واحد جهده (V_{Th}) ومقاومة واحدة متصلة معه تسلسلياً (R_{Th}). في هذه الحالة تُحسب قيمة منبع الجهد المكافئ (V_{Th}) من حالة الدارة المفتوحة بقياس الجهد على الأقطاب (ab)، وتُحسب قيمة المقاومة المكافئة (R_{Th}) بقصر جميع منابع التغذية (منابع الجهد) بأسلاك عديمة المقاومة (مقاومتها تساوي الصفر)، وفتح الدارة في مكان وجود منابع التيار.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



(a)

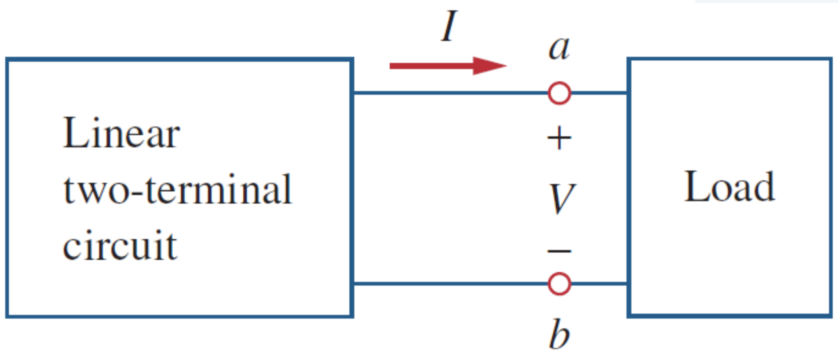


(b)

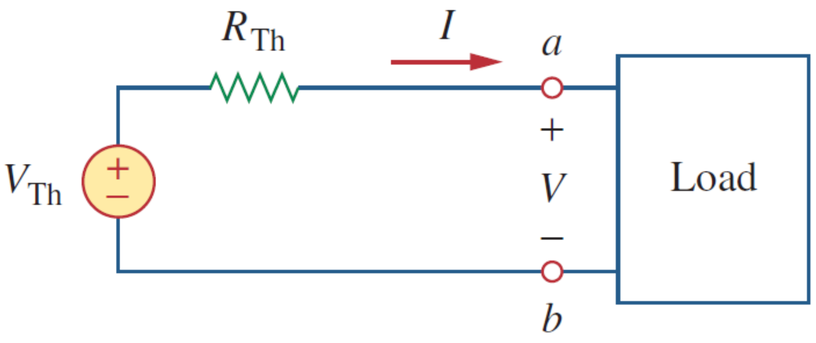
According to Thevenin's theorem, the linear circuit in Fig. (a) can be replaced by that in Fig. (b). (The load in Fig. may be a single resistor or another circuit) The circuit to the left of the terminals in Fig. (b) is known as the Thevenin equivalent circuit; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

Our major concern right now is how to find the Thevenin equivalent voltage V_{Th} and resistance R_{Th}.

To do so, suppose the two circuits in Fig. are equivalent. Two circuits are said to be equivalent if they have the same voltage-current relation at their terminals. Let us find out what will make the two circuits in Fig. equivalent.

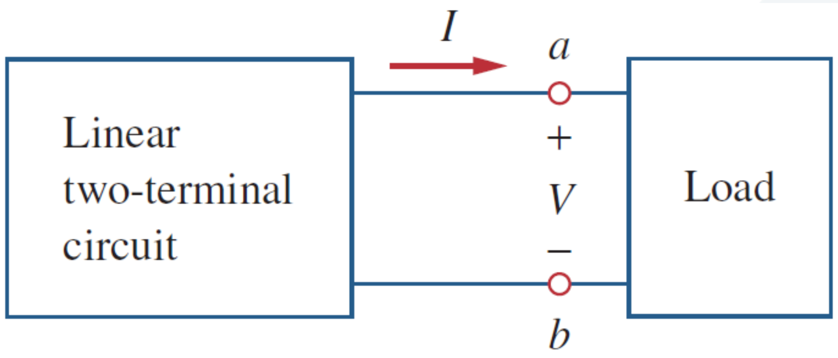


(a)

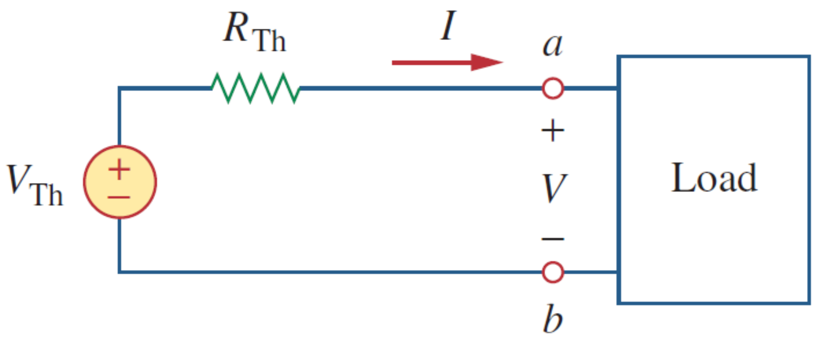


(b)

If the terminals a-b are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals a-b in Fig. (a) must be equal to the voltage source V_{Th} in Fig. (b), since the two circuits are equivalent.



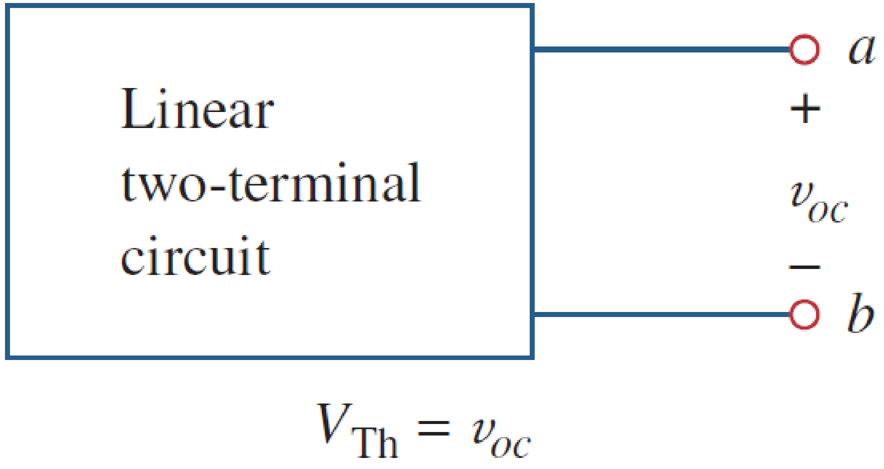
(a)



(b)

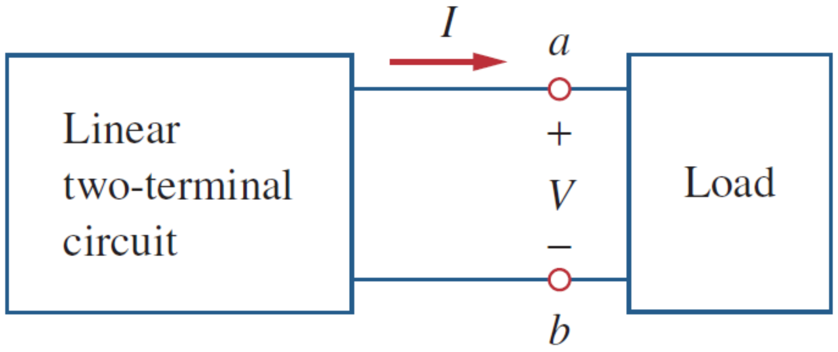
Thus V_{Th} is the open-circuit voltage across the terminals as shown in Fig. (c); that is,

$$v_{\mathrm{Th}} = v_{\mathrm{oc}}$$

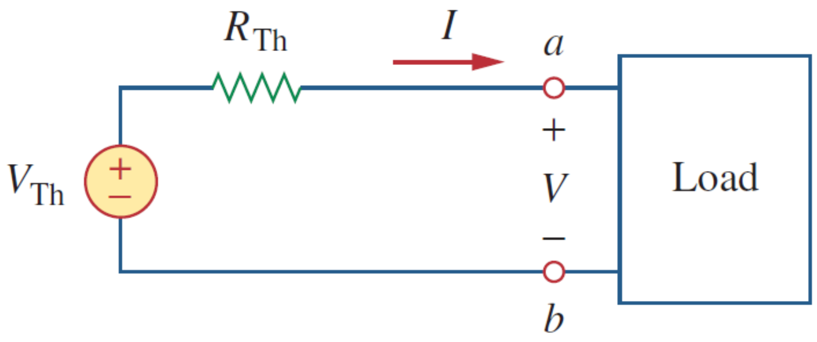


(c)

Again, with the load disconnected and terminals a-b open circuited, we turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals a-b in Fig. (a) must be equal to in Fig. (b) because the two circuits are equivalent.



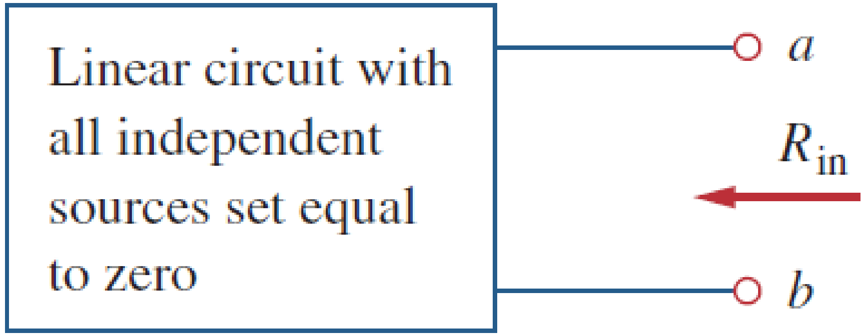
(a)



(b)

Thus, is the input resistance at the terminals when the independent sources are turned off, as shown in Fig. (d); that is,

$$R_{Th} = R_{in}$$

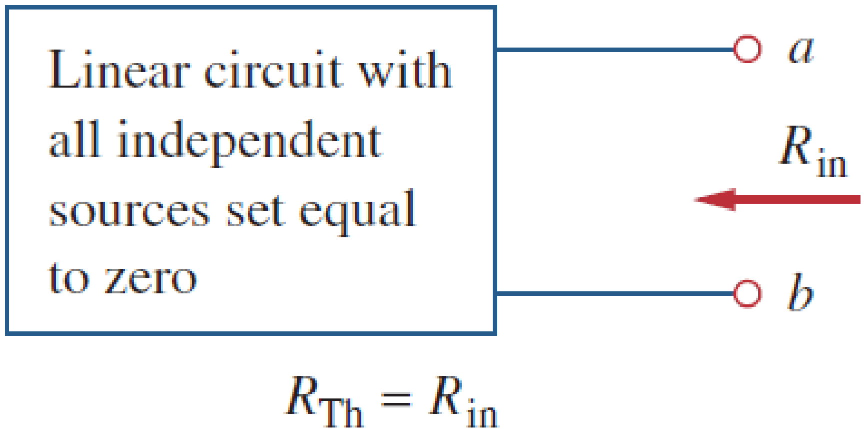


$R_{Th} = R_{in}$

(d)

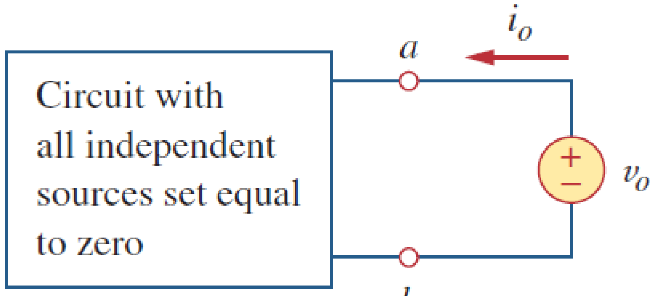
To apply this idea in finding the Thevenin resistance R_{Th} , we need to consider two cases.

■ CASE 1 If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b, as shown in Fig. (d).



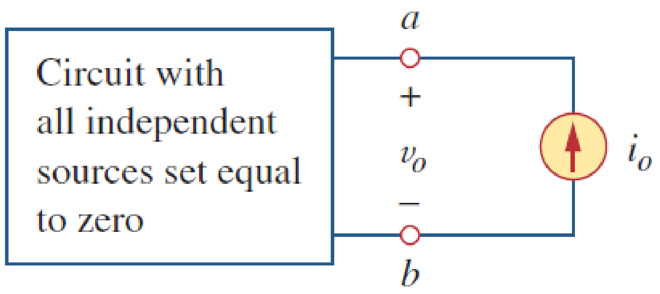
(d)

■ CASE 2 If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_0 at terminals a and b and determine the resulting current i_0. Then $R_{Th}=v_0/i_0$, as shown in Fig. (e). Alternatively, we may insert a current source i_0 at terminals a-b as shown in Fig. (f) and find the terminal voltage v_0. Again $R_{Th}=v_0/i_0$. Either of the two approaches will give the same result. In either approach we may assume any value of v_0 and i_0. For example, we may use $v_0=1V$ or $i_0=1A$, or even use unspecified values of v_0 or i_0 .



$$R_{Th} = \frac{v_o}{i_o}$$

(e)



$$R_{Th} = \frac{v_o}{i_o}$$

(f)

It often occurs that R_{Th} takes a negative value. In this case, the negative resistance ($v=-iR$) implies that the circuit is supplying power. This is possible in a circuit with dependent sources.

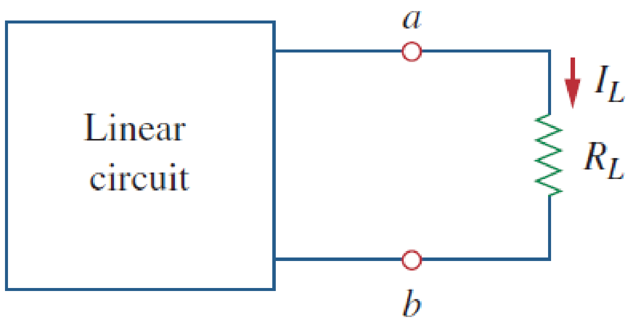
Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load. The equivalent network behaves the same way externally as the original circuit.

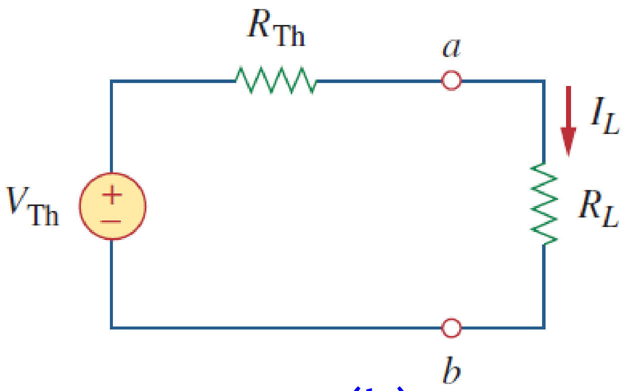
Consider a linear circuit terminated by a load , as shown in Fig. (g). The current through the load and the voltage across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in Fig. (h). From Fig. (h), we obtain

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L \cdot I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

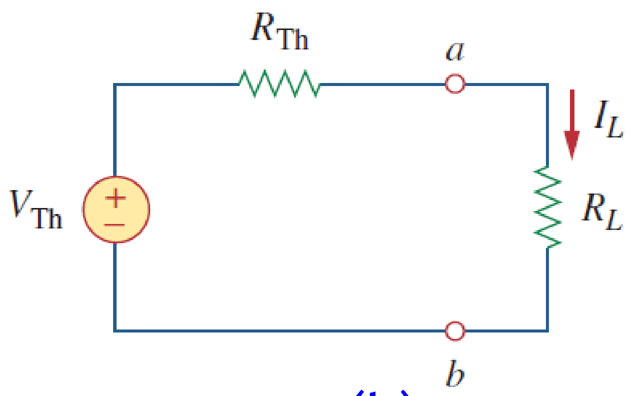


(g)



(h)

Note from Fig. (h) that the Thevenin equivalent is a simple voltage divider, yielding V_L by mere inspection.

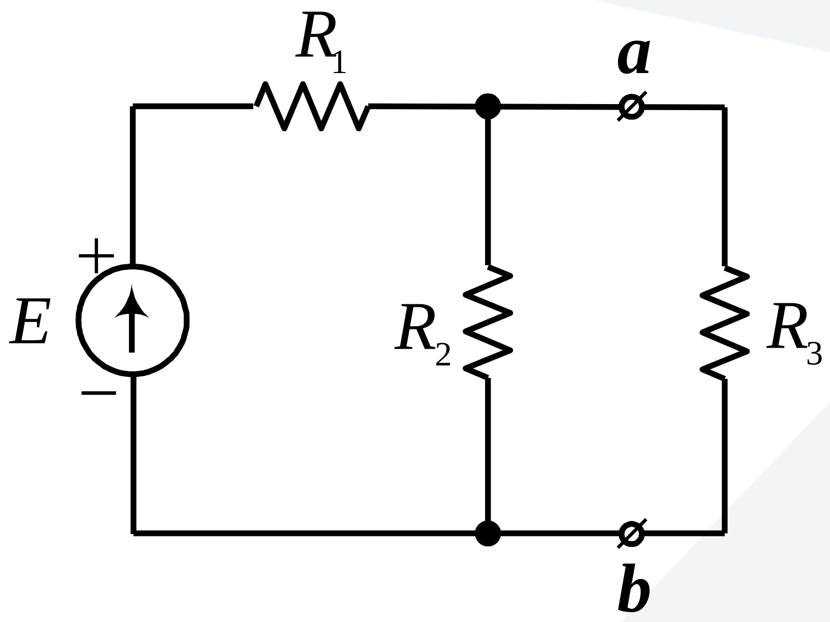


(h)

مثال:

لتكن لدينا الدارة المبينة بالشكل، فإذا علمت أن:

$E=120\ [V]\ ,\ R_1=40\ [\Omega]\ ,\ R_2=20\ [\Omega]\ ,\ R_3=10\ [\Omega]$

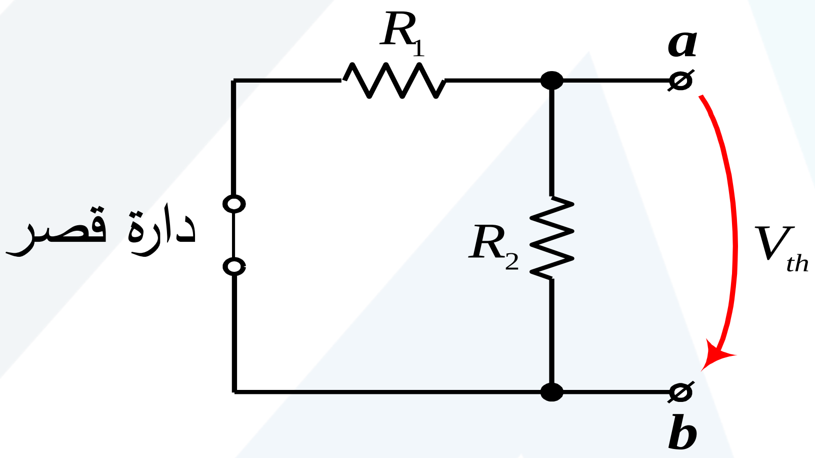


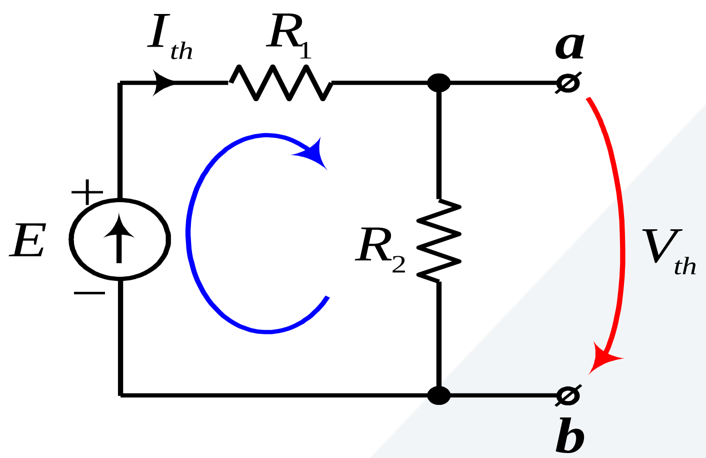
المطلوب:

1. احسب قيمة التيار في المقاومة R_3 بتطبيق نظرية ثيفينين.
2. ما هي قيمة هذا التيار إذا أستعيض عن هذه المقاومة بمقاومة قيمتها $30\ [\Omega]$؟.

الحل:

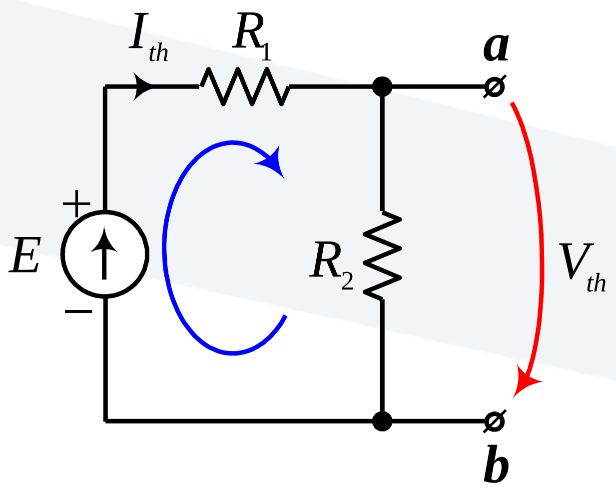
نقوم بعزل الفرع الحاوي على المقاومة R_3. وحسب نظرية ثيفينين يمكن مكافئة الدارة بدارة مكونة من منبع جهد قيمته تساوي الجهد على الأقطاب a، b أي جهد الدارة المفتوحة، ومن مقاومة موصولة تسلسلياً معه قيمتها تساوي قيمة المقاومة المكافئة للدارة بعد عَدّ القوى المحركة الكهربائية تساوي الصفر، كما في الشكل.





المقاومة المكافئة تُحسب من حالة دارة القصر الناتجة عن عزل منبع الجهد واستبداله بسلك كما هو مبين بالدارة السابقة، وبالتالي:

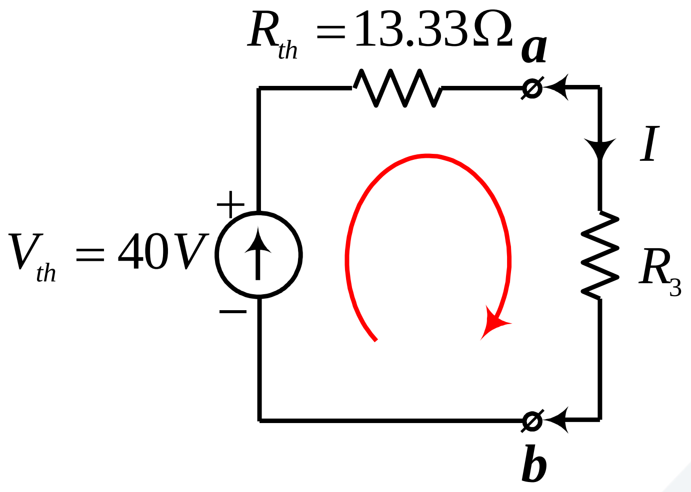
$$R_{th} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{40 \times 20}{40 + 20} = 13.33[\Omega]$$



لحساب جهد ثيفينين يجب حساب التيار I_{th} في الحلقة، كون هذا التيار لا يخرج من الحلقة لأن الدارة مفتوحة. فحسب قانون كيرشوف الثاني في الحلقة يكون:

$$I_{th} = \frac{E}{R_1 + R_2} = \frac{120}{60} = 2[A]$$

وبالتالي جهد ثيفينين: $V_{th} = I_{th} \cdot R_2 = 2 \times 20 = 40[V]$



تصبح دارة ثيفينين المكافئة للدارة الرئيسة كما هو مبين بالشكل جانباً، حيث نعيد المقاومة R_3 إلى الدارة، وعندها يمكن بتطبيق فانون كيرشوف الثاني في الحلقة حساب قيمة التيار المارفي هذه المقاومة:

$$I = \frac{V_{th}}{R_{th} + R_3} = \frac{40}{13.33 + 10} = 1.714[A]$$

عند تغيير قيمة المقاومة R_3 يكفي أن نغيرهذه القيمة في العلاقة السابقة فنحصل على قيمة التيار المطلوب:

$$I = \frac{40}{13.33 + 30} = 0.92[A]$$

نظرية نورتون (منبع التيار المكافئ) Norton's Theorem:

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

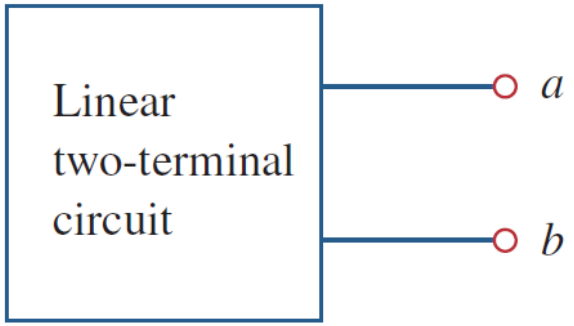
Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

يمكن استبدال أي دارة كهربائية خطية لها نهايتان خارجيتان (a) ، (b) بمنبع تيار مكافئ واحد تياره (I_N) ومقاومة واحدة متصلة معه تفرعياً (R_N). يتم حساب قيمة منبع نورتون المكافئ (I_N) من خلال قصر الدارة بين الأقطاب (a) و (b) (Short Circuit)، أي هو قيمة التيار المار بين القطبين a و b عند وصلهما بسلك مقاومته معدومة (تيار القصر). أما مقاومة نورتون المكافئة (R_N) فهي قيمة المقاومة المحسوبة بين القطبين (a) و (b) بقصر جميع منابع التغذية الداخلية بأسلاك عديمة المقاومة (مقاومتها تساوي الصفر)، وفتح منابع التيار إن وجدت في الدارة. أي أن المقاومة المكافئة حسب نظريتي ثيفينين ونورتون هي نفسها.

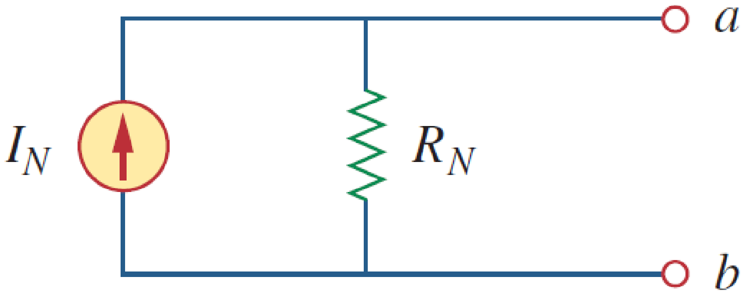
Thus, the circuit in Fig. (i) can be replaced by the one in Fig. (j).

For now, we are mainly concerned with how to get and We find in the same way we find In fact, from what we know about source transformation, the Thevenin and Norton resistances are equal; that is,

$$R_N = R_{Th}$$



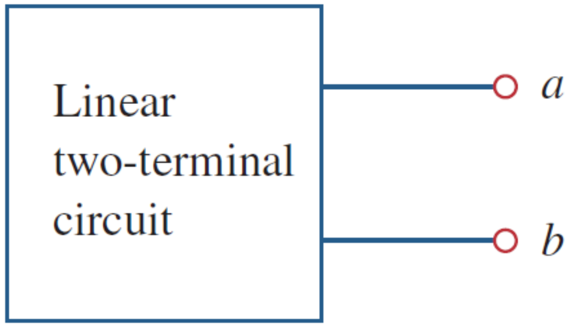
(i)



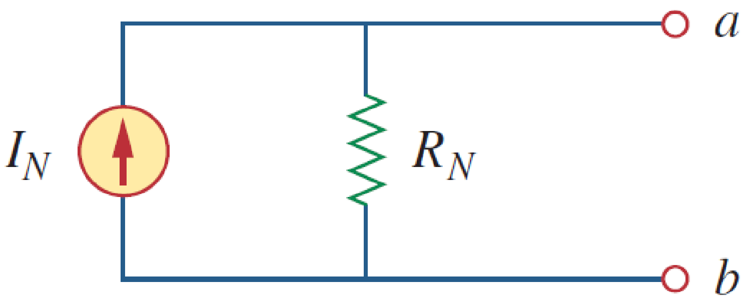
(j)

To find the Norton current I_N we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. It is evident that the short-circuit current in Fig. (j) is This must be the same short-circuit current from terminal a to b in Fig. (i), since the two circuits are equivalent. Thus,

$$I_N = I_{SC}$$

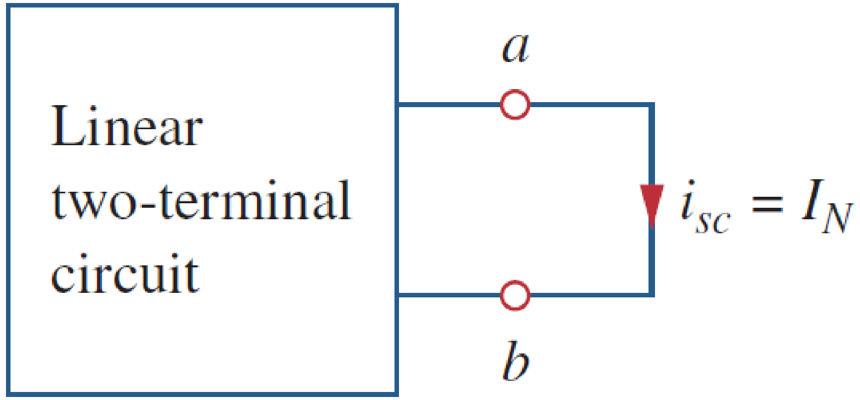


(i)

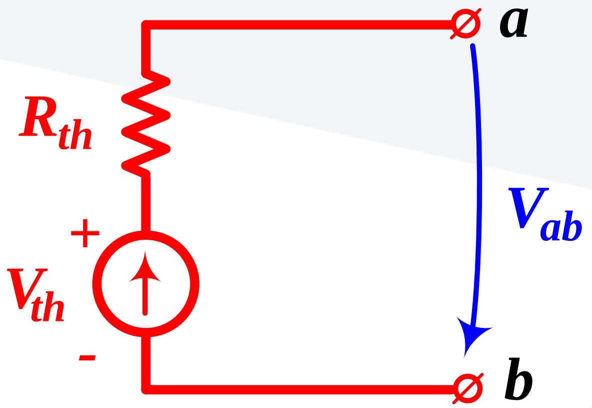


(j)

shown in Fig. Dependent and independent sources are treated the same way as in Thevenin's theorem.

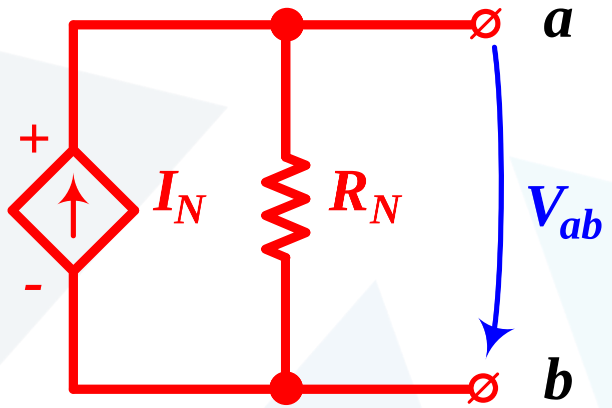


التكافؤ بين نظريتي ثيفينين ونورتون:



Thevenin

$$V_{ab} = Const.$$



Norton

Thevenin: $V_{ab} = V_{th}$

Norton: $V_{ab} = I_N \cdot R_N$

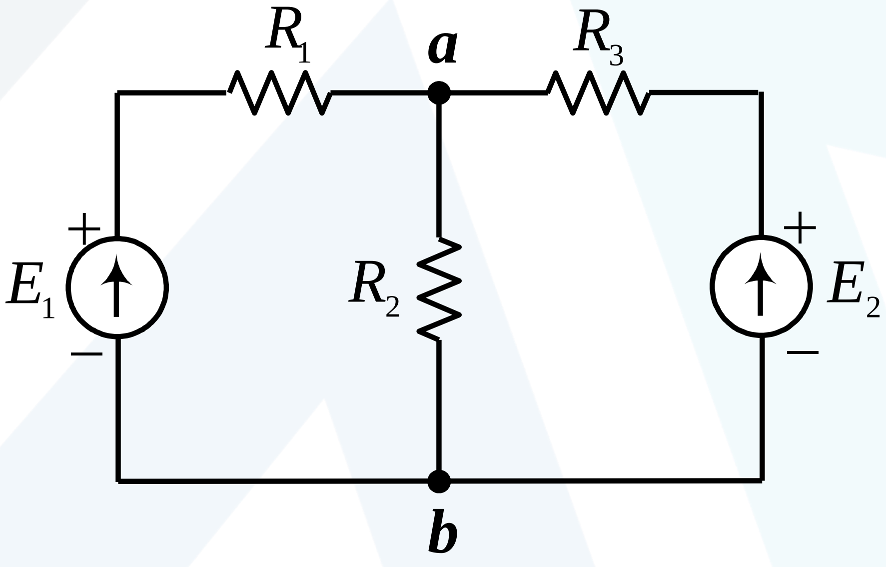
$$\left.\begin{aligned} V_{ab} &= V_{th} \\ V_{ab} &= I_N \cdot R_N \end{aligned}\right\} \Rightarrow V_{th} = I_N \cdot R_N \Rightarrow I_N = \frac{V_{th}}{R_N}$$

$$R_{th} = R_N \Rightarrow I_N = \frac{V_{th}}{R_{th}}$$

مثال:

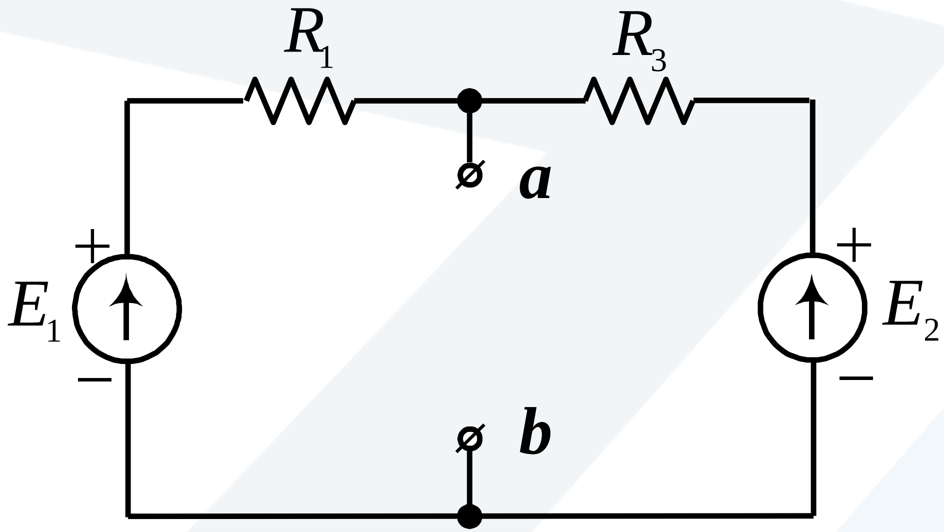
أوجد دارة نورتون المكافئة بالنسبة للأقطاب a و b للدارة المبينة بالشكل، ثم احسب قيمة التيار المار بالمقاومة R_2، علماً أن:

$$E_1 = 70[V], \qquad E_2 = 20[V]$$
$$R_1 = R_2 = 5[\Omega], \quad R_3 = 15[\Omega]$$



الحل:

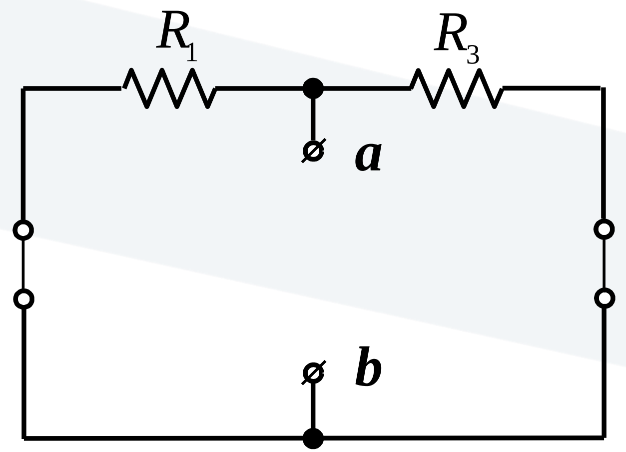
نعزل المقاومة R_2 بين القطبين a و b، كما في الشكل.



حسب نظرية نورتون: يمكن مكافئة الدارة بدارة مكونة من منبع تيار تُحسب قيمته من قصر الدارة بين القطبين بسلك عديم المقاومة وحساب تيار القصر المار في هذا السلك، ومن مقاومه موصولة تفرعياً مع منبع التيار قيمتها تساوي قيمة المقاومة المكافئة للدارة بعد عَدّ القوى المحركة الكهربائية تساوي الصفر.

حساب قيمة المقاومة المكافئة:

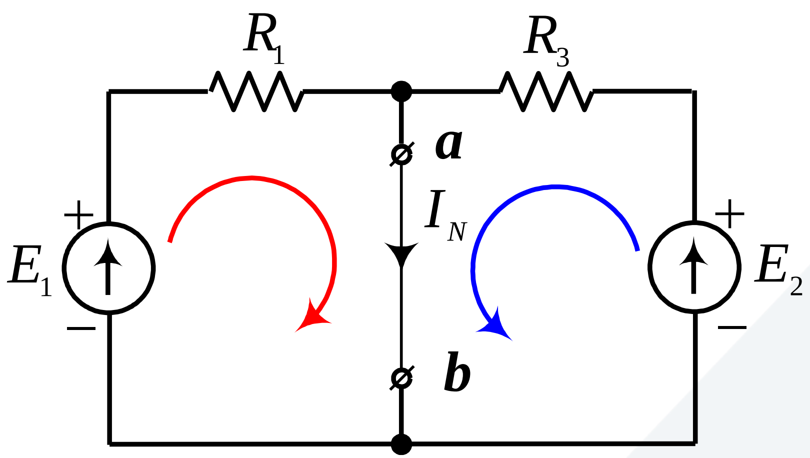
من الدارة المبينة بالشكل نجد أن:



$$R_N = \frac{R_1 \cdot R_3}{R_1 + R_3}$$

$$R_N = \frac{5 \times 15}{5 + 15} = 3.75[\Omega]$$

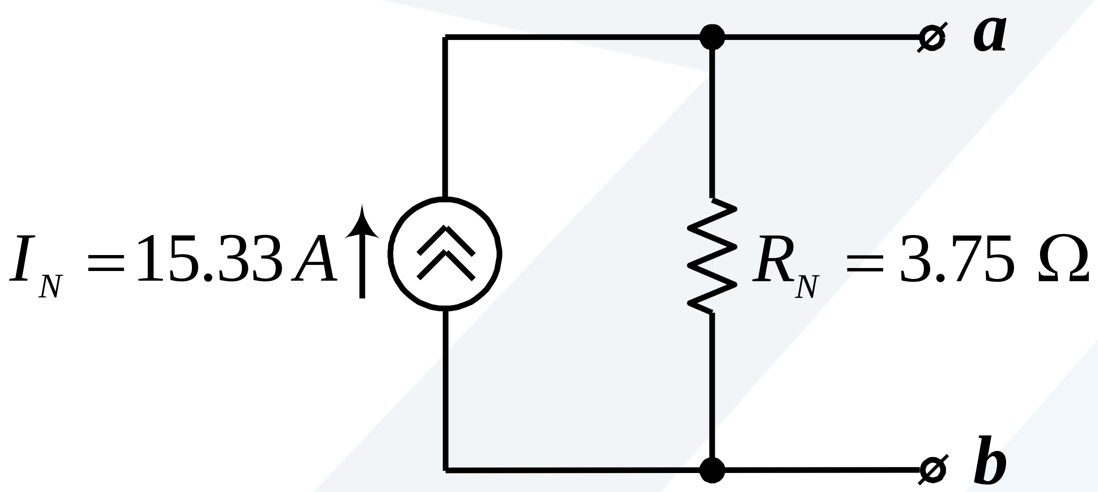
حساب قيمة تيار نورتون:



نقصر بين القطبين **a** و **b** بسلك عديم المقاومة فتصبح الدارة كما في الشكل. وبالتالي وحسب طريقة التيارات الحلقية:

$$I_N = \frac{E_1}{R_1} + \frac{E_2}{R_3} = \frac{70}{5} + \frac{20}{15} = 14 + 1.33 = 15.33[A]$$

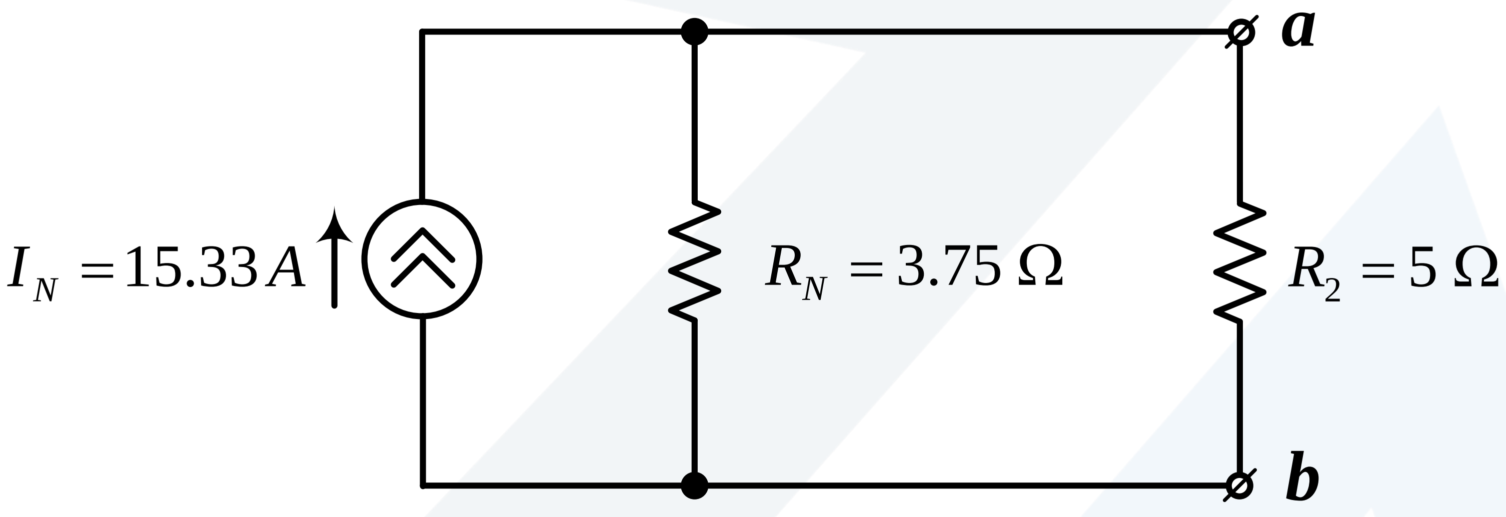
وفقاً لذلك تصبح دارة نورتون كما في الشكل. يمكن في هذه الحالة حساب قيمة الجهد بين القطبين **a** و **b** كما يأتي:



$$V_{ab} = I_N \cdot R_N$$

$$V_{ab} = 15.33 \times 3.75 \approx 57.5[V]$$

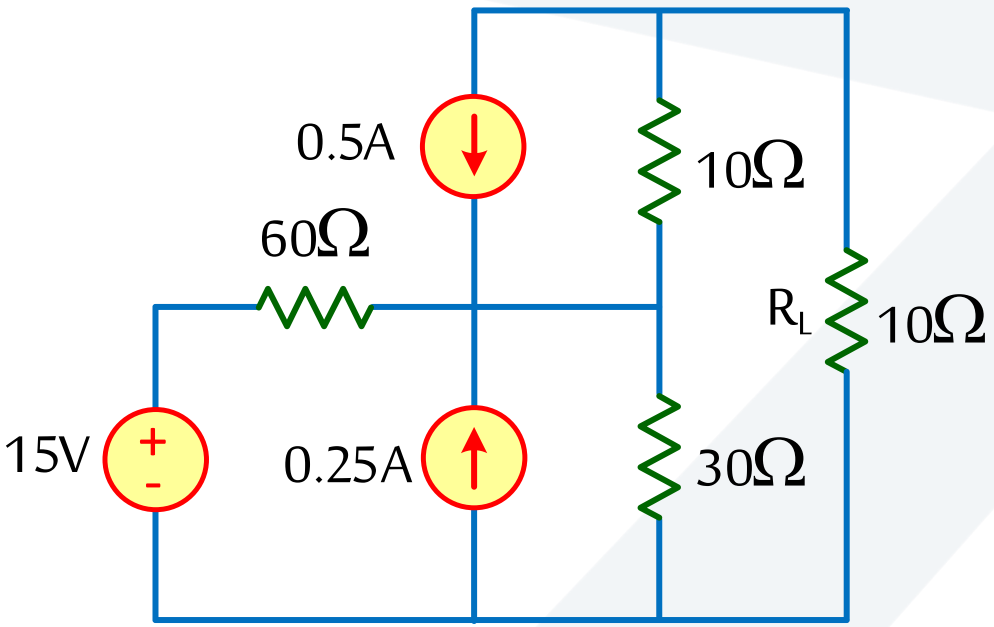
لحساب قيمة التيار المار في المقاومة R_2 نعيدها إلى دارة نورتون بين القطبين **a** و **b**. ثم نحسب قيمة التيار حسب قاعدة مجزئ التيار كما يلي:



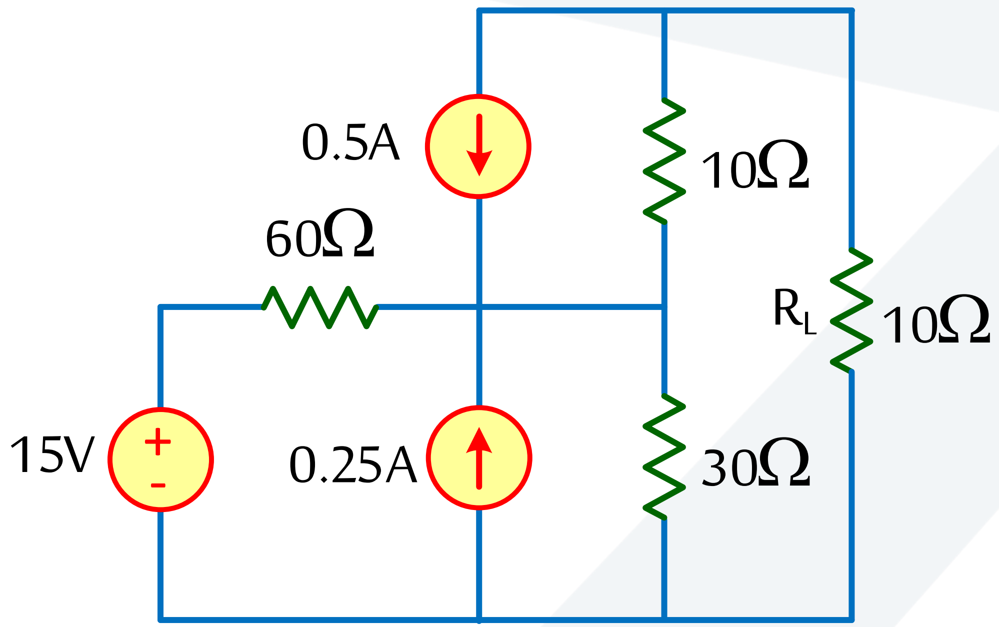
$$I_2 = I_N \cdot \frac{R_N}{R_N + R_2} = 15.33 \times \frac{3.75}{3.75 + 5} = 6.57[A]$$

مسائل

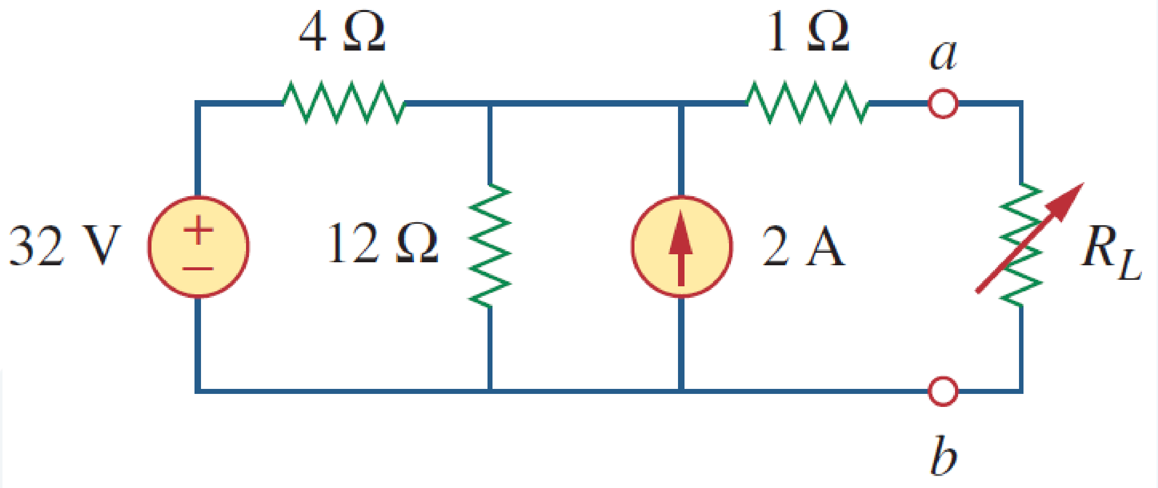
1. احسب قيمة التيار المار في المقاومة $R_L=10\Omega$ باستخدام نظرية ثيفينين، ثم استنتج دارة نورتون المكافئة.



2. احسب قيمة التيار المار في المقاومة $R_L=10\Omega$ باستخدام نظرية نورتون، ثم استنتج دارة ثيفينين المكافئة.



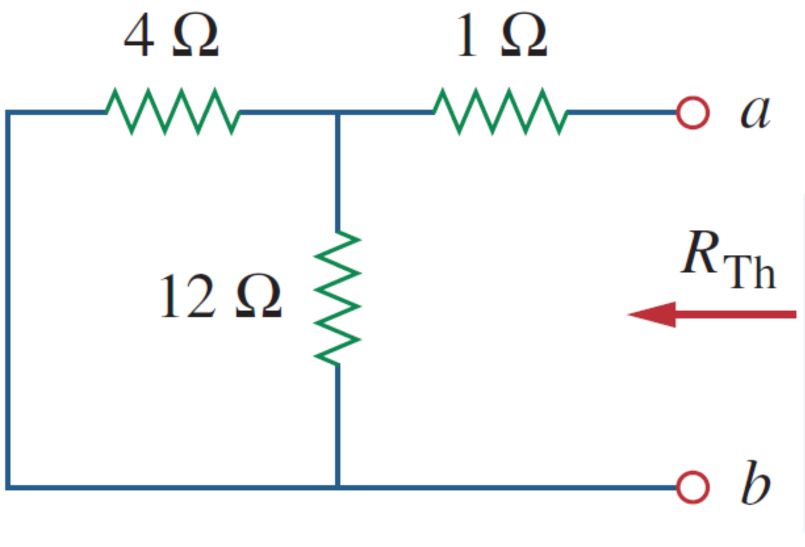
3. Find the Thevenin equivalent circuit of the circuit shown in Fig. to the left of the terminals a-b. Then find the current through R_L=6 , 16 and 36 Ω



Solution:

We find by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an R_{Th} open circuit). The circuit becomes what is shown in Fig.(a). Thus,

$$R_{Th} = 4//\,12 + 1 = \frac{4 \times 12}{2 + 12} + 1 = 4\Omega$$



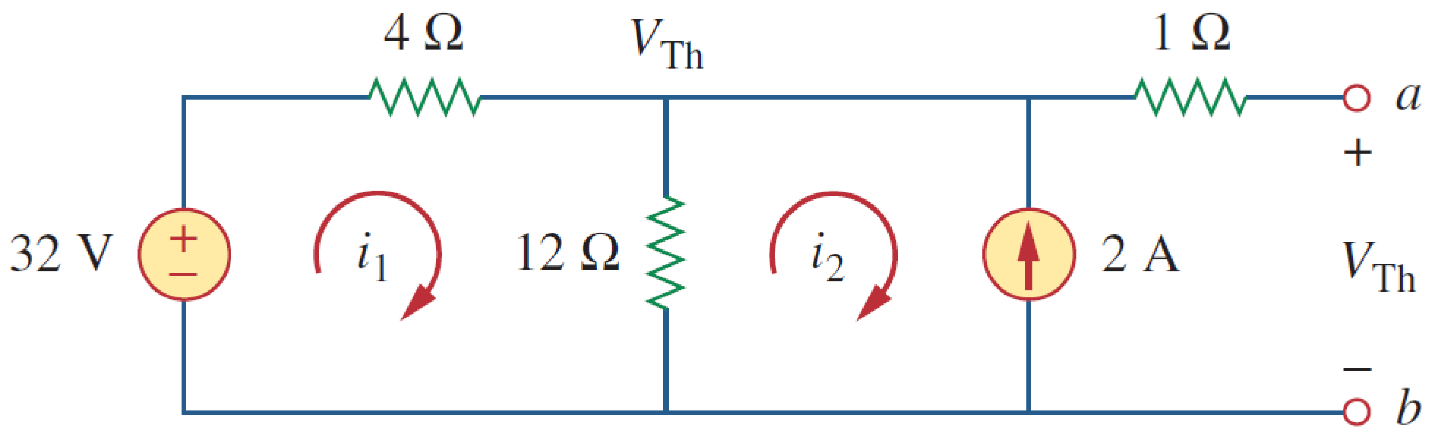
(a)

To find consider the circuit in Fig. (b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0\,, \qquad i_2 = -2A$$

Solving for i_1 , we get i_1=0.5A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12 \times (0.5 + 2) = 30V$$



(b)

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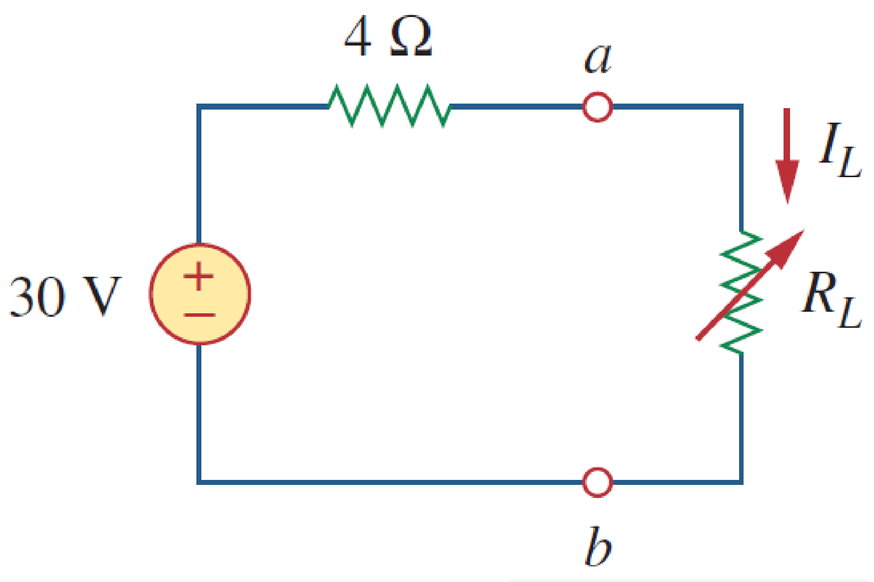
The Thevenin equivalent circuit is shown in Fig. The current through R_L is



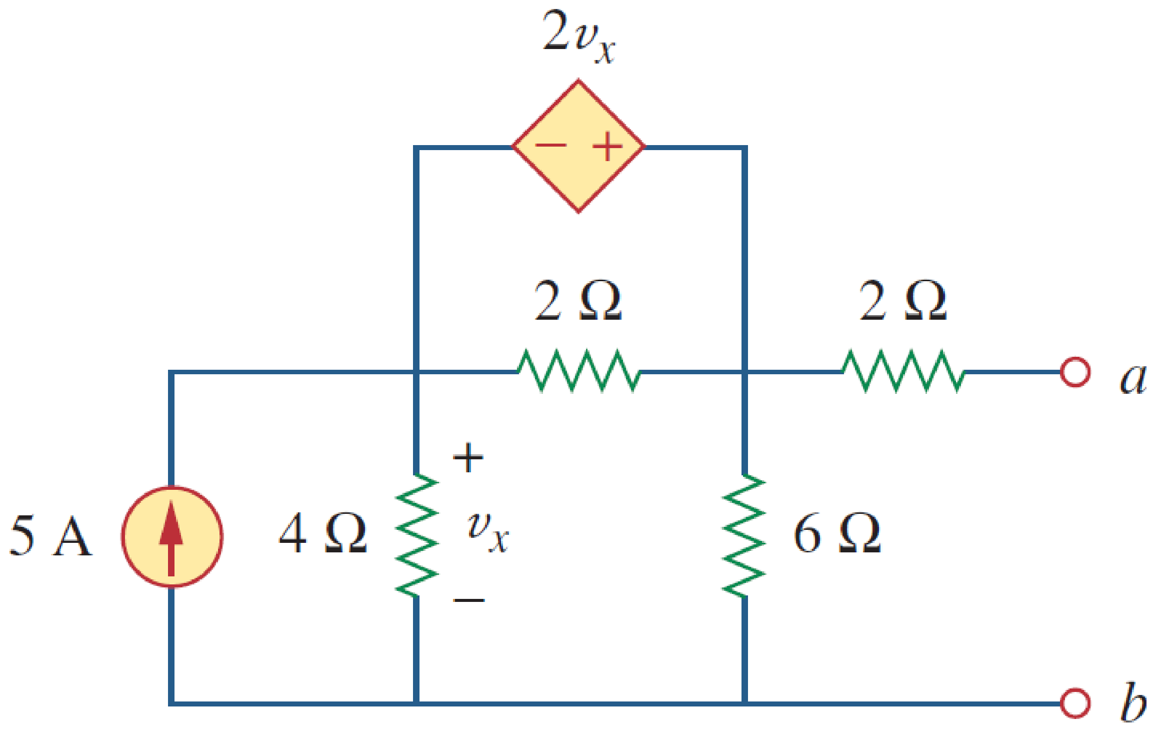
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L} = \frac{30}{10} = 3A$$

When R_L=6Ω $\quad I_L = \frac{30}{10} = 3A$

When R_L=16Ω $\quad I_L = \frac{30}{20} = 1.5A$

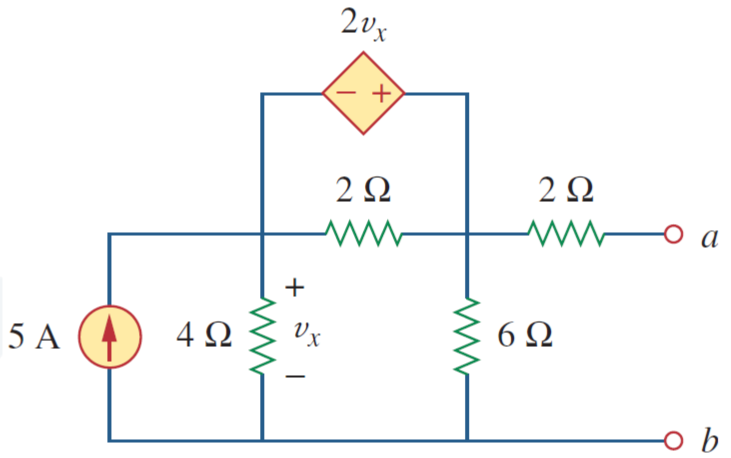
When R_L=36Ω $\quad I_L = \frac{30}{40} = 0.75A$

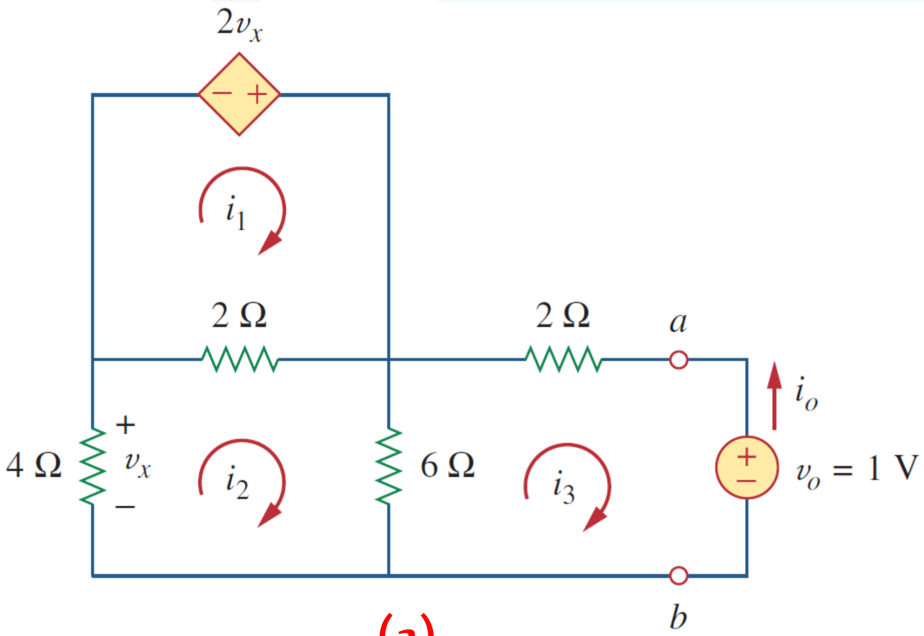
4. Find the Thevenin equivalent of the circuit in Fig. at terminals a-b.



Solution:

This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th} we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_0 connected to the terminals as indicated in Fig. (a). We may set v_0=1V to ease calculation, since the circuit is linear. Our goal is to find the current i_0 through the terminals, and then obtain $R_{Th}=1/i_o$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage v_0 and obtain $R_{Th}=v_o/1$).





(a)

Applying mesh analysis to loop 1 in the circuit of Fig. (a) results in

$$-2v_x + 2(i_1 - i_2) = 0$$

or $$v_x = i_1 - i_2$$

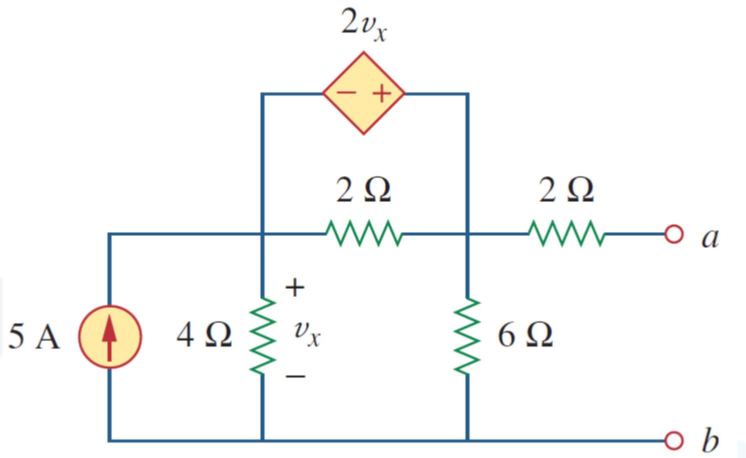
But $$-4i_2 = v_x = i_1 - i_2$$

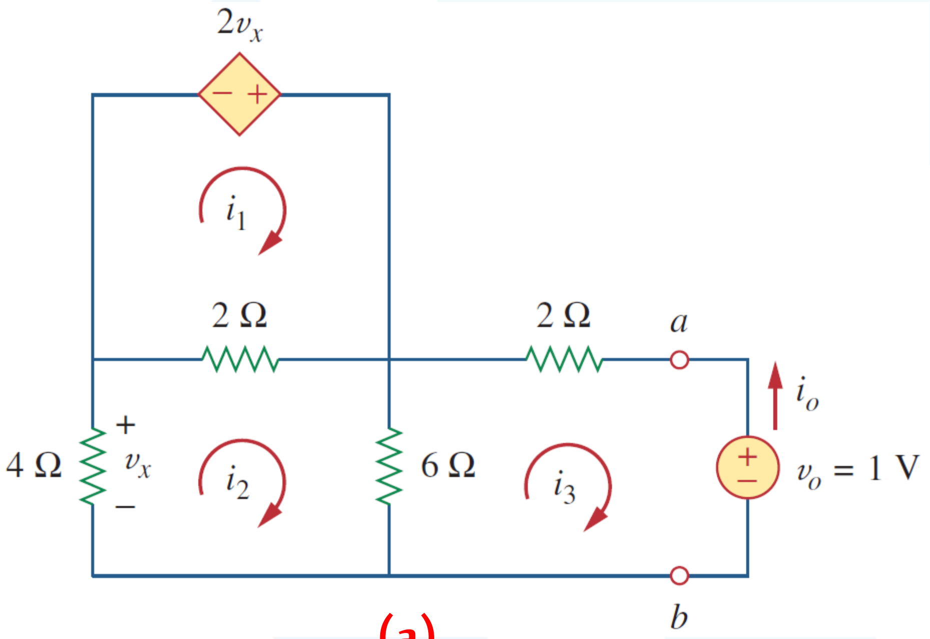
Hence, $$i_1 = -3i_2 \qquad \textbf{(1)}$$

For loops 2 and 3, 1 Applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \qquad \textbf{(2)}$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \qquad \textbf{(3)}$$



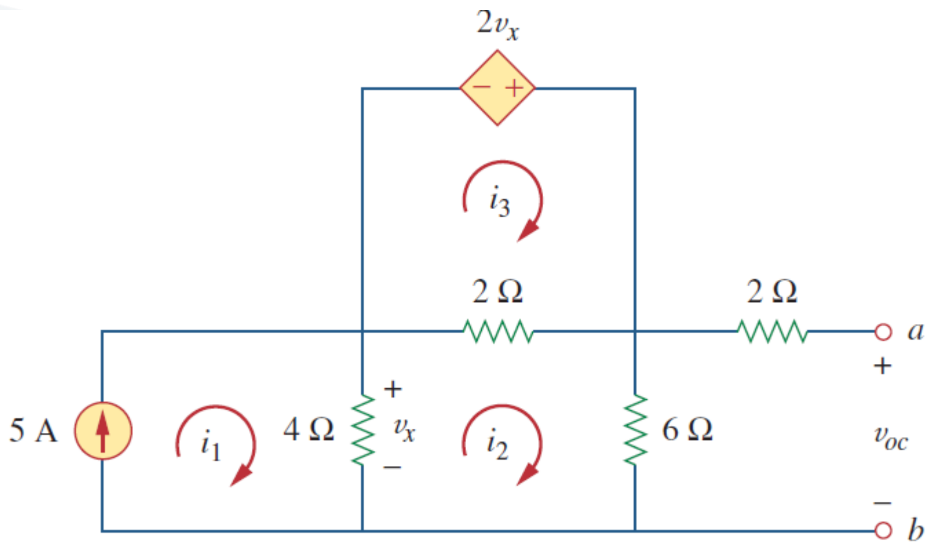


(a)

Solving these equations gives $i_3 = -\frac{1}{6}\text{A}$

But $i_0 = -i_3 = \frac{1}{6}\text{A}$ Hence, $R_{Th} = \frac{1V}{i_0} = 6\Omega$

To get V_{Th}. We find v_{oc} in the circuit of Fig (b). Applying mesh analysis, we get



(b)

$$i_1 = 5\text{A} \quad (4)$$

$$-2v_x + 2(i_3 - i_2) = 0 \;\Rightarrow\; v_x = i_3 - i_2 \quad (5)$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

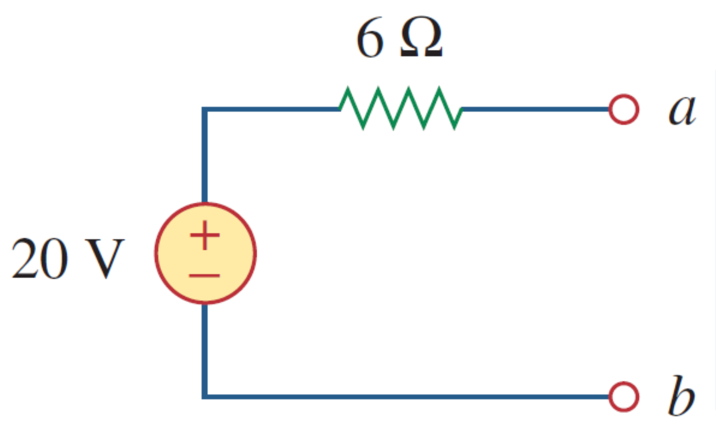
or

$$12i_2 - 4i_1 - 2i_3 = 0 \quad (6)$$

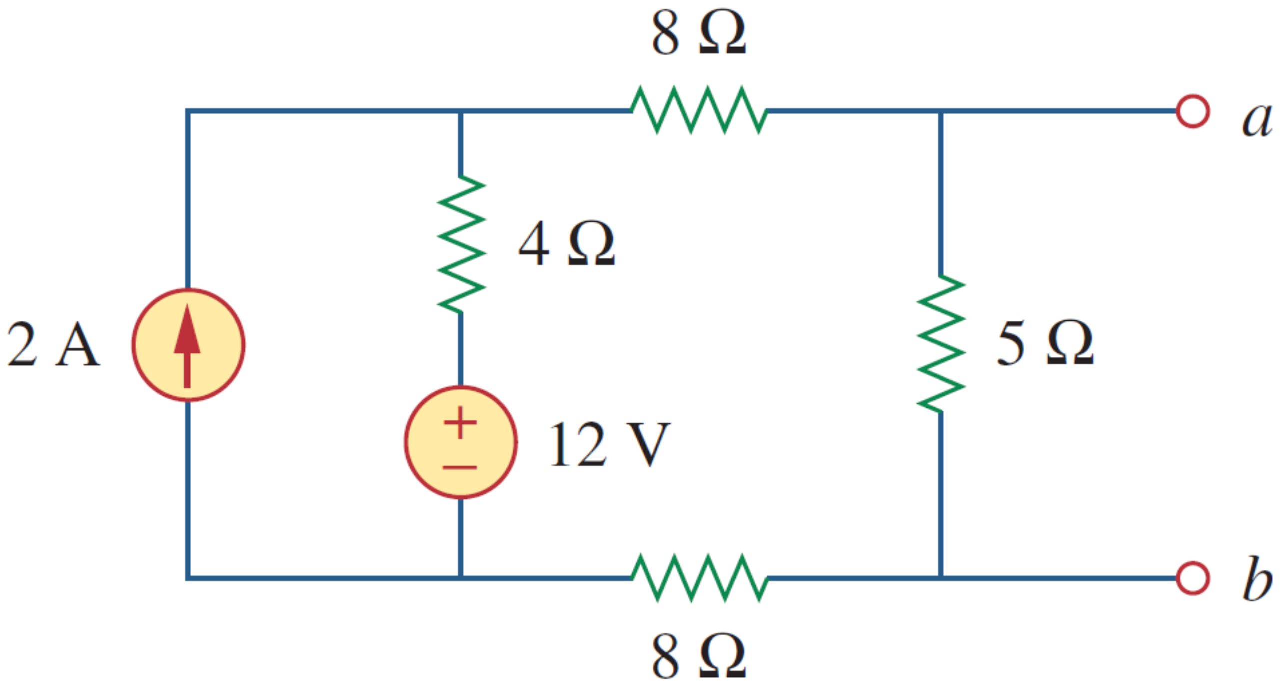
But $4(i_1-i_2)=v_x$. Solving these equations leads to $i_3=10/3$. Hence,

$$V_{Th} = V_{oc} = 6i_2 = 20V$$

The Thevenin equivalent is as shown in Fig.



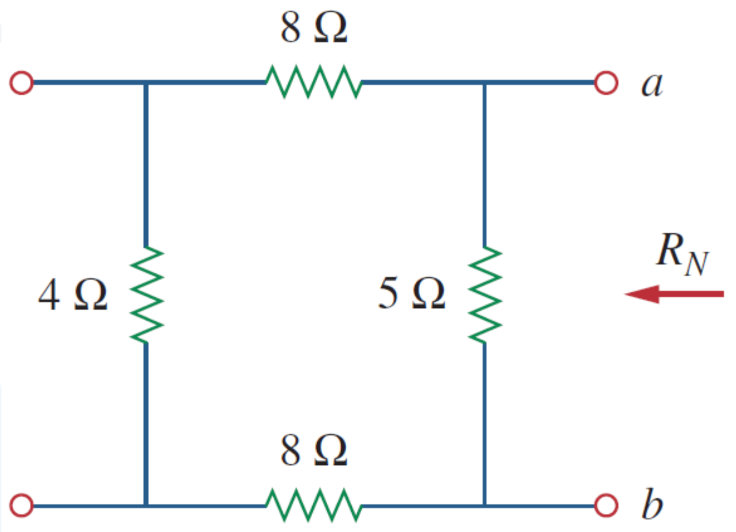
5. Find the Norton equivalent circuit of the circuit in Fig. at terminals a-b.



Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. (a), from which we find R_N, Thus,

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{5 \times 20}{5 + 20} = 4\Omega$$



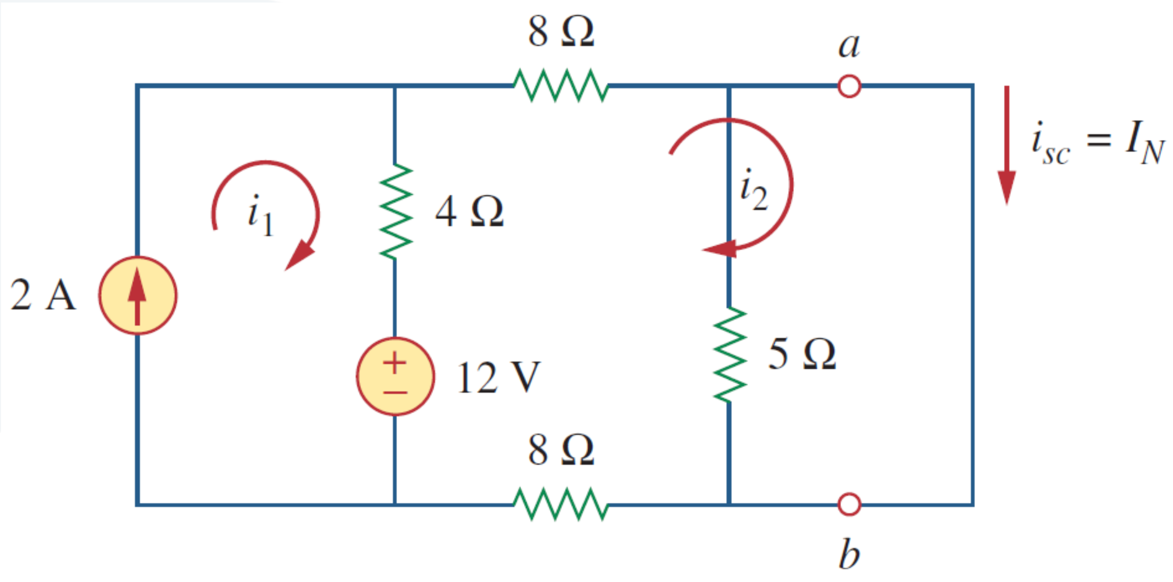
(a)

To find I_N we short-circuit terminals a and b, as shown in Fig. (b). We ignore 5-Ω the resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2A \quad , \qquad 20i_2 - 4i_1 - 12 = 0$$

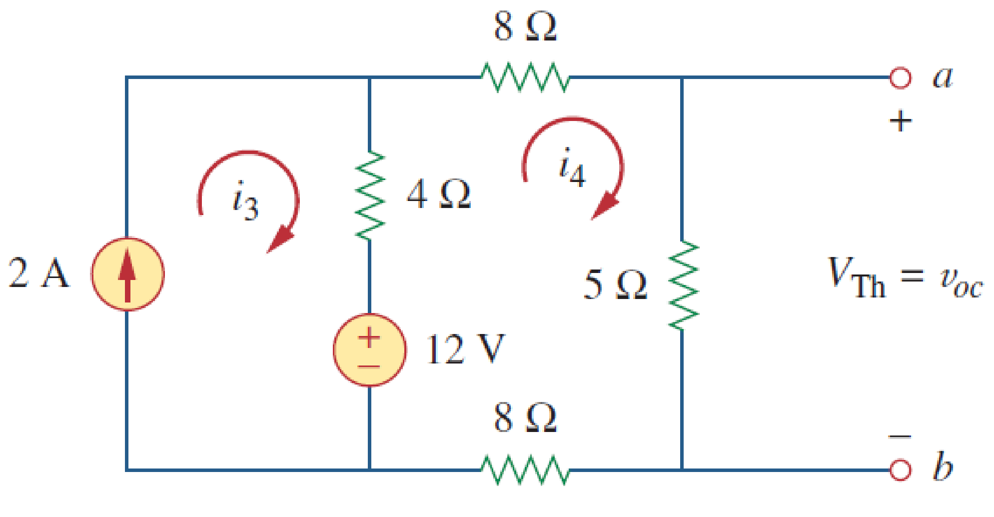
From these equations, we obtain

$$i_2 = 1A = i_{sc} = I_N$$



(b)

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. (c). Using mesh analysis, we obtain



(c)

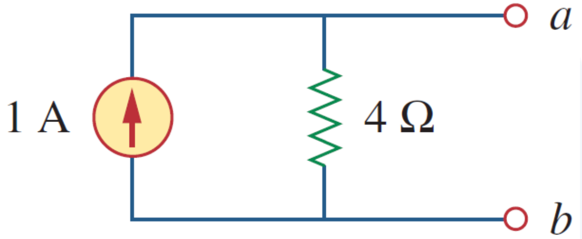
$$i_3 = 2A$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8A$$

and $v_{oc} = v_{Th} = 5i_4 = 4V$ Hence $I_N = \dfrac{V_{Th}}{R_{Th}} = \dfrac{4}{4} = 1A$

as obtained previously. This also serves to confirm Eq. $R_{Th} = \dfrac{V_{oc}}{i_{sc}}$ that $R_{Th} = \dfrac{V_{oc}}{i_{sc}} = \dfrac{4}{1} = 4\Omega$

Thus, the Norton equivalent circuit is as shown in Fig.



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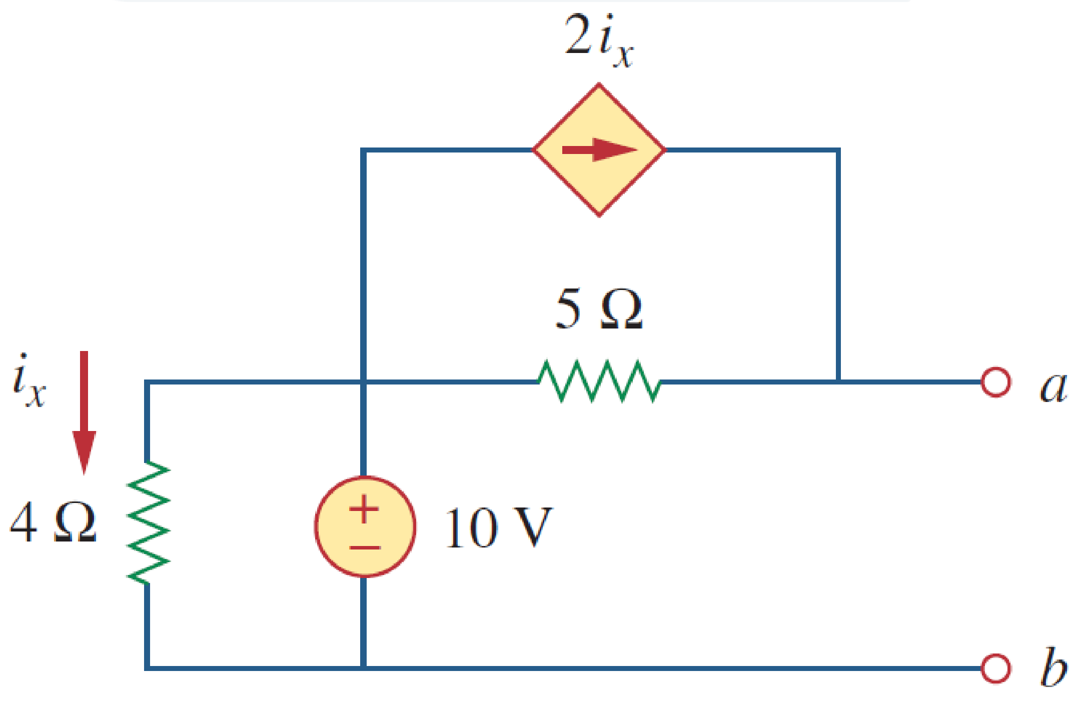
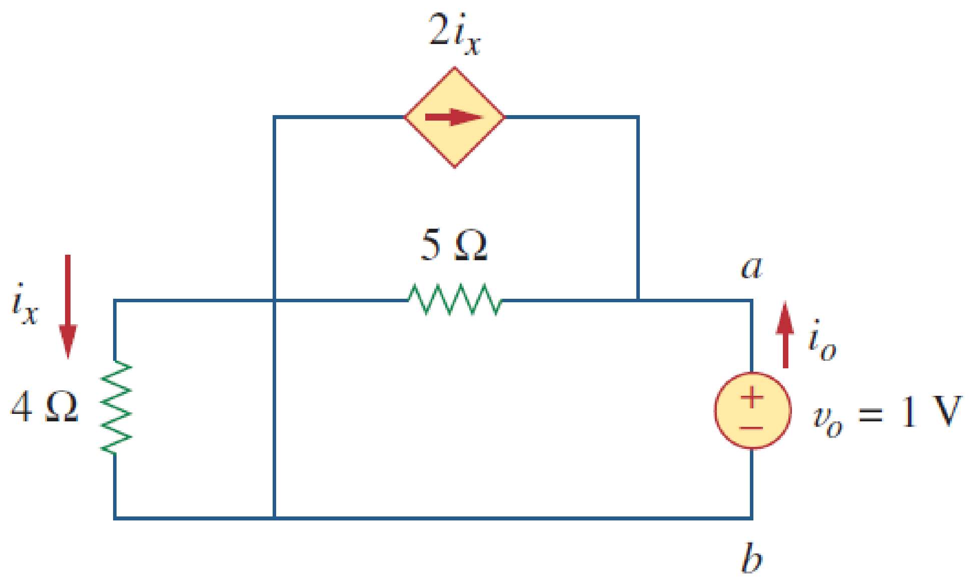
6. Using Norton's theorem, find and of the circuit in Fig. at terminals a-b.



Solution:

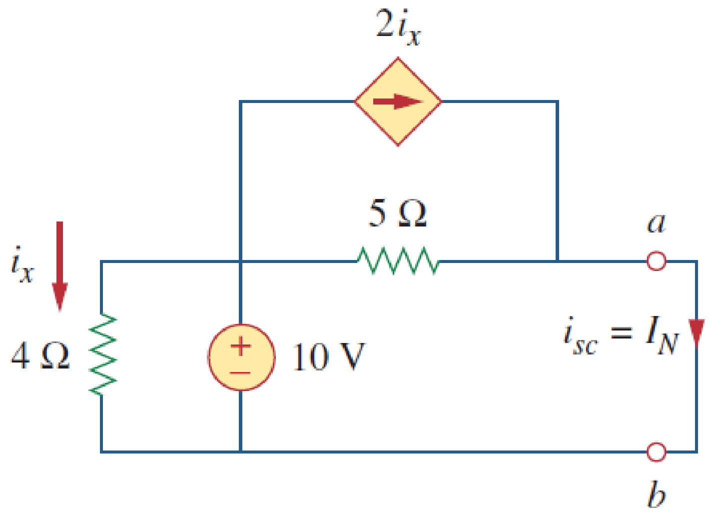
To find R_N we set the independent voltage source equal to zero and connect a voltage source of v_0=1V (or any unspecified voltage v_0) to the terminals. We obtain the circuit in Fig. (a). We ignore the 4-Ω resistor because it is short-circuited. Also due to the short circuit, the 5-Ω resistor, the voltage source, and the dependent current source are all in parallel. Hence, i_x=0, At node a, i_0=1V/5Ω=0.2A, and

$$R_N = \frac{v_0}{i_o} = \frac{1}{0.2} = 5\Omega$$



(a)

To find I_N we short-circuit terminals a and b and find the current i_{sc} as indicated in Fig. (b). Note from this figure that the 4-Ω resistor, the 10-V voltage source, the 5-Ω resistor, and the dependent current source are all in parallel. Hence,



(b)

$$i_x = \frac{10}{4} = 2.5\text{A}$$

At node a, KCL gives $$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2 \times (2.5) = 7\text{A}$$

Thus $$I_N = 7\text{A}$$

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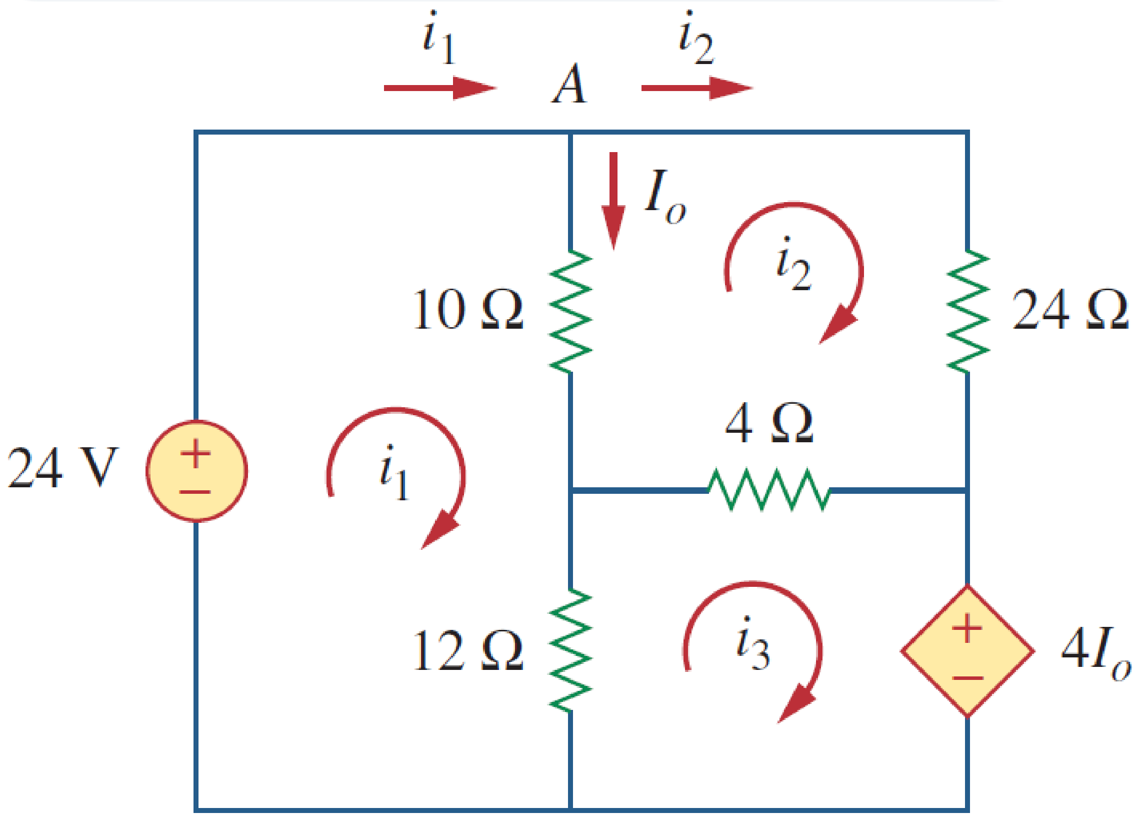
7. Use mesh analysis to find the current I_o in the circuit of Fig.



Solution:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

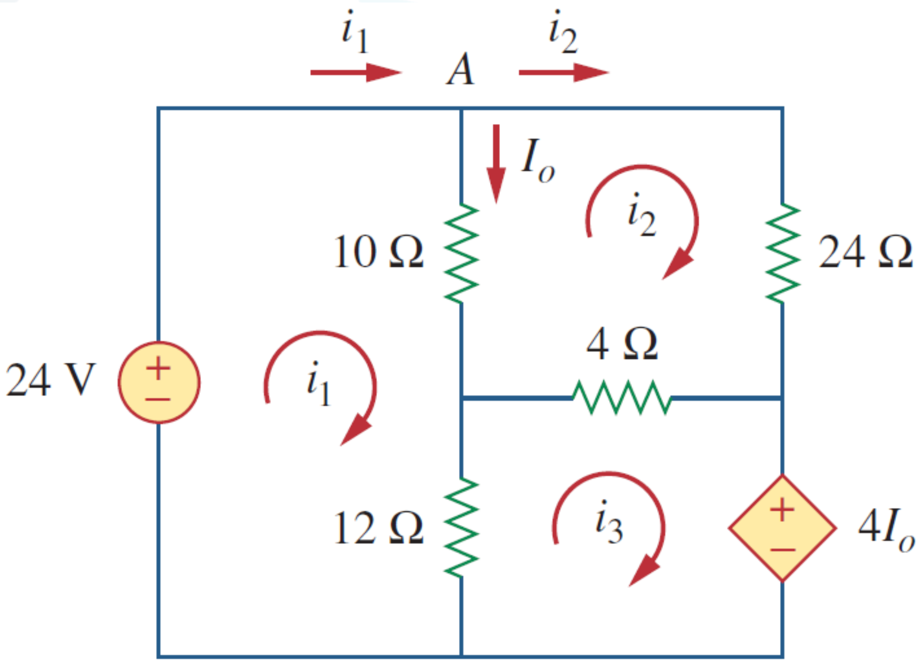
$$11i_1 - 5i_2 - 6i_3 = 12 \qquad (1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

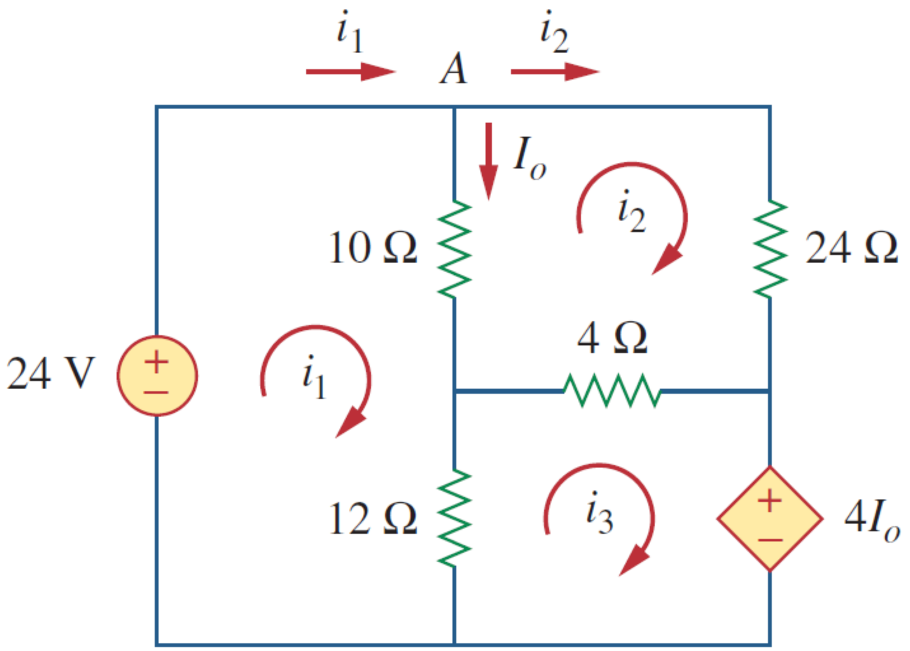
or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (2)$$



For mesh 3,

$$4I_0 + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$



But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3)$$

In matrix from, Eqs. (1) to (3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{bmatrix}$$

$- \quad - \quad -$ $\qquad + \quad + \quad +$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{bmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{bmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{bmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{bmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{bmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{bmatrix} \begin{matrix} - & + \\ - & + \\ - & + \end{matrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25\text{A} \;, i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75\text{A} \,, i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5\text{A}$$

Thus, $\text{I}_0 = i_1 - i_2 = 1.5\text{A}.$

8. Find the Thevenin equivalent of the circuit in Fig. at terminals a-b.