Tension and Compression in Bars

1. Stress الإجهاد
2. Strain التشوه (الانفعال)
3. Constitutive Law قانون السلوك
4 .Single Bar under Tension or Compression قضيب مفرد: شد أو ضغط
5. Systems of Bars جمل القضبان
6. Supplementary Examples أمثلة إضافية

Objectives: *Mechanics of Materials* investigates the stressing and the deformations of structures subjected to applied loads, starting by the simplest structural members, namely, bars in tension or compression.

يدرس ميكانيك المواد إجهادات وتشوهات الجمل الإنشائية (الهياكل الحاملة) الناتجة عن الحمولات الخارجية، مبتدأً بالعناصر الأبسط أي القضبان (العناصر الطولية) المشدودة أو المضغوطة.

In order to treat such problems, the kinematic relations and a constitutive law are needed to complement the equilibrium conditions which are known from Engineering Mechanics (Statics).

تقوم هذه الدراسة على:

(1) معادلات التوازن التي دُرست في الميكانيك الهندسي (علم السكون)

(2) العلاقات الكينماتيكية التي ستدرس وهي تصف التشوهات كمياً أي تحدد شكل ومقدار تغيرات الشكل الجيومتري.

(3) قوانين سلوك مادة الجملة وهي كما ستُعرض لاحقاً، قوانين تجريبية تعرّف السلوك الميكانيكي لمادة الهيكل الحامل.

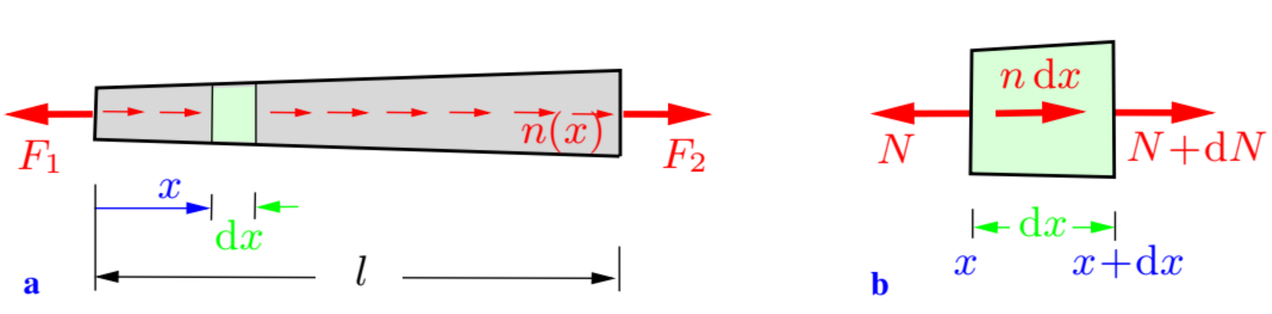
The kinematic relations represent the geometry of the deformation, whereas the behavior of the material is described by the constitutive law. The students will learn how to apply these equations and how to solve determinate as well as statically indeterminate problems. يعالج الطلبة مسائل مقررة سكونياً وأخرى غير مقررة سكونياً ؟؟

4 Single Bar under Tension or Compression

There are three different types of equations that allow us to determine the stresses & the strains in a bar: the equilibrium condition, the kinematic relation and Hooke's law.

Depending on the problem, the equilibrium condition may be formulated for the entire bar, a portion of the bar or for an element of the bar.

We will derive the equilibrium condition for an element. For this purpose we consider a bar which is subjected to two forces F_1 & F_2 at its ends and to a line load $n = n(x)$, see Fig.a.



The forces are assumed to be in equilibrium. We imagine a slice element of infinitesimal length dx separated from the bar as shown in Fig.b.

The F. B. D. shows the normal forces N and $N + dN$, respectively, at the ends of the element; the line load is replaced by its resultant ndx (note that n may be considered to be constant over the length dx).

Equilibrium of the forces in the direction of the axis of the bar

$$\Delta l = \int_0^l \frac{N(x)}{EA(x)} dx$$

In the special case of a bar (length l) with constant axial rigidity ($EA = const$) which is subjected only to forces at its end ($n \equiv 0, N = F$) the elongation is given by

$$\Delta l = \frac{l}{EA} F \Leftrightarrow F = \frac{EA}{l} \Delta l$$

Quantity $\frac{EA}{l}$ is the *axial rigidity (Stiffness) of the bar*.

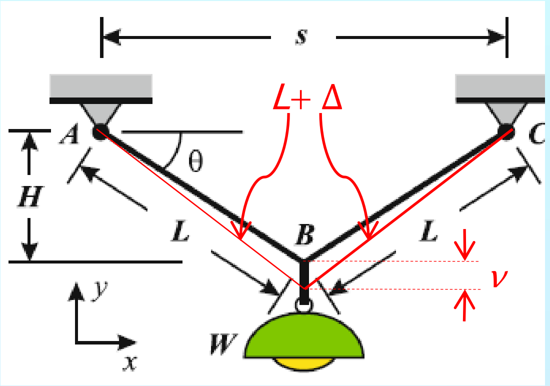
The Inverse $\frac{l}{EA}$ is the axial *flexibility* of the bar

جملة مقررة ستاتيكيا

Statically Determinate Systems

Reactions & internal forces can be calculated using only

Statics: *Eq. Eqs*.



2 Unknowns (T_A & T_C)

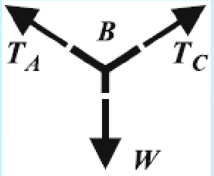
with 2 Eq. Eqs.

$\sum F_x = 0$:

$-T_A \cos\theta + T_C \cos\theta = 0$

$\sum F_y = 0$:

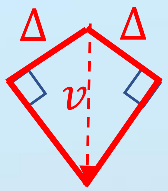
$T_A \sin\theta + T_C \sin\theta - W = 0$



$\Rightarrow T_A = T_C = T = W/2\sin\theta$

$\Rightarrow \sigma_{AB} = \sigma_{CB} = W/2A\sin\theta$

Behavior (Elastic) *Eqs*.: $\varepsilon = \dfrac{\sigma}{E}, \Delta = \dfrac{l}{EA} N$



$\Rightarrow \varepsilon_{AB} = \varepsilon_{CB} = W/2EA\sin\theta$

$\Rightarrow \Delta_{AB} = \Delta_{CB} = \Delta = WL/2EA\sin\theta$

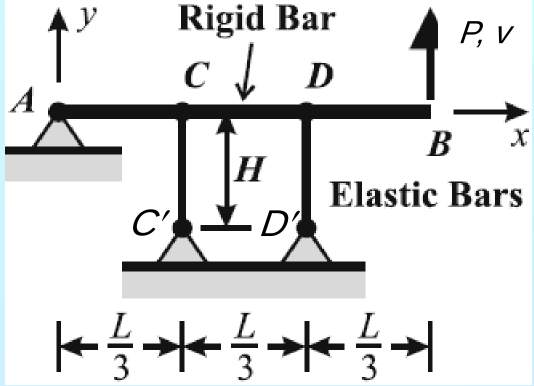
Kinematic *Eqs*. (displacement-elongation):

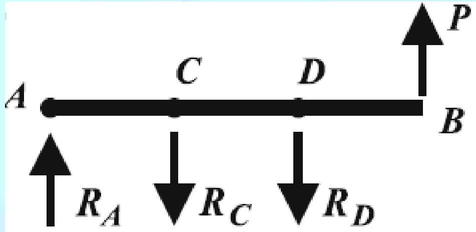
$\Rightarrow v = \Delta/\sin\theta = WL/2EA\sin^2\theta$

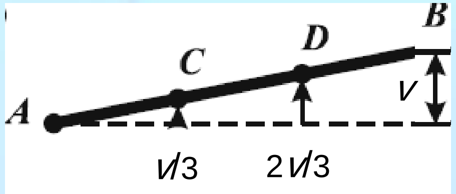
جملة غير مقررة ستاتيكيا

Statically Indeterminate (Redundant) Systems

Statics alone (*Eq. Eqs.*) is not enough to calculate reactions and internal forces.







3 Unknowns (R_A, R_C & RD) with 2 Eq. Eqs. (impossible)

$\sum F_y = 0$: $R_A - R_C - R_D + P = 0$

$\sum MA = 0$: $-R_C(L/3) - R_D\,(2L/3) + PL = 0$

Additional *compatibility equation* is needed.

$$\left.\begin{aligned}\Delta_{C'C} &= v/3 \Rightarrow RC = EA\delta_{C'C}/H = EAv/3H \\ \Delta_{D'D} &= 2v/3 \Rightarrow R_D = EA\delta_{D'D}/H = 2EAv/3H\end{aligned}\right\} \Rightarrow \boxed{R_D = 2R_C}$$

3Eqs solution gives: $R_A = 0.8P, R_C = 0.6P, R_D = 1.2P$

$\Rightarrow \sigma_{C'C} = 0.6P/A$ & $\sigma_{D'D} = 1.2P/A$.

$\Rightarrow \Delta_{C'C} = 0.6PH/EA, \Delta_{D'D} = 1.2PH/EA$ & $v = 1.8PH/EA$

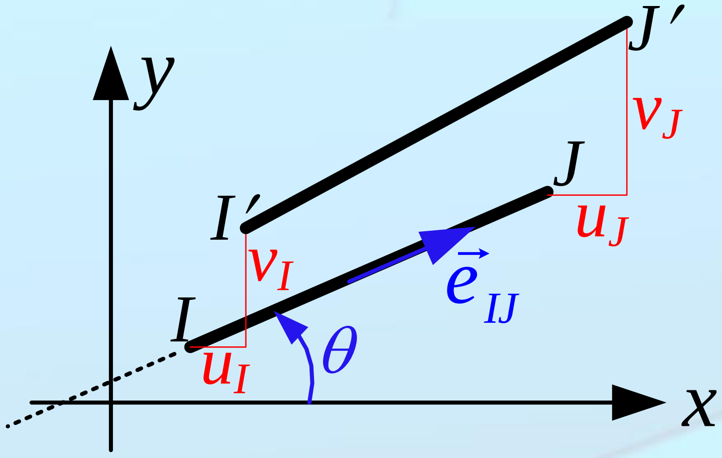
Kinematics: Elongation-Displacement of an axially loaded member.

The axial member IJ of length L, directed in x,y, by the unit vector $\vec{e}_{IJ}$

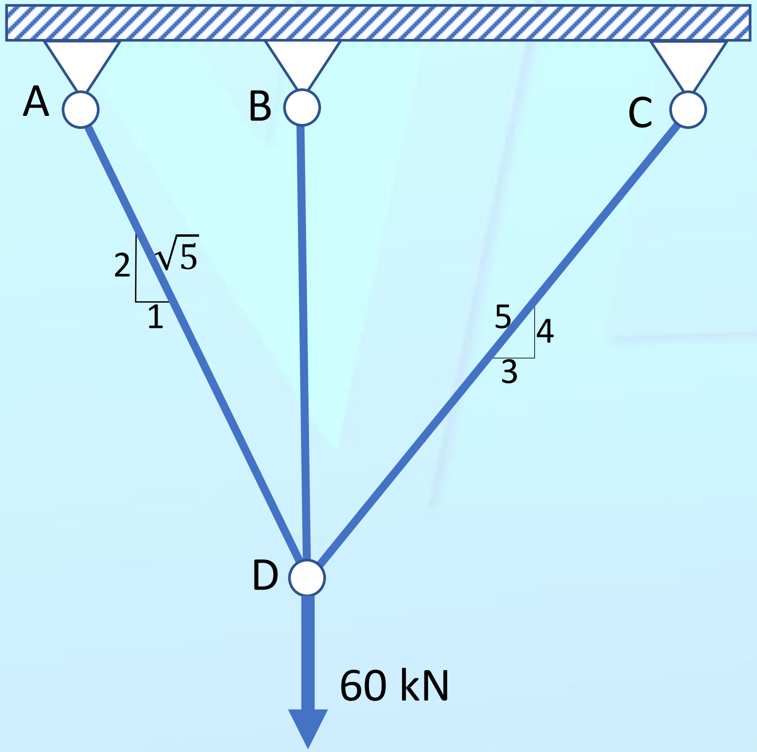
moves under loading to $I'J'$ by two *small displacement vectors*

getting a new length $L'=L+\Delta_{IJ}$, and a new direction, to determine Δ_{IJ},
we observe from the figure that: $\overrightarrow{I'J'}=\overrightarrow{IJ}+\overrightarrow{JJ'}-\overrightarrow{II'}$

$$\boxed{\Delta_{IJ}=(u_J-u_I)\cos\theta+(v_J-v_I)\sin\theta}$$



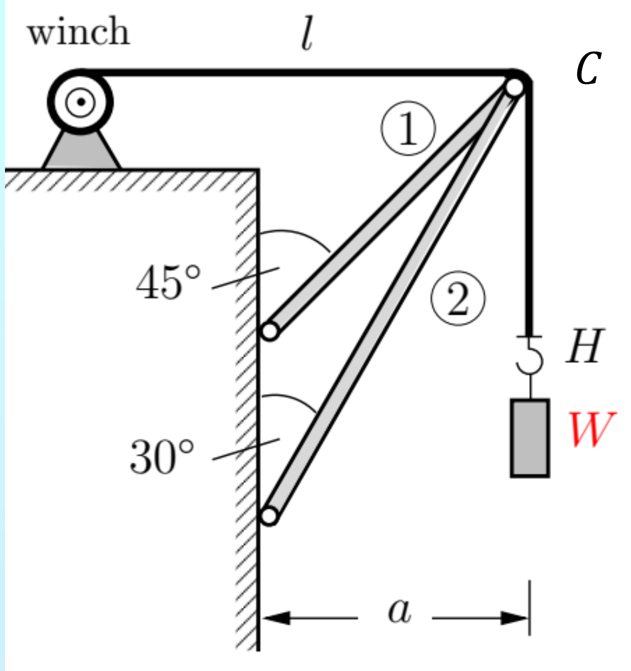
Problem 3. An aluminum truss *ABCD* is loaded at joint *D* by a point load $F = 60$ kN. The cross-sectional areas of the bars are: $A_{AD} = 325\ mm^2$, $A_{BD} = 390\ mm^2$, and $A_{CD} = 420\ mm^2$. The modulus of aluminum is $E = 70$ GPa.



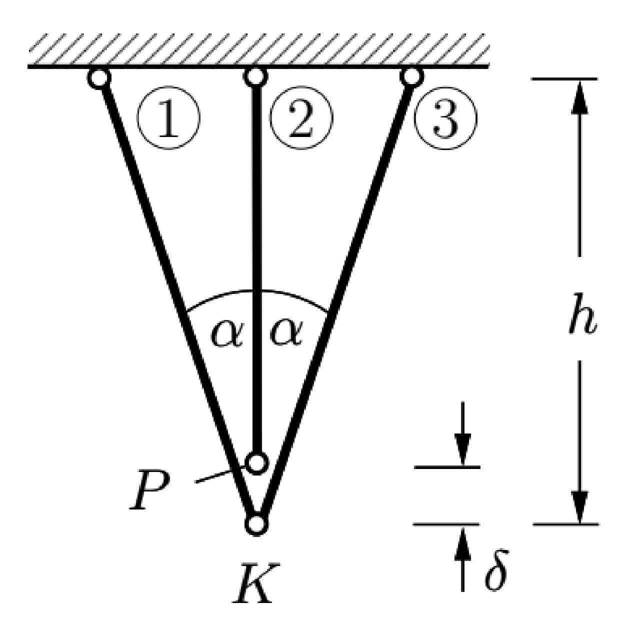
Determine

1) the horizontal and vertical displacements of joint *D, u, and v.*
2) The axial stress in members *AD, BD,* and *CD.*

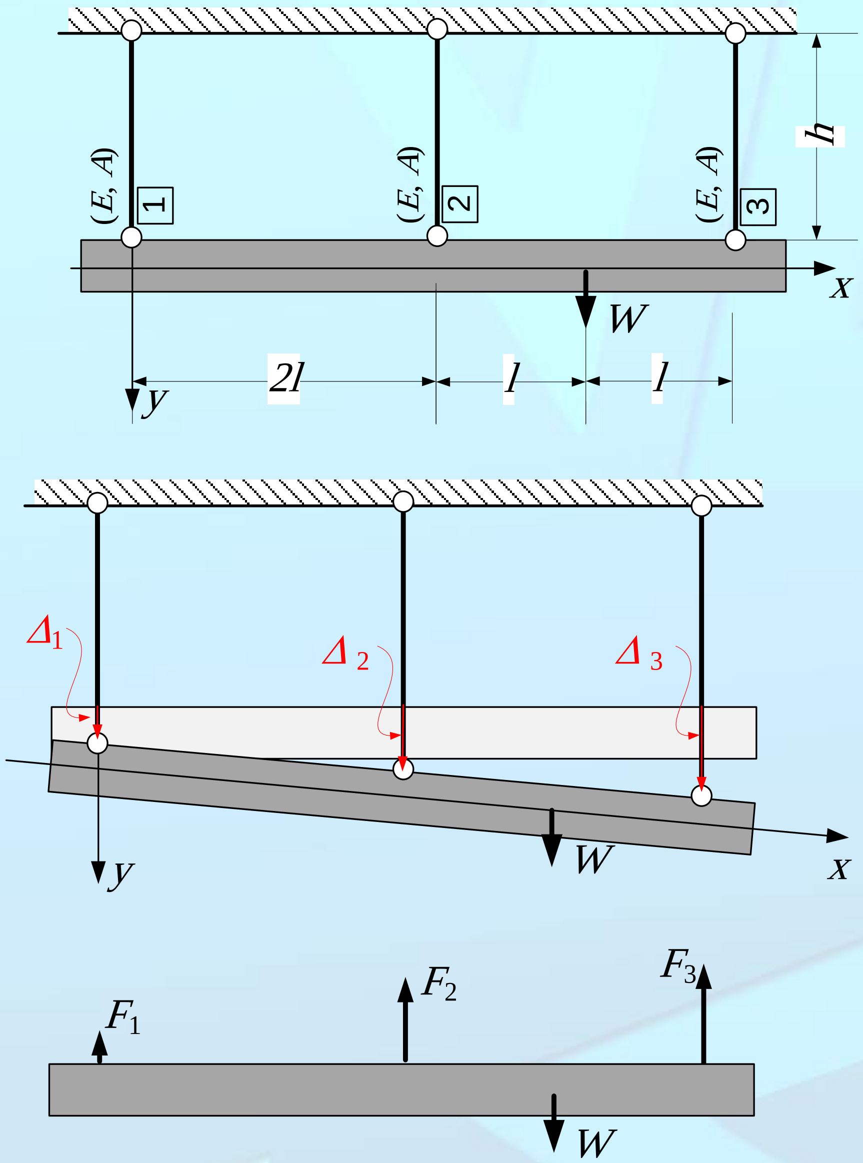
Problem 4. The Fig. shows a freight elevator. The cable (of total length is l and axial rigidity K) passes over a smooth pin C. A crate (of weight W) is suspended at the end of the cable. The axial rigidity EA of the two rods BC and DC is given. Determine the displacements of pin C and of the end of the cable (point H) due to the weight of the crate.



Problem 5. To assemble the truss (axial rigidity EA of the three bars) in Fig. the end point P of bar 2 has to be connected with pin C. Assume $\delta \ll h$. Determine the forces in the bars after the truss has been assembled.

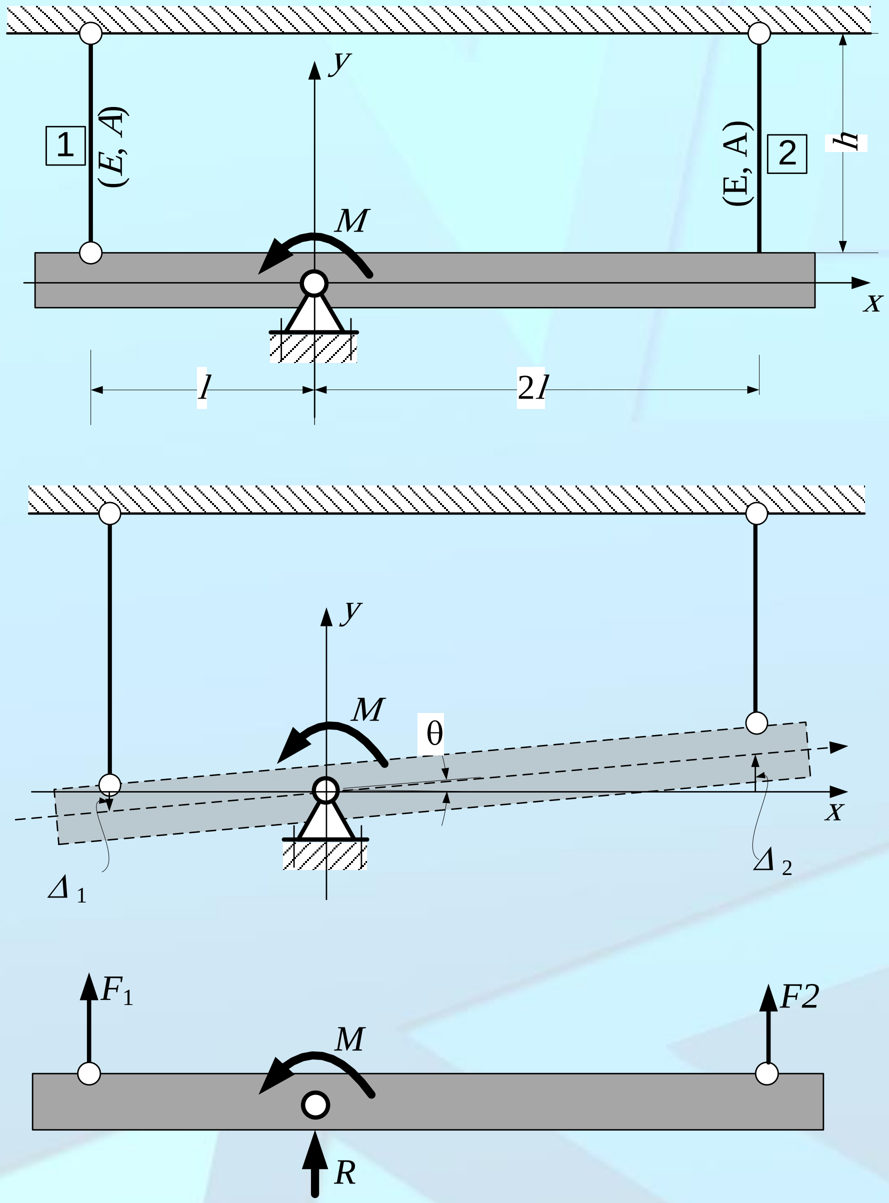


Problem 6. Consider a rigid horizontal beam suspended to the roof by three identical bars as indicated on the figure. Under the action of load, determine the internal force in each bar.



Solution:

Problem 7. Consider a rigid horizontal beam suspended to the roof by two different bars as indicated on the figure, and rotates about the shown hinge by a small angle θ, under the action of a moment M, applied at the hinge. determine the internal force in each bar.



Solution: