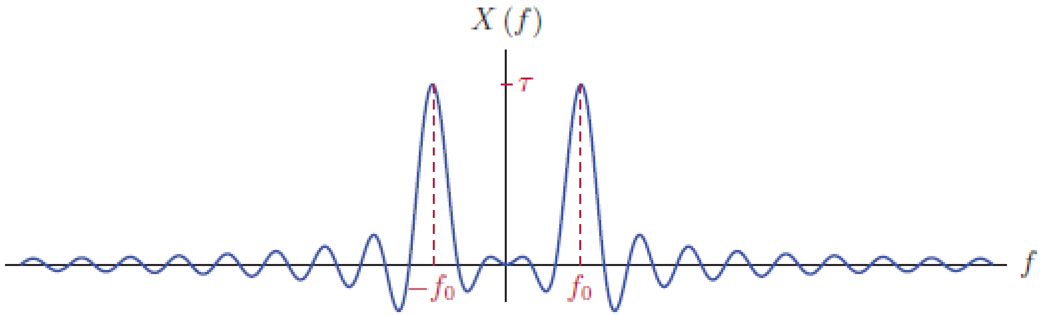
CEDC403: Signals and Systems

Exercises 3: Analyzing Continuous Time Systems in the Time Domain



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1. **For each case, determine if the system is linear and/or time-invariant:**

$a.\ y(t) = |x(t)| + x(t)$

$b.\ y(t) = tx(t)$

$c.\ y(t) = e^{-t}x(t)$

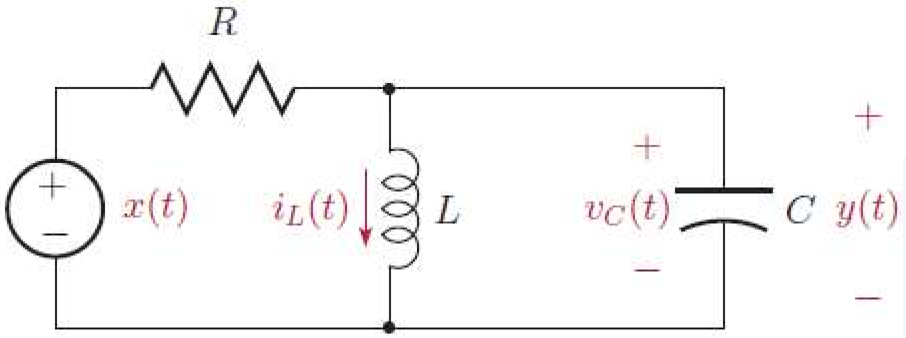
$d.\ y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

$e.\ y(t) = \int_{t-1}^{t} x(\tau)d\tau$

$f.\ y(t) = (t+1)\int_{-\infty}^{t} x(\tau)d\tau$

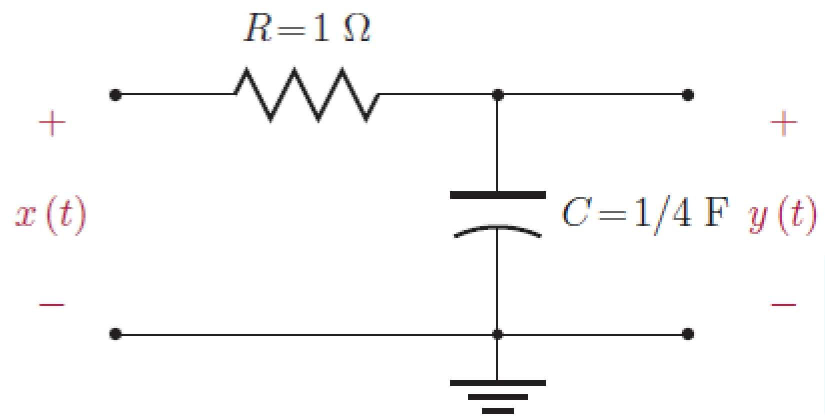
2. Differential equation for RLC circuit:

Find a DE between the input $x(t)$ and the output $y(t)$. At $t = 0$ the initial values are $i_L(0) = 1$ A, $v_C(0) = 2$ V. Express the initial conditions for $y(t)$ and $dy(t)/dt$.



3. Consider the differential equation for the RC circuit :

$$\frac{dy(t)}{dt} + 4y(t) = 4x(t)$$



Let the input signal be a unit step, that is, $x(t) = u(t)$. Using the first-order differential equation solution technique find the solution $y(t)$ for $t \geq 0$ subject to each initial condition specified below:

a. $y(0) = 0$

b. $y(0) = 5$

4. **Solve each of the first-order differential equations given below for the specified input signal and subject to the specified initial condition:**

a. $\dfrac{dy(t)}{dt} + 2y(t) = 2x(t), \quad x(t) = u(t) - u(t-5), \quad y(0) = 2$

b. $\dfrac{dy(t)}{dt} + 5y(t) = 3x(t), \quad x(t) = \delta(t), \quad y(0) = 0.5$

c. $\dfrac{dy(t)}{dt} + 5y(t) = 3x(t), \quad x(t) = tu(t), \quad y(0) = -4$

d. $\dfrac{dy(t)}{dt} + y(t) = 2x(t), \quad x(t) = e^{-2t}u(t), \quad y(0) = -1$

5. For each homogeneous DE given below, find the homogeneous solution for $t \geq 0$ in each case subject to the initial conditions specified condition:

$$a.\ \frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 0, \quad y(0) = 3, \quad \left.\frac{dy(t)}{dt}\right|_{t=0} = 0$$

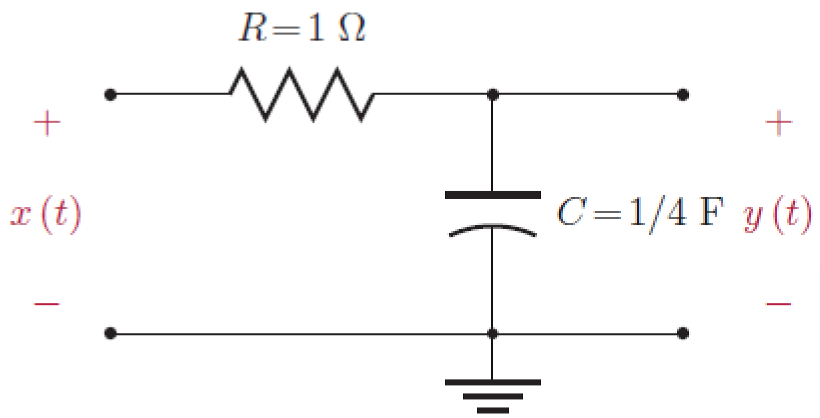
$$b.\ \frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = 0, \quad y(0) = 2, \left.\frac{dy(t)}{dt}\right|_{t=0} = -1, \left.\frac{d^2y(t)}{dt^2}\right|_{t=0} = 1$$

$$c.\ \frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} = 0, \quad y(0) = 2, \quad \left.\frac{dy(t)}{dt}\right|_{t=0} = 0$$

$$d.\ \frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = 0, \quad y(0) = -2, \quad \left.\frac{dy(t)}{dt}\right|_{t=0} = -1$$

6. Consider the differential equation for the RC circuit :

$$\frac{dy(t)}{dt} + 4y(t) = 4x(t)$$



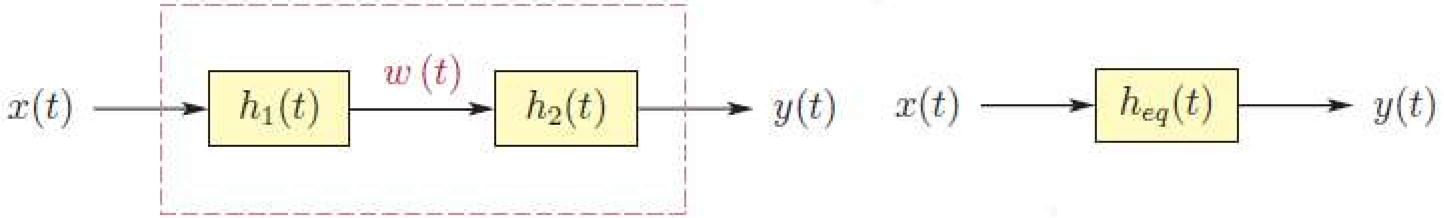
Assuming the circuit is initially relaxed, compute and sketch the response to a sinusoidal input signal in the form $x(t) = 5\cos(8t)$

7. A system is described by the differential equation:

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} - 2x(t), \quad y(0) = -2, \quad \left.\frac{dy(t)}{dt}\right|_{t=0} = 1$$

Draw a block diagram

8. Two CTLTI systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade:

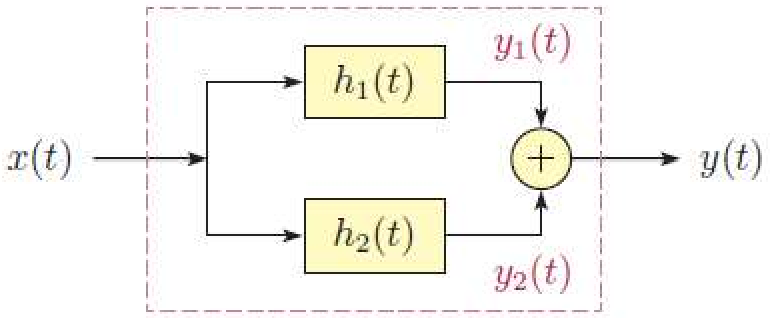


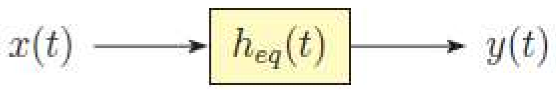
a. Determine the impulse response $h_{eq}(t)$ of the equivalent system, in terms of $h_1(t)$ and $h_2(t)$.
   Hint: Use convolution to express $w(t)$ in terms of $x(t)$. Afterwards use convolution again to express $y(t)$ in terms of $w(t)$.

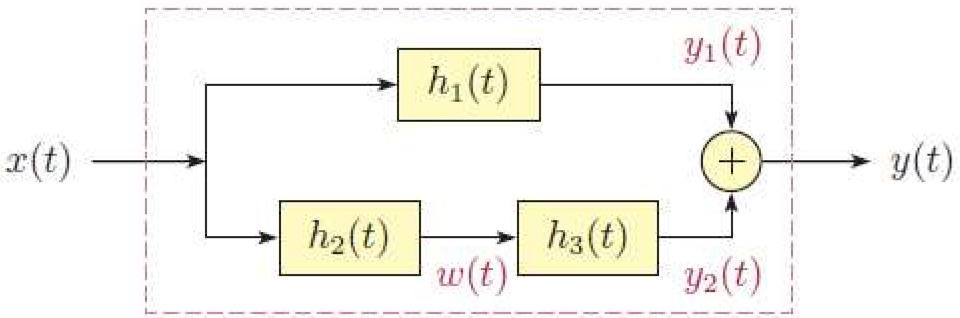
b. Let $h_1(t) = h_2(t) = \Pi(t - 0.5)$ where $\Pi(t)$ is the unit pulse. Determine and sketch $h_{eq}(t)$ for the equivalent system.

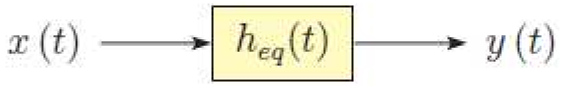
c. With $h_1(t)$ and $h_2(t)$ as specified in part (b), let the input signal be a unit step, that is, $x(t) = u(t)$. Determine and sketch the signals $w(t)$ and $y(t)$.

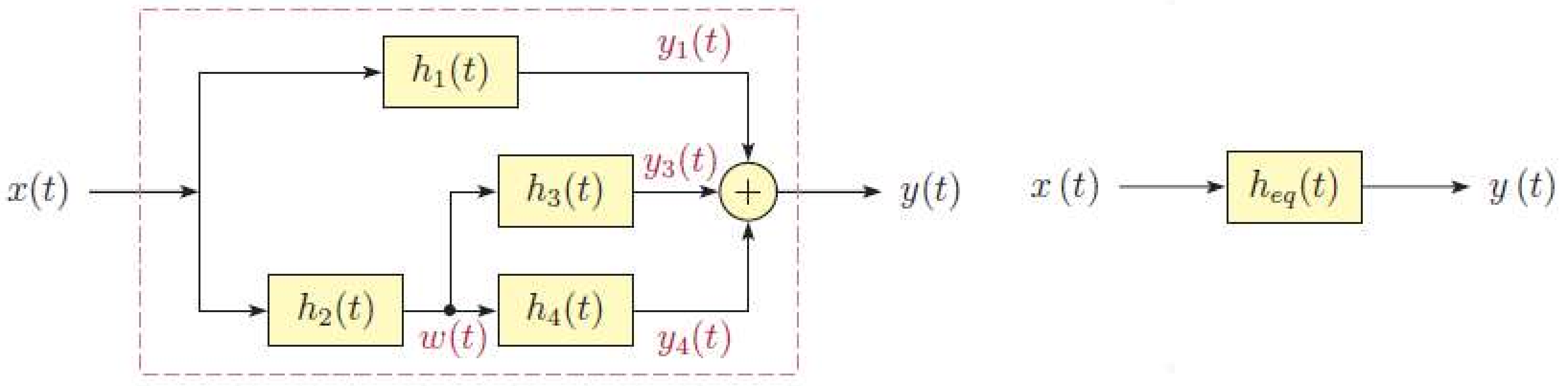
9. Determine the impulse response $h_{eq}(t)$ of the equivalent system:







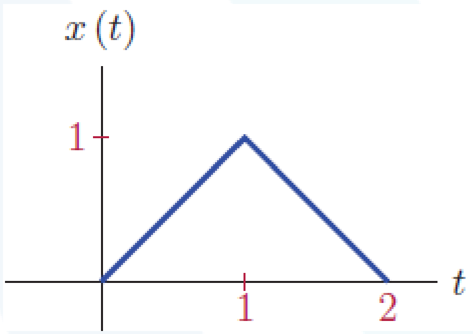




10. The impulse response of a CTLTI system is:

$$h(t) = \delta(t) - \delta(t-1)$$

Determine sketch the response of this system to the triangular waveform



11. For each pair of signals $x(t)$ and $h(t)$ given below, find the convolution $y(t) = x(t) * h(t)$.

$a.\ x(t) = u(t), \quad h(t) = e^{-2t}u(t)$

$b.\ x(t) = u(t-2), \quad h(t) = e^{-2t}u(t)$

$c.\ x(t) = u(t) - u(t-2), \quad h(t) = e^{-2t}u(t)$

$d.\ x(t) = e^{-t}u(t), \quad h(t) = e^{-2t}u(t)$

12. For each pair of signals $x(t)$ and $h(t)$ given below, find the convolution $y(t) = x(t) * h(t)$ first graphically, and then analytically.

$$a.\ x(t) = \Pi\left(\frac{t-2}{4}\right), \quad h(t) = u(t)$$

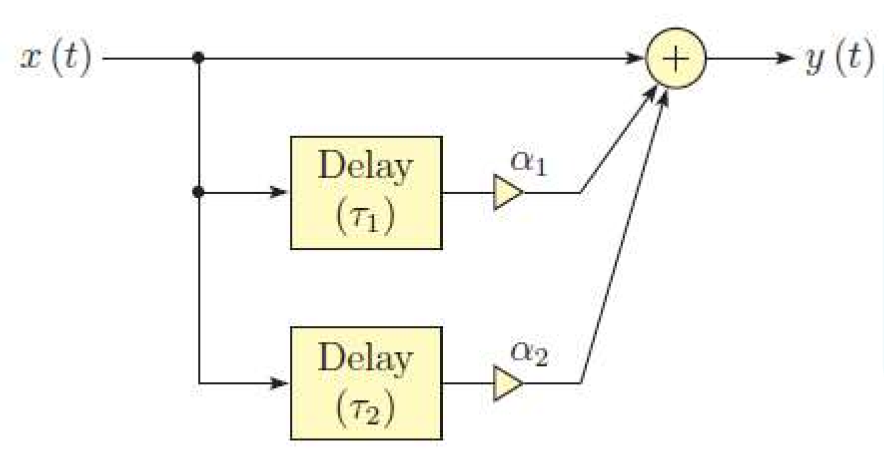
$$b.\ x(t) = 3\Pi\left(\frac{t-2}{4}\right), \quad h(t) = e^{-t}u(t)$$

$$c.\ x(t) = \Pi\left(\frac{t-2}{4}\right), \quad h(t) = \Pi\left(\frac{t-3}{6}\right)$$

13. The system shown represents addition of echos to the signal $x(t)$:

$$y(t) = x(t) + \alpha_1 x(t-\tau_1) + \alpha_2 x(t-\tau_2)$$

Comment on the system's
a. Linearity
b. Time invariance
c. Causality
d. Stability



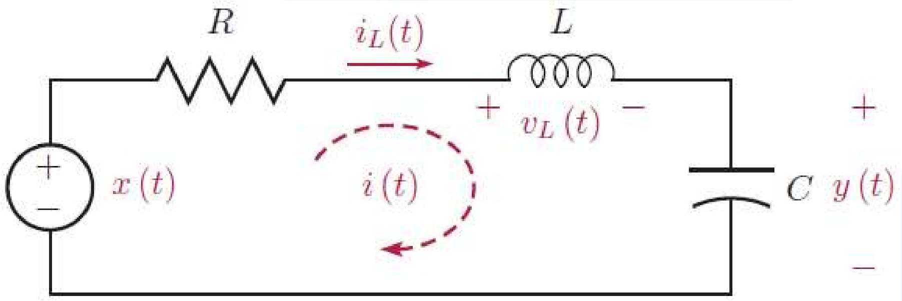
14. For each system described below find the impulse response. Afterwards determine if the system is causal and/or stable.

a. $y(t) = T\{x(t)\} = \int_{-\infty}^{t} x(\tau)d\tau$

b. $y(t) = T\{x(t)\} = \int_{t-T}^{t} x(\tau)d\tau, \quad T > 0$

c. $y(t) = T\{x(t)\} = \int_{t-T}^{t+T} x(\tau)d\tau, \quad T > 0$

15. Consider the RLC circuit shown, where the input is a voltage source $x(t)$ and the output the voltage $y(t)$ across the capacitor. Let LC = 1 and R/L = 2. Find the impulse response $h(t)$ of the circuit.



16. Determine the step response of the RC circuit using the convolution operation.

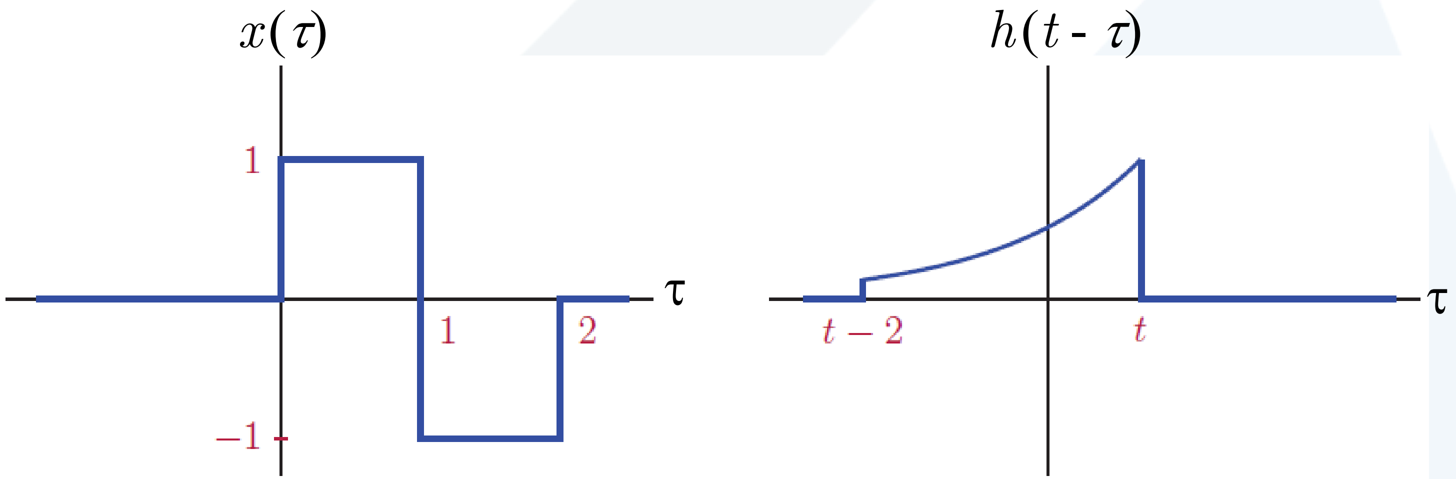
17. Using convolution, determine the response of the RC circuit to a unit-pulse input signal $x(t) = \Pi(t)$.

18. Consider a system with the impulse response $h(t) = e^{-t}[u(t) - u(t-2)]$

Let the input signal applied to this system be

$$x(t) = \prod(t-0.5) - \prod(t-1.5) = \begin{cases} 1 & 0 \le t < 1 \\ -1 & 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Determine the output signal $y(t)$ using convolution.



19. A voltage $x(t) = 10e^{-3t}u(t)$ is applied at the input of the RLC circuit. Find the output voltage $v_C(t) = y(t)$ for $t \geq 0$ if the initial inductor current is $i_L(0^-) = 0$, and the initial capacitor voltage $v_C(0^-) = 5$ V. Use $R = 3\ \Omega$, $L = 1$ H and $C = 1/2$ F.

20. Find the output loop current $i_L(t) = y(t)$ for $t \geq 0$ for the circuit described in exercise 19.