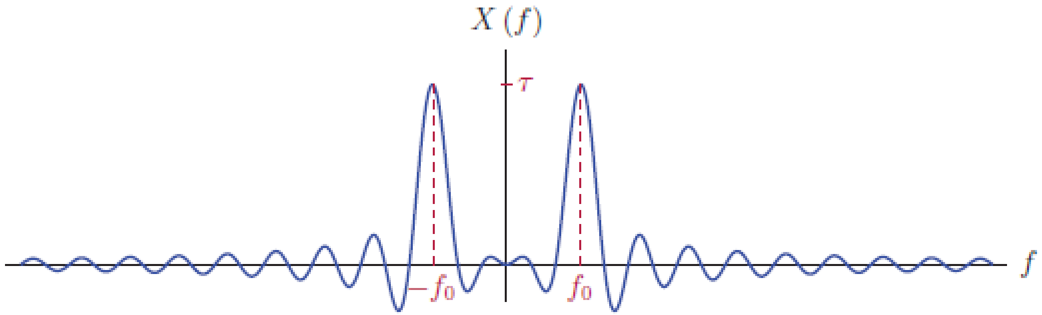
CEDC403: Signals and Systems

Exercises 4: Analyzing Discrete Time Systems in the Time Domain



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1. For each case, determine if the system is linear and/or time-invariant:

   a. $y[n] = x[n]\ u[n]$

   b. $y[n] = 3x[n] + 5$

   c. $y[n] = 3x[n] + 5u[n]$

   d. $y[n] = nx[n]$

   e. $y[n] = \cos(0.2\pi n)\ x[n]$

   f. $y[n] = x[n] + 3x[n-1]$

   g. $y[n] = x[n] + 3x[n-1]\ x[n-2]$

2. The response of a linear and time-invariant system to the input signal $x[n] = \delta[n]$ is given by:

$$T\{\delta[n]\} = \{\underset{\substack{\uparrow \\ n=0}}{2}, 1, -1\}$$

Determine the response of the system to the following input signals:

a. $x[n] = \delta[n] + \delta[n-1]$

b. $x[n] = u[n] - u[n-5]$

c. $x[n] = n(u[n] - u[n-5])$

3. Consider a system that is known to be linear but not necessarily time-invariant. Its responses to three impulse signals are given below:

$$T\{\delta[n]\} = \{\underset{\substack{\uparrow \\ n=0}}{1}, 2, 3\}, \quad T\{\delta[n-1]\} = \{\underset{\substack{\uparrow \\ n=1}}{3}, 3, 2\}, \quad T\{\delta[n-2]\} = \{\underset{\substack{\uparrow \\ n=2}}{3}, 2, 1\}$$

For each of the input signals listed below, state whether the response of the system can be determined from the information given. If the response can be determined, find it. If not, explain why it cannot be done.

a. $x[n] = 3\delta[n] + 2\delta[n-1]$

b. $x[n] = \delta[n] - 2\delta[n-1] + 4\delta[n-2]$

c. $x[n] = u[n] - u[n-3]$

d. $x[n] = u[n] - u[n-4]$

4. For each homogeneous DE given below, find the homogeneous solution for $n \geq 0$ in each case subject to the initial conditions specified condition:

$a.\ y[n] + 0.2y[n-1] - 0.63y[n-2] = 0, \quad y[-1] = 5, \quad y[-2] = -3$

$b.\ y[n] + 0.6y[n-1] - 0.51y[n-2] - 0.28y[n-3] = 0,\ y[-1] = 3,\ y[-2] = 2,\ y[-3] = 1$

$c.\ y[n] - 1.4y[n-1] + 0.85y[n-2] = 0, \quad y[-1] = 2, \quad y[-2] = -2$

$d.\ y[n] + y[n-2] = 0, \quad y[-1] = 3, \quad y[-2] = 2$

5. Solve each difference equation given below for the specified input signal and initial conditions:

a. $y[n] = 0.6y[n-1] + x[n]$, $x[n] = u[n]$, $y[-1] = 2$

b. $y[n] = 0.8y[n-1] + x[n]$, $x[n] = 2\sin(0.2n)$, $y[-1] = 1$

c. $y[n] - 0.2y[n-1] - 0.63y[n-2] = x[n]$, $x[n] = e^{-0.2n}$, $y[-1] = 0$, $y[-2] = 3$

6. Consider the exponential smoother. It is modeled with the difference equation

$$y[n] = (1 - \alpha)\, y[n - 1] + \alpha x[n]$$

Let $y[-1] = 0$ so that the system is linear.

a. Let the input signal be a unit step, that is, $x[n] = u[n]$. Determine the response of the linear exponential smoother as a function of α.

b. Let the input signal be a unit ramp, that is, $x[n] = nu[n]$. Determine the response of the linear exponential smoother as a function of α.

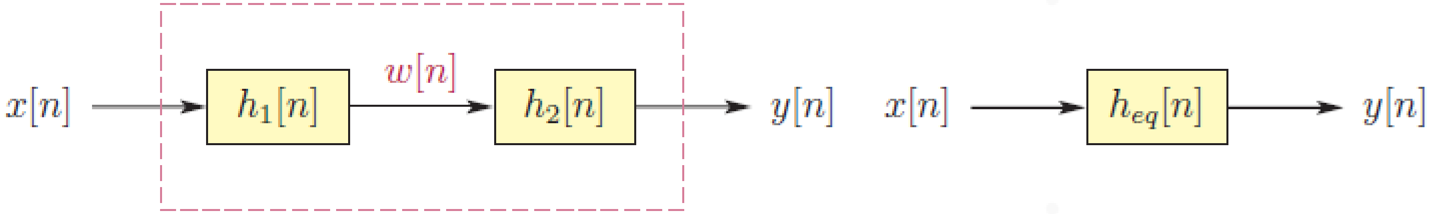
7. Construct a block diagram for each difference equation given below:

a. $y[n] + 0.2y[n-1] - 0.63y[n-2] = x[n] + x[n-2]$

b. $y[n] - 2.5y[n-1] + 2.44y[n-2] - 0.9y[n-3] = x[n] - 3x[n-1] + 2x[n-2]$

c. $y[n] + 0.6y[n-1] - 0.51y[n-2] - 0.28y[n-3] = x[n] - 2x[n-2]$

8. Two DTLTI systems with impulse responses $h_1[n]$ and $h_2[n]$ are connected in cascade:

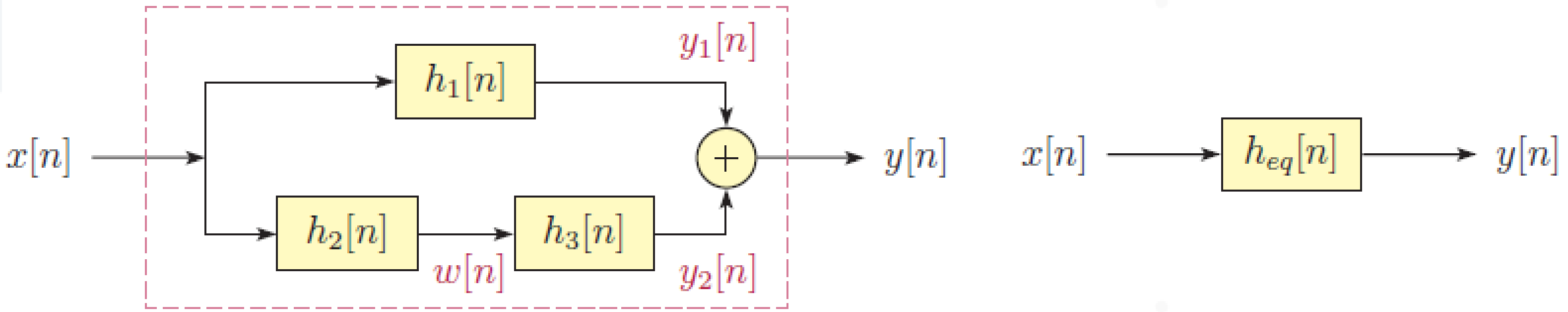


a. Determine the impulse response $h_{eq}[n]$ of the equivalent system, in terms of $h_1[n]$ and $h_2[n]$.
Hint: Use convolution to express $w[n]$ in terms of $x[n]$. Afterwards use convolution again to express $y[n]$ in terms of $w[n]$.

b. Let $h_1[n] = h_2[n] = u[n] - u[n-5]$. Determine and sketch $h_{eq}(t)$ for the equivalent system.

c. With $h_1[n]$ and $h_2[n]$ as specified in part (b), let the input signal be a unit step, that is, $x[n] = u\,[n]$. Determine and sketch the signals $w[n]$ and $y[n]$.

9. Three DTLTI systems with impulse responses $h_1[n]$, $h_2[n]$ and $h_3[n]$ are connected as shown:

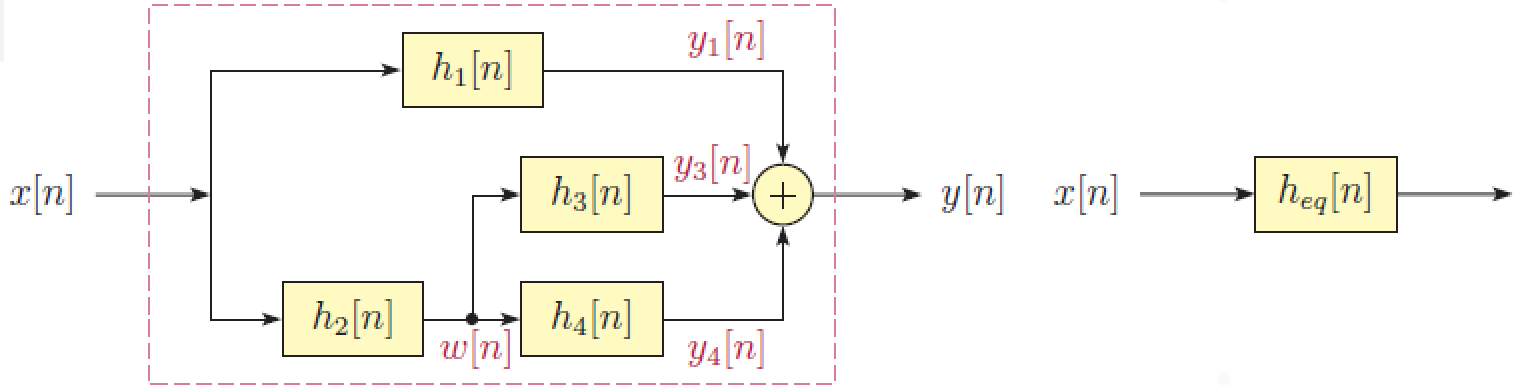


a. Determine the impulse response $h_{eq}[n]$ of the equivalent system in terms of $h_1[n]$, $h_2[n]$ and $h_3[n]$.

b. Let $h_1[n] = e^{-0.1n}u[n]$, $h_2[n] = \delta[n-2]$, and $h_3[n] = e^{-0.2n}u[n]$. Determine and sketch $h_{eq}[n]$ for the equivalent system.

c. With $h_1[n]$, $h_2[n]$ and $h_3[n]$ as specified in part (b), let the input signal be a unit step. Determine and sketch the signals $w[n]$ and $y_1[n]$, $y_1[n]$ and $y[n]$.

10. Consider the DTLTI system shown



a. Express the impulse response of the system as a function of the impulse responses of the subsystems.

b. Let $h_1[n] = e^{-0.1n}u[n]$, $h_2[n] = h_3[n] = u[n] - u[n-3]$ and $h_4[n] = \delta[n-2]$. Determine the impulse response $h_{eq}[n]$ of the equivalent system.

c. Let the input signal be a unit-step, that is, $x[n] = u[n]$. Determine and sketch the signals $w[n]$, $y_1[n]$, $y_3[n]$ and $y_4[n]$.

11. **For each system described below find the impulse response. Afterwards determine if the system is causal and/or stable.**

$$a.\ y[n] = T\{x[n]\} = \sum_{k=-\infty}^{n} x[k] \qquad b.\ y[n] = T\{x[n]\} = \sum_{k=-\infty}^{n} e^{-0.1(n-k)} x[k]$$

$$c.\ y[n] = T\{x[n]\} = \sum_{k=0}^{n} x[k] \quad \text{for } n \geq 0$$

$$d.\ y[n] = T\{x[n]\} = \sum_{k=n-10}^{n} x[k] \qquad e.\ y[n] = T\{x[n]\} = \sum_{k=n-10}^{n+10} x[k]$$