جامعة المنارة
قسم هندسة الروبوت والأنظمة الذكية

الدارات الكهربائية

الدكتور المهندس
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5

الحالات العابرة في دارات التيار المستمر

المكثفات والوشائع

DC Transient

Capacitors and Inductors

Introduction

So far we have limited our study to resistive circuits. We shall introduce two new and important passive linear circuit elements: the capacitor and the inductor.

Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called storage elements.

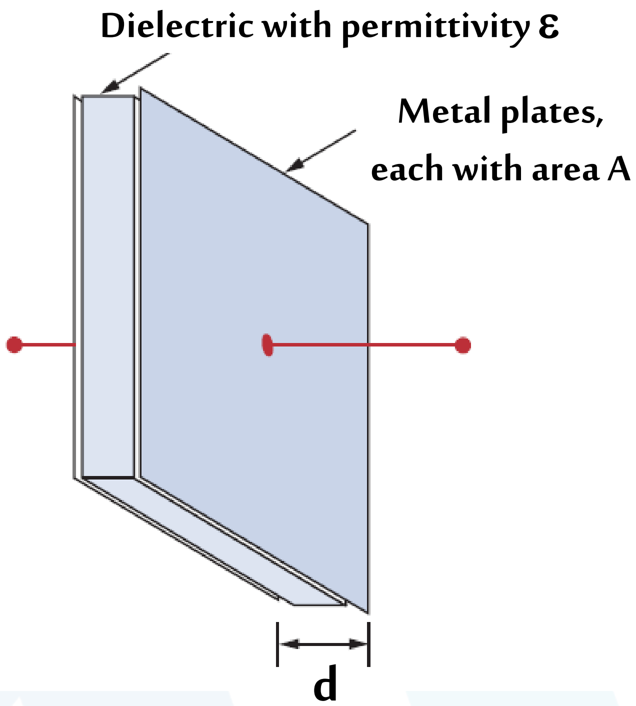
The application of resistive circuits is quite limited. With the introduction of capacitors and inductors in this chapter, we will be able to analyse more important and practical circuits.

We begin by introducing capacitors and describing how to combine them in series or in parallel. Later, we do the same for inductors.

As typical applications, we explore how capacitors are combined with op amps to form integrators, differentiators, and analog computers.

Capacitors

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

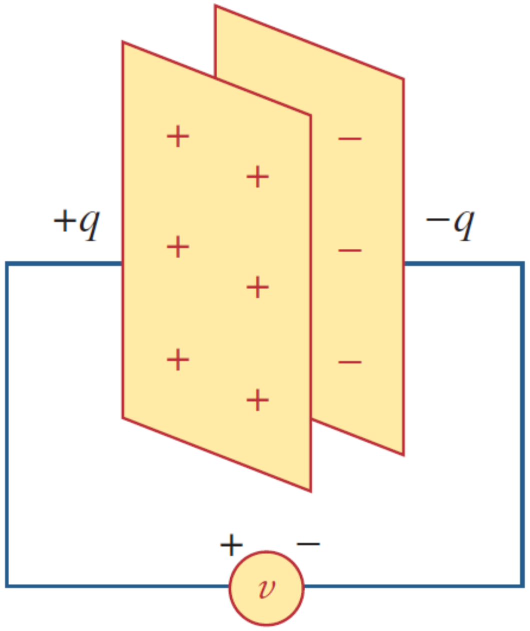


A capacitor is typically constructed as depicted in Fig.

A capacitor consists of two conducting plates separated by an insulator (or dielectric).

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.

When a voltage source v is connected to the capacitor, as in Fig. the source deposits a positive charge q on one plate and a negative charge −q on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that



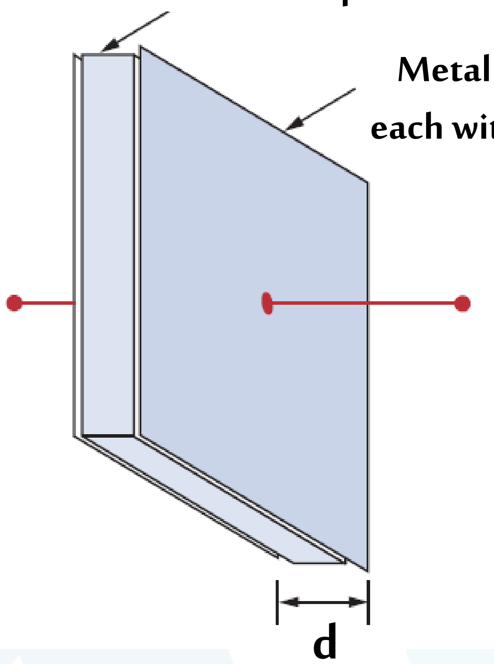
$$q = Cv$$

where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F), in honor of the English physicist Michael Faraday (1791–1867). From Eq. we may derive the following definition

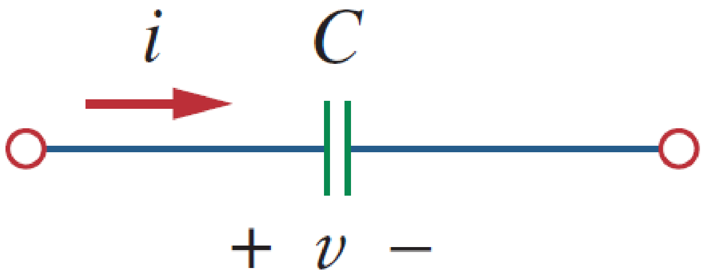
Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

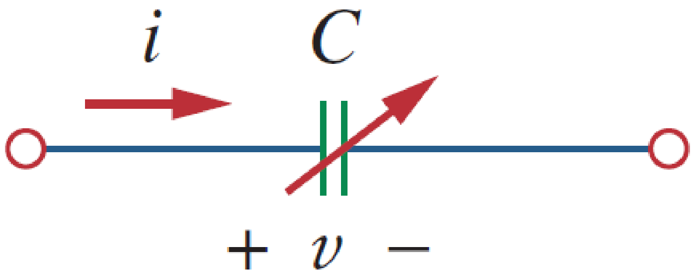
1 farad = 1 coulomb/volt.

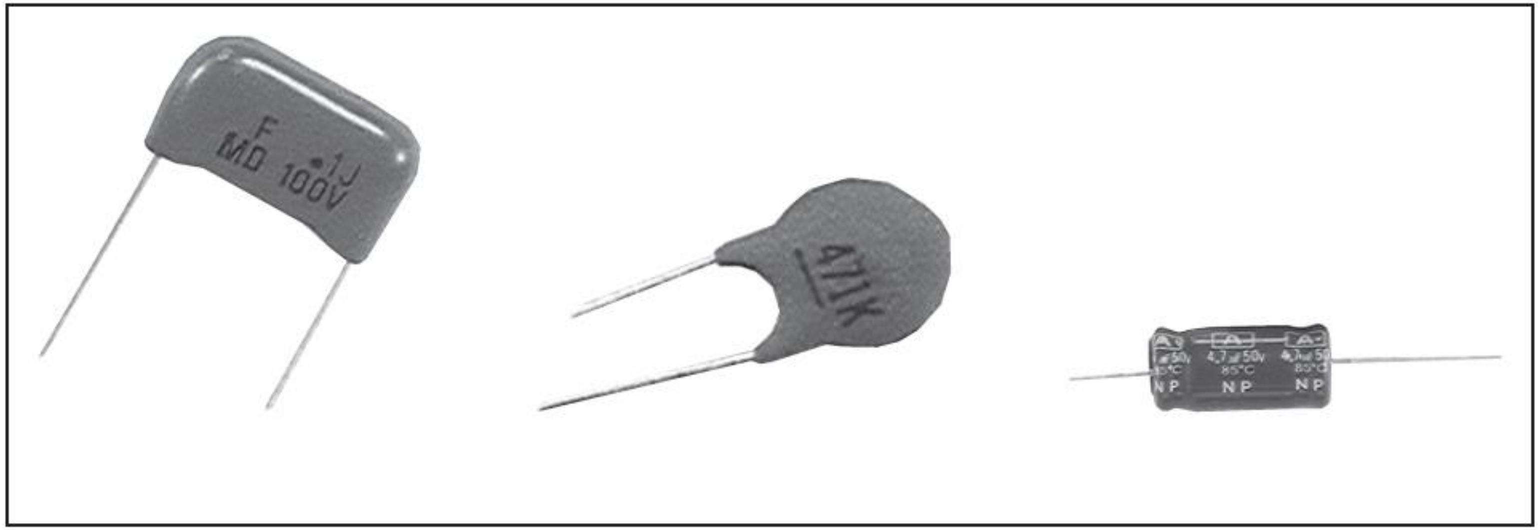
Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage it does not depend on q or v It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig. the capacitance is given by

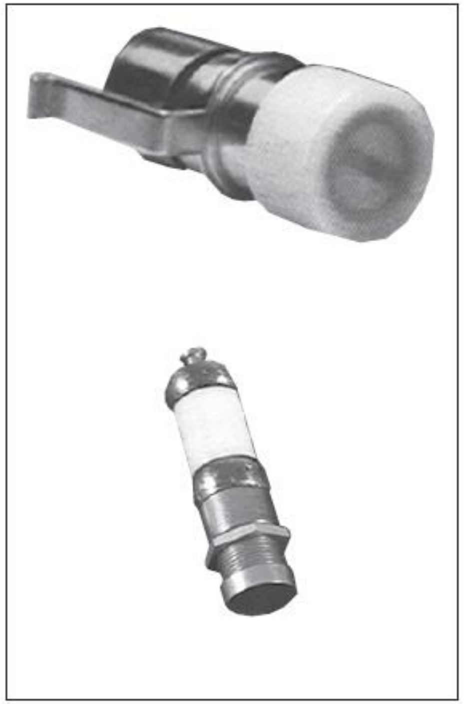


$$C = \frac{\varepsilon A}{d}$$


i
C
$+ \ v \ -$


i
C
$+ \ v \ -$

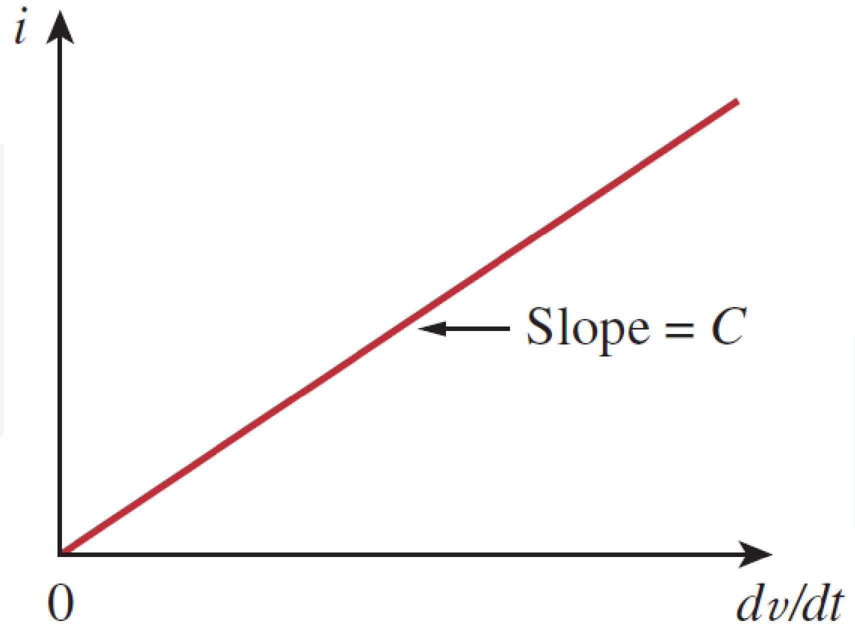

F
MD .1J
100V
471K



$$q = Cv$$

$$\frac{dq}{dt} = C\frac{dv}{dt} \qquad \left(i = \frac{dq}{dt}\right)$$

$$i = C\frac{dv}{dt}$$



$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\cdot dt + v(t_0)$$

where $v(t_0) = \frac{q(t_0)}{C}$ is the voltage across the capacitor at time t_0.

Equation shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory-a property that is often exploited.

The instantaneous power delivered to the capacitor is $p = v \cdot i = v \cdot \mathrm{C} \cdot \frac{\mathrm{d}v}{\mathrm{dt}}$

The energy stored in the capacitor is therefore

$$\mathrm{w} = \int_{-\infty}^{\mathrm{t}} p(\mathrm{t}) \cdot \mathrm{dt} = \mathrm{C} \int_{-\infty}^{\mathrm{t}} v \cdot \frac{\mathrm{d}v}{\mathrm{dt}} \cdot \mathrm{dt} = \mathrm{C} \int_{v(-\infty)}^{v(\mathrm{t})} v \cdot \mathrm{d}v = \frac{1}{2}\mathrm{C}v^2 \Big|_{v(-\infty)}^{v(\mathrm{t})}$$

We note that $v(-\infty)=0$, because the capacitor was uncharged at $t=-\infty$. Thus,

$$\mathrm{w} = \frac{1}{2}\mathrm{C}v^2$$

$$\mathrm{q} = \mathrm{C}v \Rightarrow v = \frac{\mathrm{q}}{\mathrm{C}} \Rightarrow \mathrm{w} = \frac{\mathrm{q}^2}{2\mathrm{C}}$$

Last Equation represents the energy stored in the electric field that exists between the plates of the capacitor. This energy can be retrieved, since an ideal capacitor cannot dissipate energy. In fact, the word capacitor is derived from this element's capacity to store energy in an electric field.

We should note the following important properties of a capacitor:

1. Note from Eq. ($i=C dv/dt$) that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus,

A capacitor is an open circuit to dc.

However, if a battery (dc voltage) is connected across a capacitor, the capacitor charges.

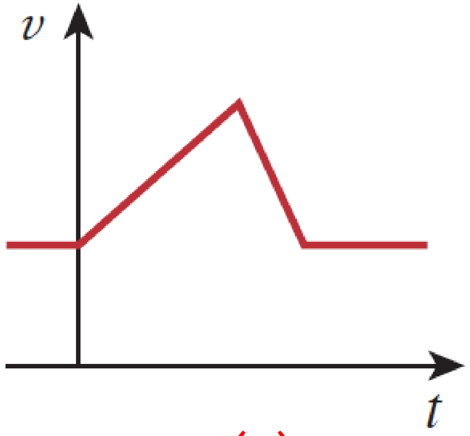
2. The voltage on the capacitor must be continuous.

The voltage on a capacitor cannot change abruptly.

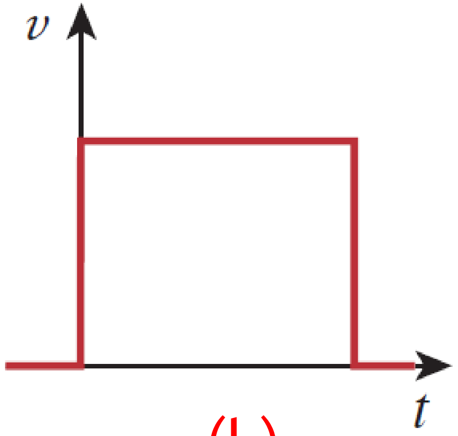
The capacitor resists an abrupt change in the voltage across it.

According to Eq. ($i=Cdv/dt$) , a discontinuous change in voltage requires an infinite current, which is physically impossible. For example, the voltage across a capacitor may take the form shown in Fig. (a), whereas it is not physically possible for the capacitor voltage to take the form shown in Fig. (b) because of the abrupt changes.

Conversely, the current through a capacitor can change instantaneously.



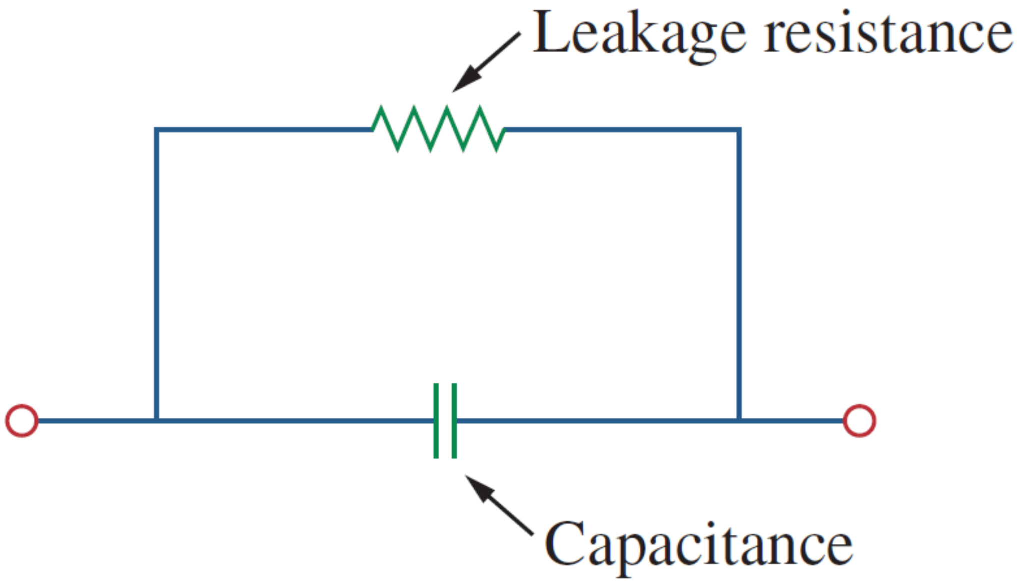
(a)



(b)

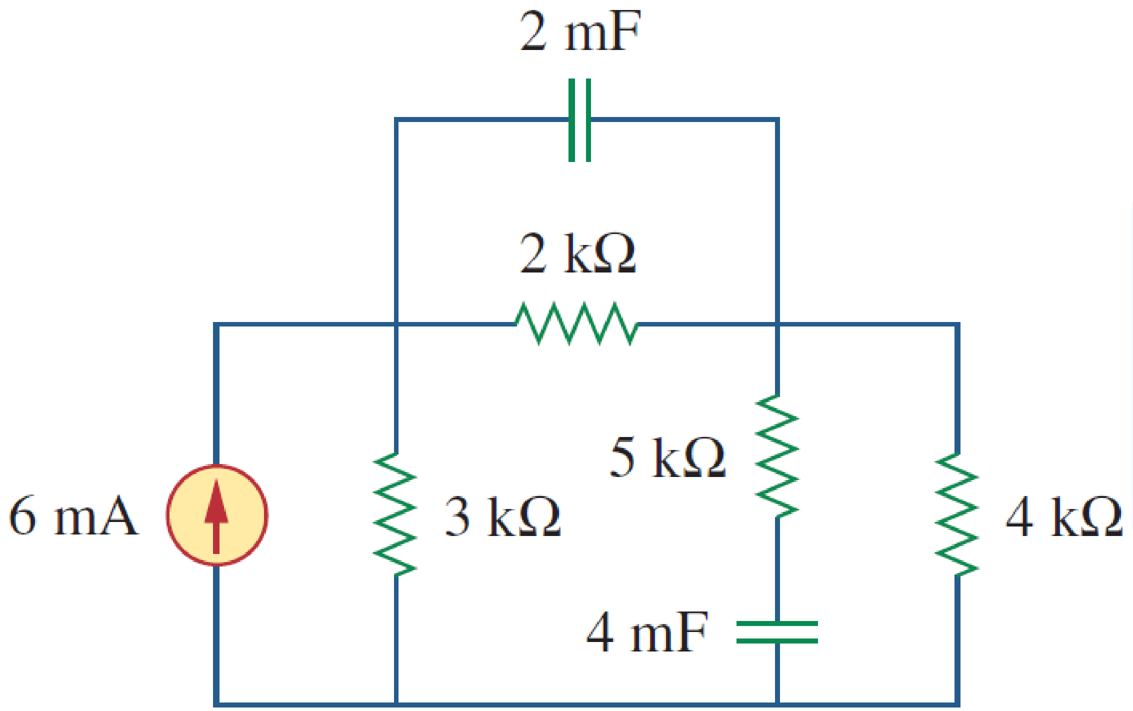
3. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

4. A real, nonideal capacitor has a parallel-model leakage resistance, as shown in Fig. The leakage resistance may be as high as 100MΩ and can be neglected for most practical applications. For this reason, we will assume ideal capacitors.



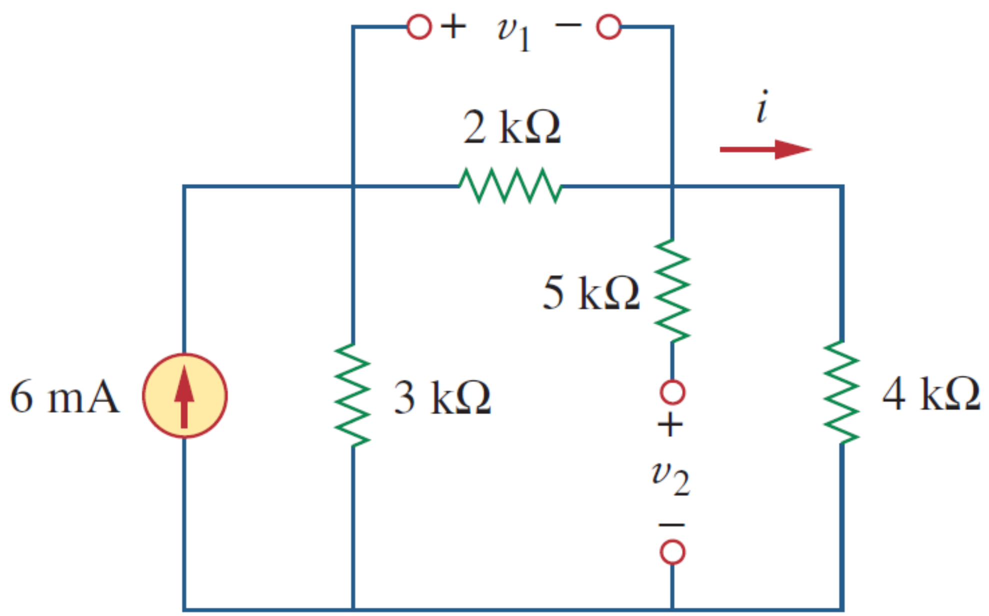
Example

Obtain the energy stored in each capacitor in Fig. under dc conditions



Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig.



The current through the series combination of the 2-KΩ and 4-KΩ resistors is obtained by current division as

$$i = 6(\text{mA}) \times \frac{3}{3+2+4} = 2\text{mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000\mathrm{i} = 4\mathrm{V} \quad , \quad v_2 = 4000\mathrm{i} = 8\mathrm{V}$$

and the energies stored in them are

$$w_1 = \frac{1}{2} C_1 \cdot v_1^2 = \frac{1}{2} \times (2 \times 10^{-3}) \times (4)^2 = 16\mathrm{mJ}$$

$$w_2 = \frac{1}{2} C_2 \cdot v_2^2 = \frac{1}{2} \times (4 \times 10^{-3}) \times (8)^2 = 128\mathrm{mJ}$$

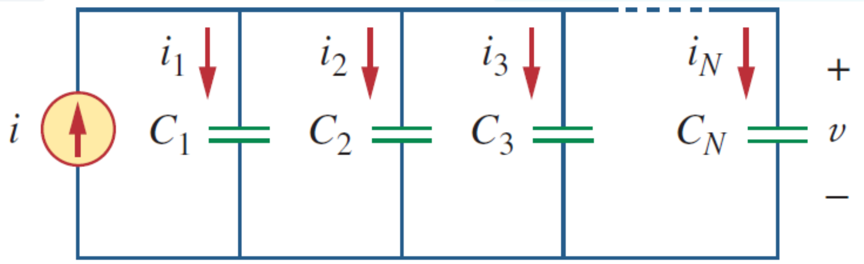
Series and Parallel Capacitors

We know from resistive circuits that the series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered.

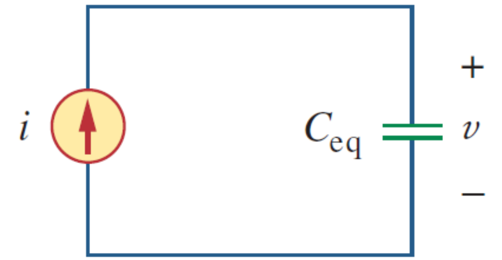
We desire to replace these capacitors by a single equivalent capacitor C_{eq}.

In order to obtain the equivalent capacitor C_{eq} of N capacitors in parallel, consider the circuit in Fig. (a) The equivalent circuit is in Fig. (b). Note that the capacitors have the same voltage across them. Applying KCL to Fig. (a),

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$



(a)



(b)

But $i_k = C_k . dv/dt$. Hence,

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

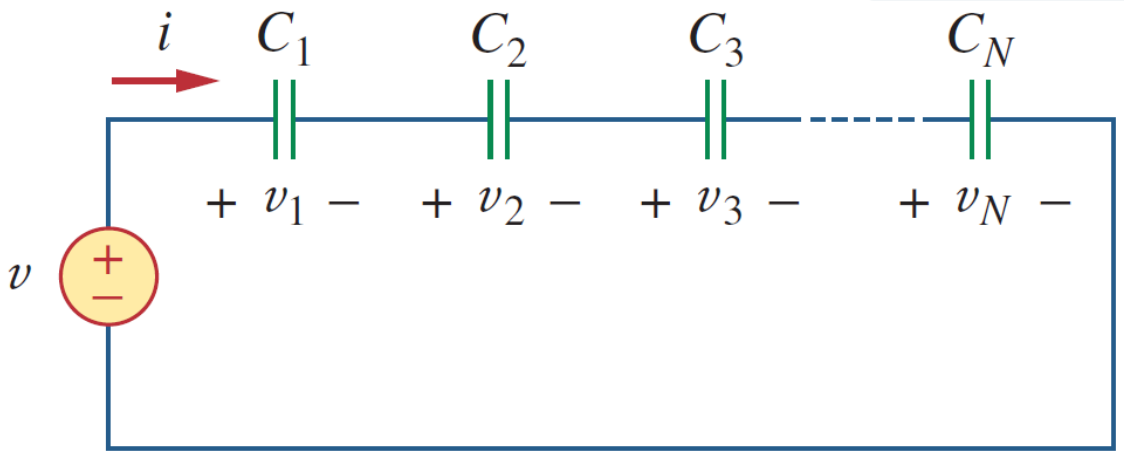
$$= \left(\sum_{k=1}^{N} C_k\right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

where $$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

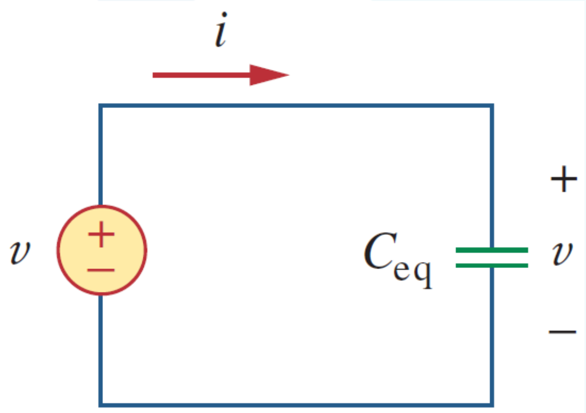
The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.

We observe that capacitors in parallel combine in the same manner as resistors in series.

We now obtain of N capacitors connected in series by comparing the circuit in Fig. (a) with the equivalent circuit in Fig. (b). Note that the same current i flows (and consequently the same charge) through the capacitors.

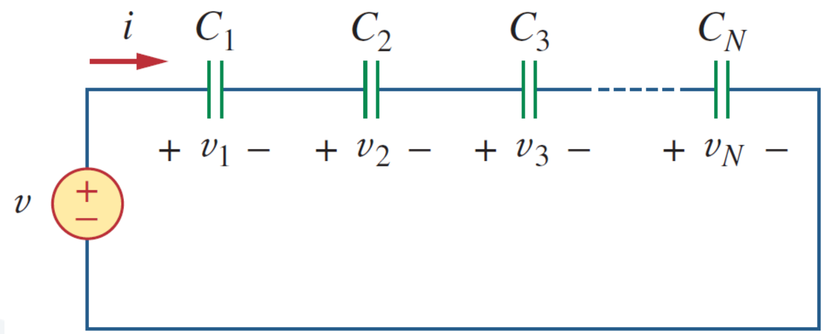


(a)



(b)

Applying KVL to the loop in Fig. (a)



$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

But $v_k = \dfrac{1}{C_k}\displaystyle\int_{t_0}^{t} i(t)\cdot dt + v_0$. Therefore

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(t)\cdot dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(t)\cdot dt + v_2(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(t)\cdot dt + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(t)\cdot dt + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(t)\cdot dt + v_0(t_0)$$

Where $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots + \dfrac{1}{C_N}$

The initial voltage $v_0(t)$ across C_{eq} is required by KVL to be the sum of the capacitor voltages at t_0 .

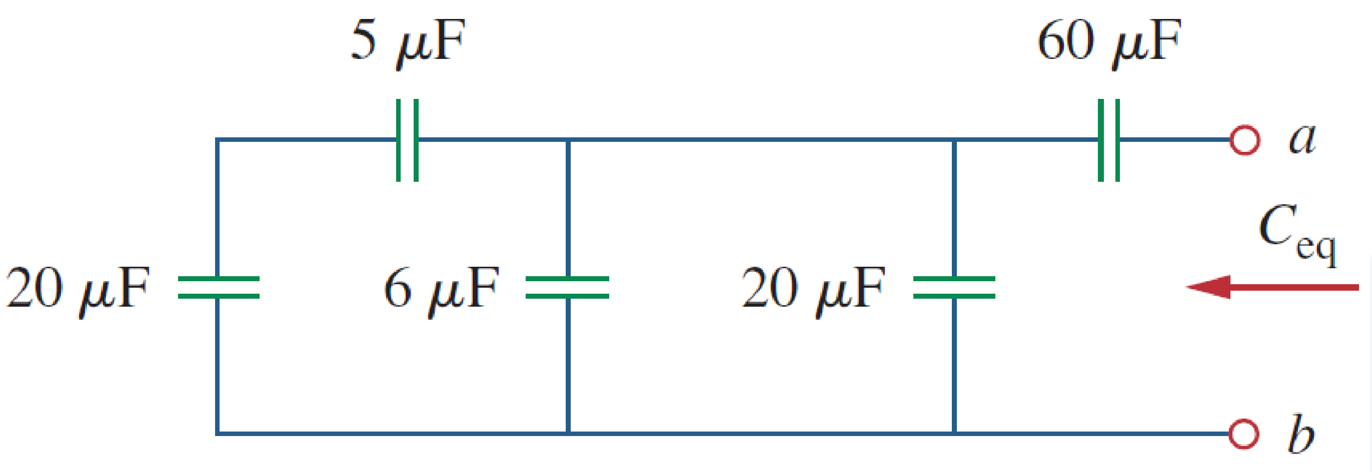
$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

Note that capacitors in series combine in the same manner as resistors in parallel. For N=2 (i.e., two capacitors in series),

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad or \qquad C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$$

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig.



Solution:

The 20-μF and 5-μF capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4\ \mu F$$

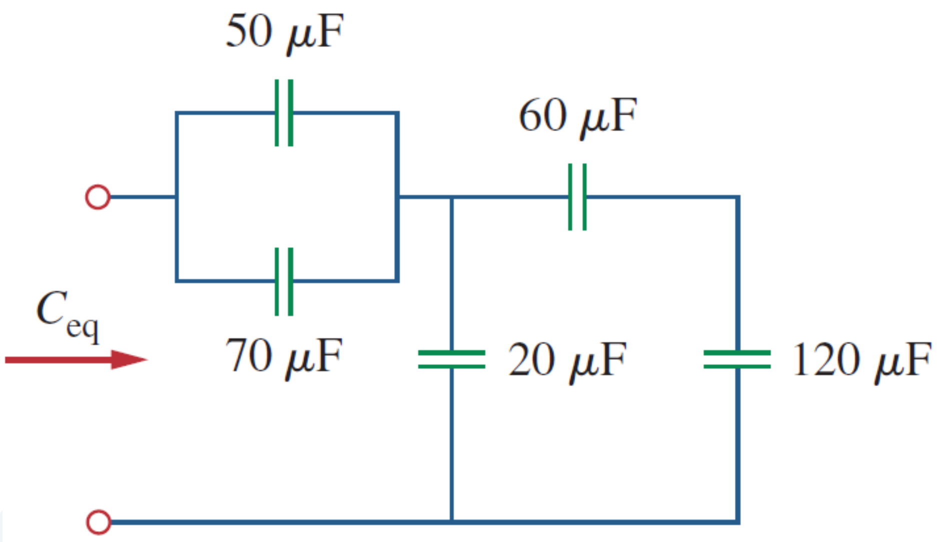
This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors; their combined capacitance is

$$4 + 6 + 20 = 30\ \mu F$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

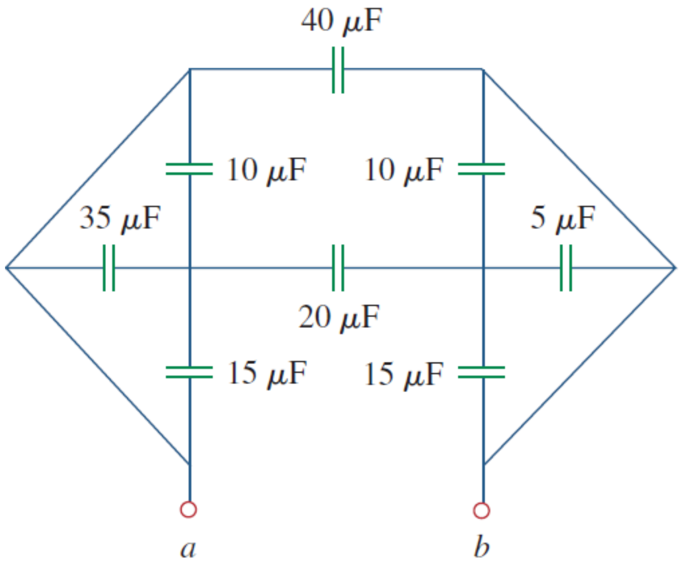
$$C_{eq} = \frac{30 \times 60}{30 + 60} = 2\ \mu\text{F}$$

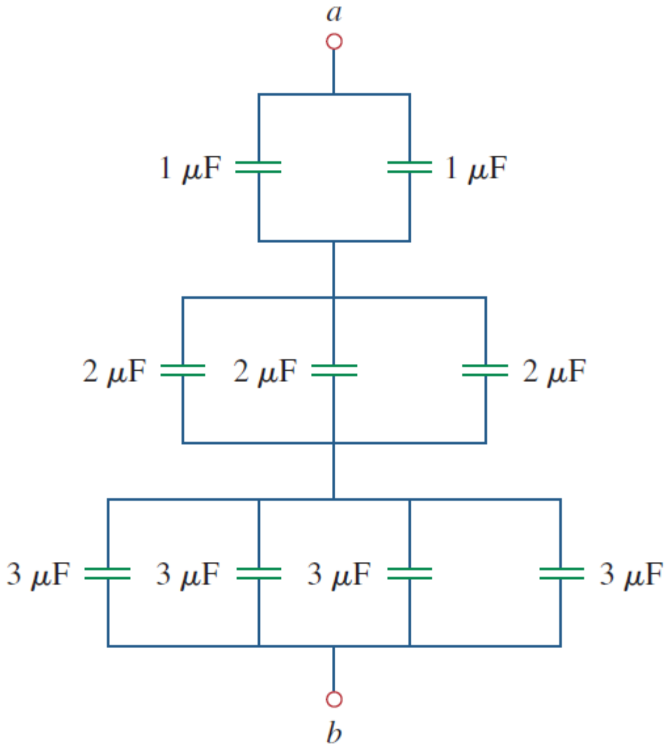
Find the equivalent capacitance seen at the terminals of the circuit in Fig.

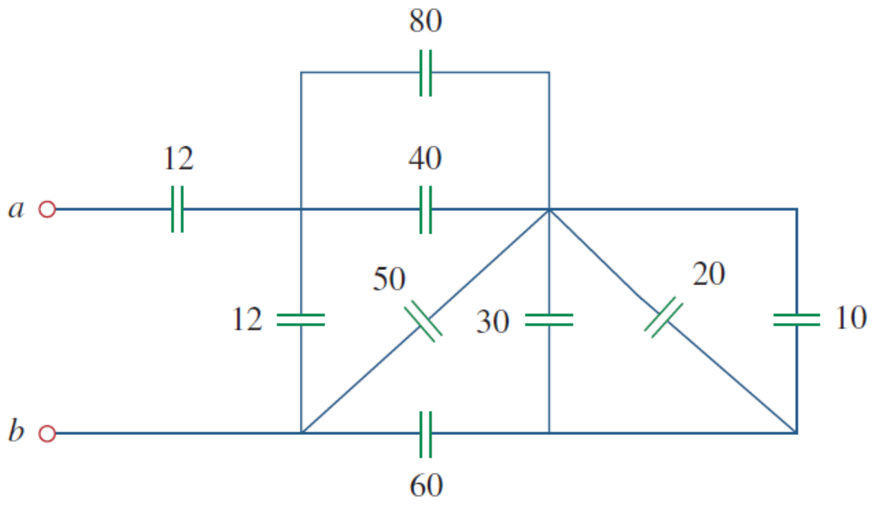


Answer: 40 mF.

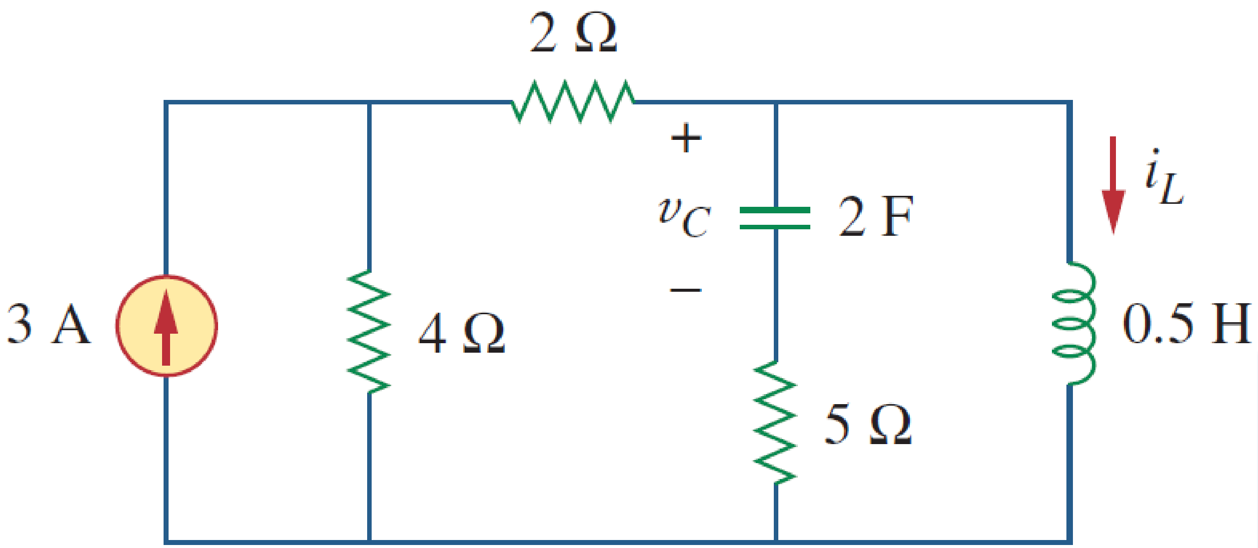
Find the equivalent capacitance between terminals a and b in the circuit of Fig.



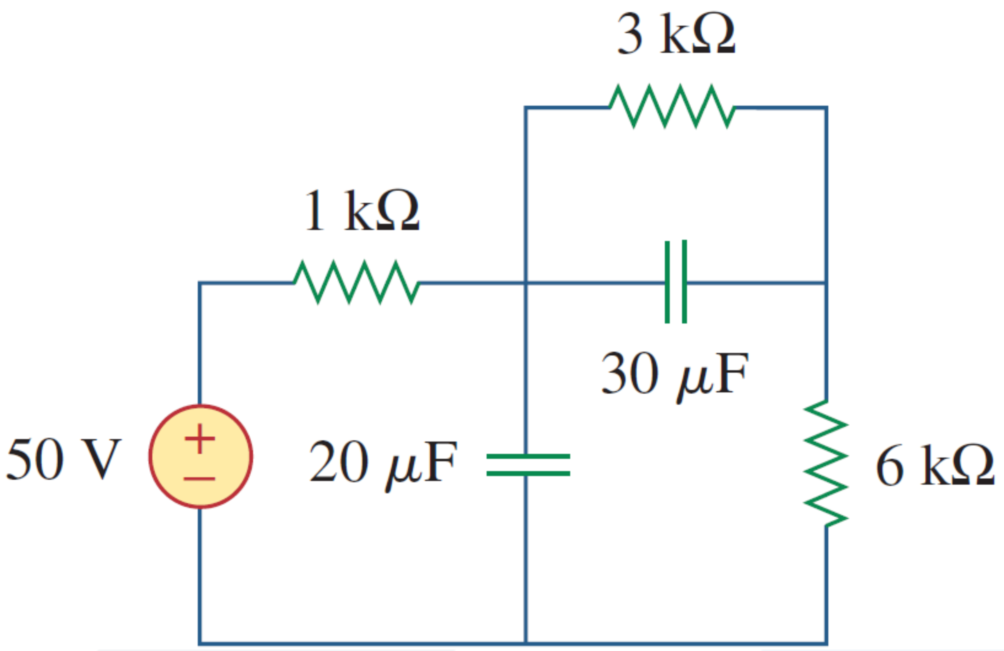




Find and the energy stored in the capacitor and inductor in the circuit of Fig. under dc conditions.

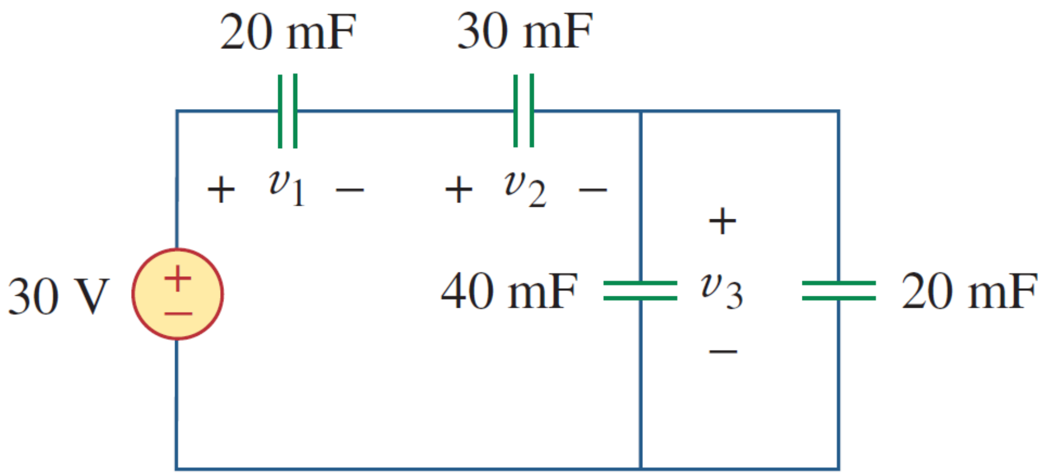


Under dc conditions, find the energy stored in the capacitors in Fig.



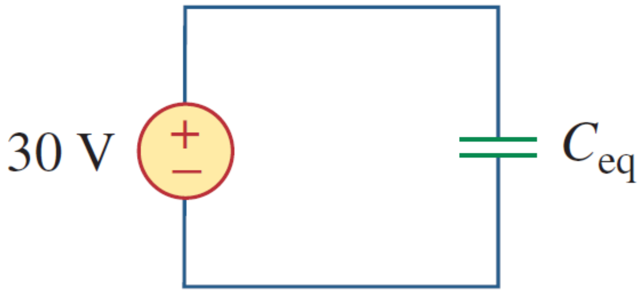
Answer: 20.25 mJ, 3.375 mJ.

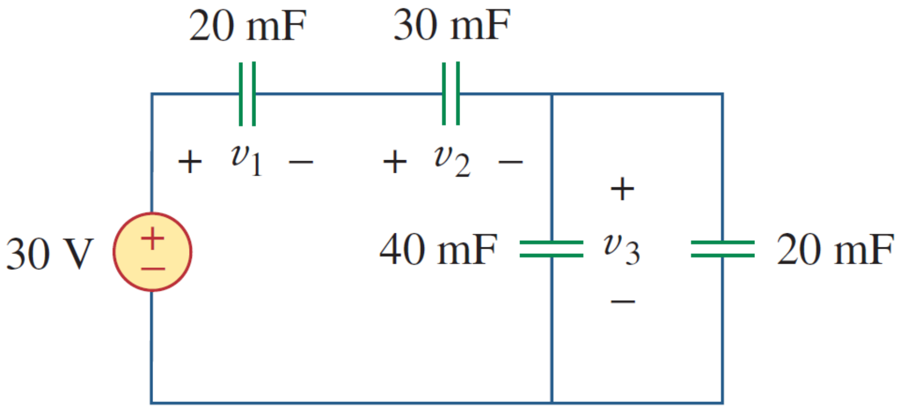
For the circuit in Fig., find the voltage across each capacitor.



Solution:

We first find the equivalent capacitance C_{eq} , shown in Fig.





The two parallel capacitors in Fig. can be combined to get 40+ 20=60 mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is $q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3\ \text{C}$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since i=dq/dt)

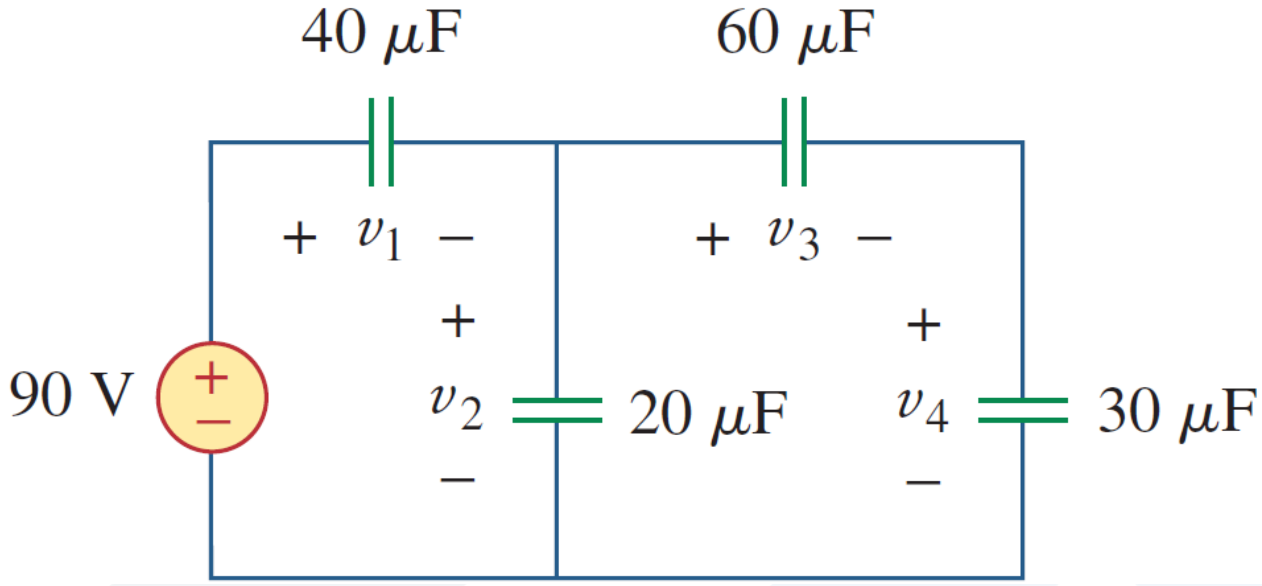
Therefore, $v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15\ \text{V}$ $\quad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10\ \text{V}$

Having determined v_1 and v_2, we now use KVL to determine v_3 by $v_3 = 30 - v_1 - v_2 = 5\ \text{V}$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is get 40+ 20=60 mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60\ \text{mF}} = \frac{0.3}{60 \times 10^{-3}} = 5\ \text{V}$$

Find the voltage across each of the capacitors in Fig.

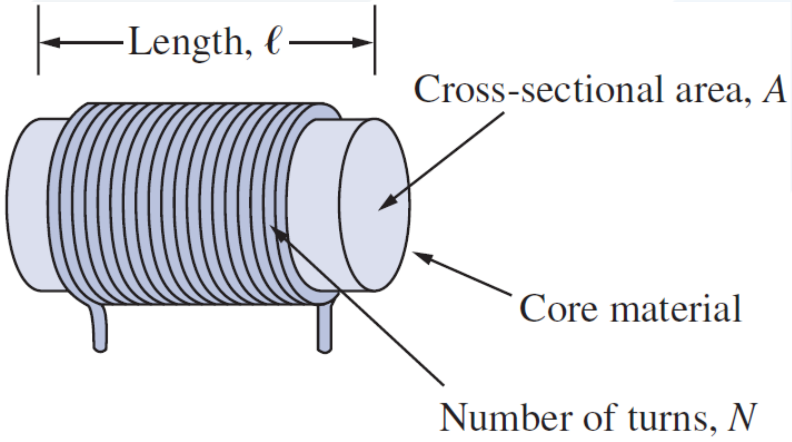


Answer: $v_1 = 45$ V, $v_2 = 45$ V, $v_3 = 15$ V, $v_4 = 30$ V.

Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in Fig.



An inductor consists of a coil of conducting wire.

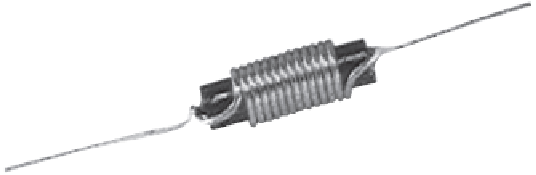
If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.

$$v = L\frac{di}{dt}$$

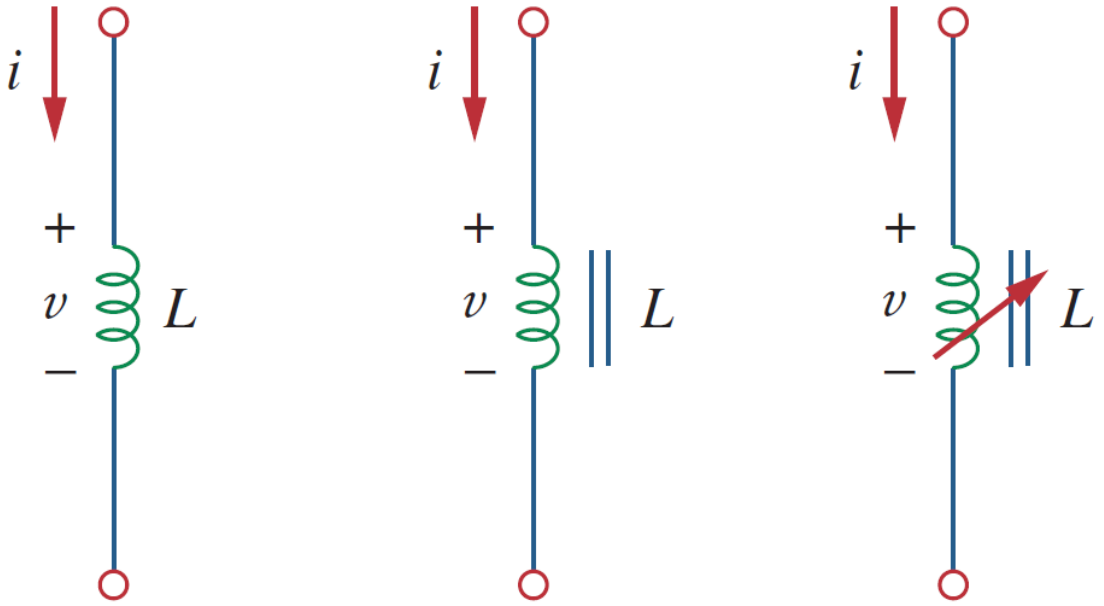
where L is the constant of proportionality called the inductance of the inductor. The unit of inductance is the henry (H), named in honor of the American inventor Joseph Henry (1797–1878). It is clear from Eq. that 1 henry equals 1 volt-second per ampere.

In view of Eq. , for an inductor to have voltage across its terminals, its current must vary with time. Hence, v=0 for constant current through the inductor.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).



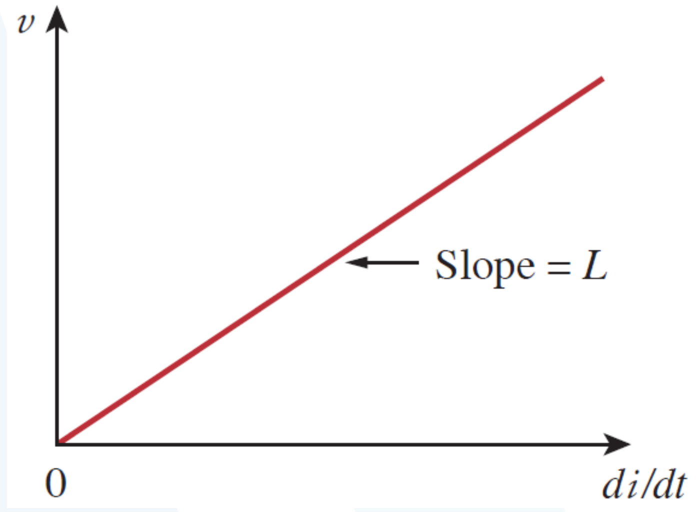
The circuit symbols for inductors are shown in Fig.



(a) (b) (c)

(a) air-core, (b) iron-core, (c) variable iron-core.

inductance is independent of current. Such an inductor is known as a linear inductor. For a nonlinear inductor, the plot of Eq. v=L.di/dt will not be a straight line because its inductance varies with current. We will assume linear inductors unless stated otherwise. The current-voltage relationship is obtained from Eq. v=L.di/dt as



$$di = \frac{1}{L} v dt$$

Integrating gives

$$i = \frac{1}{L} \int_{-\infty}^{t} v(t) dt \qquad \text{or} \qquad i = \frac{1}{L} \int_{t_o}^{t} v(t) dt + i(t_0)$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v(t)dt \qquad \text{or} \qquad i = \frac{1}{L}\int_{t_o}^{t} v(t)dt + i(t_0)$$

Where $i(t_0)$ is the total current for $-\infty < t < t_0$ and $i(-\infty)=0$. The idea of making $i(-\infty)=0$ is practical and reasonable, because there must be a time in the past when there was no current in the inductor.

The inductor is designed to store energy in its magnetic field. The energy stored can be obtained from Eq. ($v=L\,di/dt$). The power delivered to the inductor is

$$p = vi = \left(L\frac{di}{dt}\right)i$$

The energy stored is

$$w = \int_{-\infty}^{t} p(\mathrm{t})\mathrm{dt} = \mathrm{L} \int_{-\infty}^{t} \frac{di}{dt} i\mathrm{dt} = L \int_{-\infty}^{t} i\mathrm{dt} = \frac{1}{2} L i^2(t) - \frac{1}{2} L i^2(-\infty)$$

Since $i(-\infty)=0$

$$w = \frac{1}{2} L i^2$$

We should note the following important properties of an inductor.

1. Note from Eq. ($v=L di/dt$) that the voltage across an inductor is zero when the current is constant. Thus,

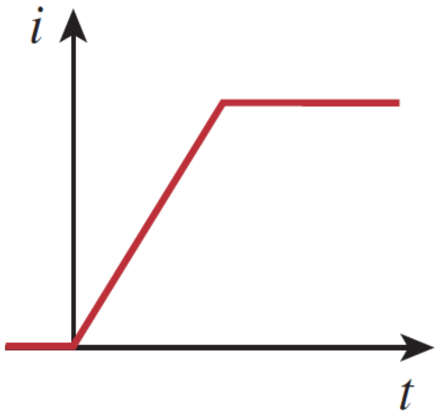
An inductor acts like a short circuit to dc.

2. An important property of the inductor is its opposition to the change in current flowing through it.

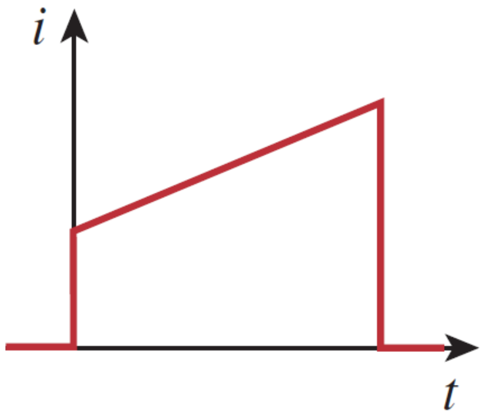
The current through an inductor cannot change instantaneously.

According to Eq. ($v = L\,di/dt$), a discontinuous change in the current through an inductor requires an infinite voltage, which is not physically possible. Thus, an inductor opposes an abrupt change in the current through it.

For example, the current through an inductor may take the form shown in Fig. (a), whereas the inductor current cannot take the form shown in Fig. (b) in real-life situations due to the discontinuities. However, the voltage across an inductor can change abruptly.



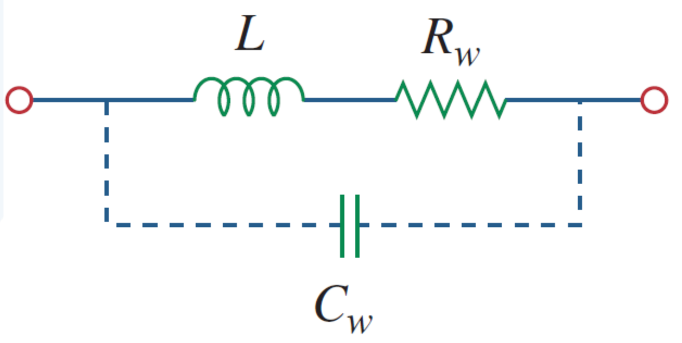
(a)



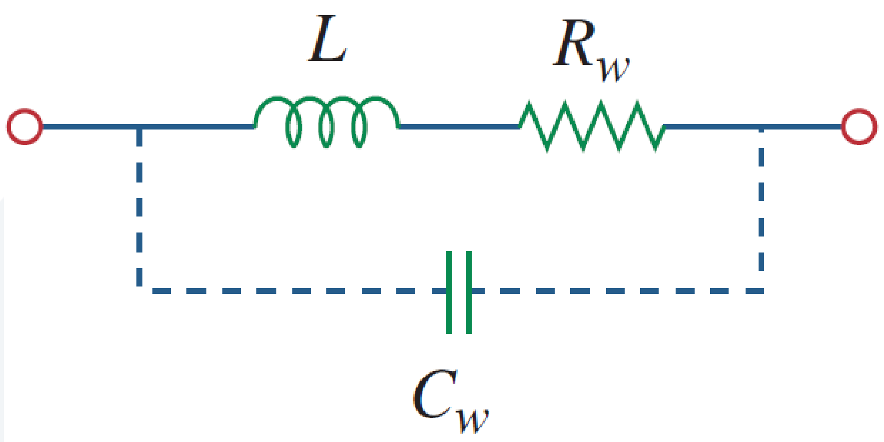
(b)

3. Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.

4. A practical, nonideal inductor has a significant resistive component, as shown in Fig. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the winding resistance R_w, and it appears in series with the inductance of the inductor.



The presence of R_w makes it both an energy storage device and an energy dissipation device. Since R_w is usually very small, it is ignored in most cases. The nonideal inductor also has a winding capacitance C_w due to the capacitive coupling between the conducting coils. C_w is very small and can be ignored in most cases, except at high frequencies. We will assume ideal inductors.



The current through a 0.1-H inductor is $i(t) = 10te^{-5t}A$. Find the voltage across the inductor and the energy stored in it.

The current through a 0.1-H inductor is $i(t) = 10te^{-5t} A$. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at t=5s, Assume i(v)>0.

Solution:

Since $i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, And the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25 \text{ kJ}$$

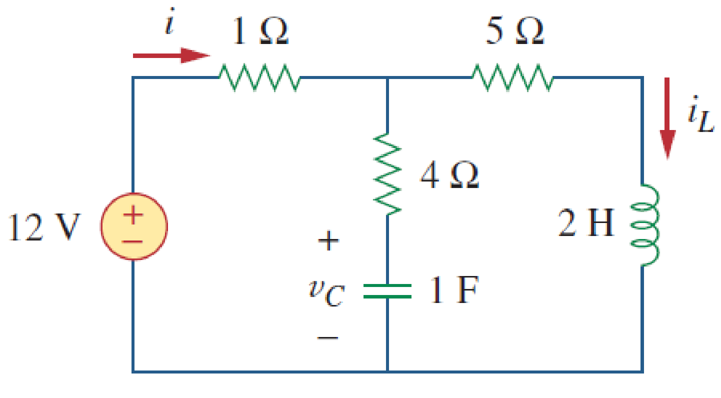
Alternatively, we can obtain the energy stored using Eq. $w=1/2\ li^2$, by writing

$$w\big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

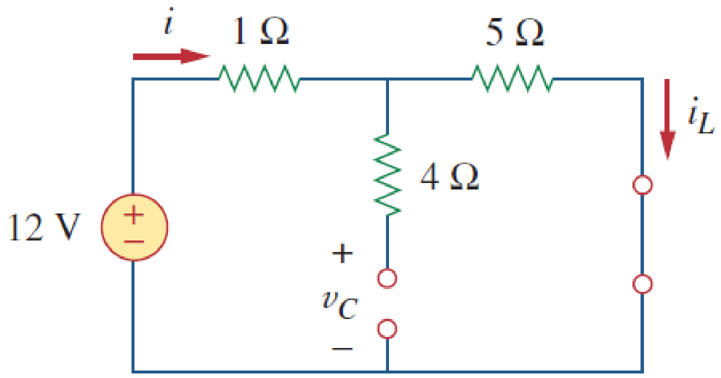
Consider the circuit in Fig. Under dc conditions, find:

(a) i

(b) the energy stored in the capacitor and inductor.



(a)



(b)

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and inductor with a short circuit, as in Fig. (b). It is evident from Fig. (b) that

$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

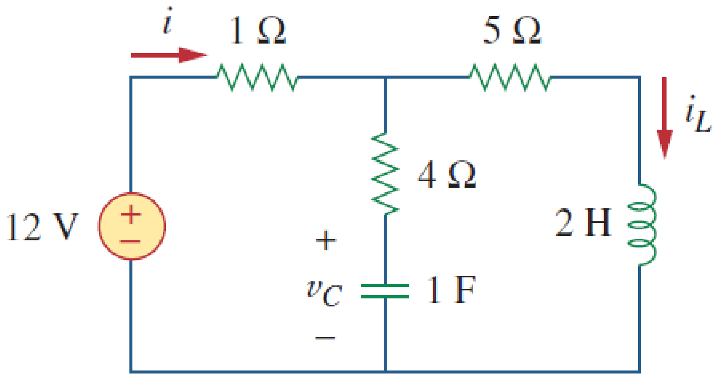
$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

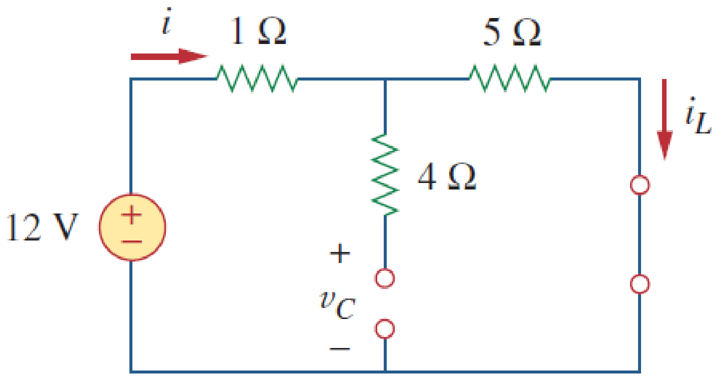
$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$



(a)



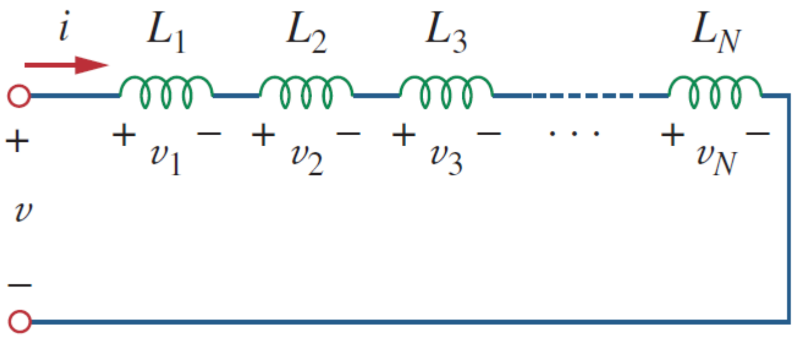
(b)

Series and Parallel Inductors

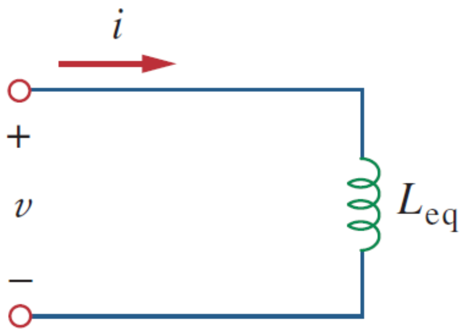
Now that the inductor has been added to our list of passive elements, it is necessary to extend the powerful tool of series-parallel combination. We need to know how to find the equivalent inductance of a series-connected or parallel-connected set of inductors found in practical circuits.

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

Substituting $v_k = L\dfrac{di}{dt}$ results in



(a)



(b)

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

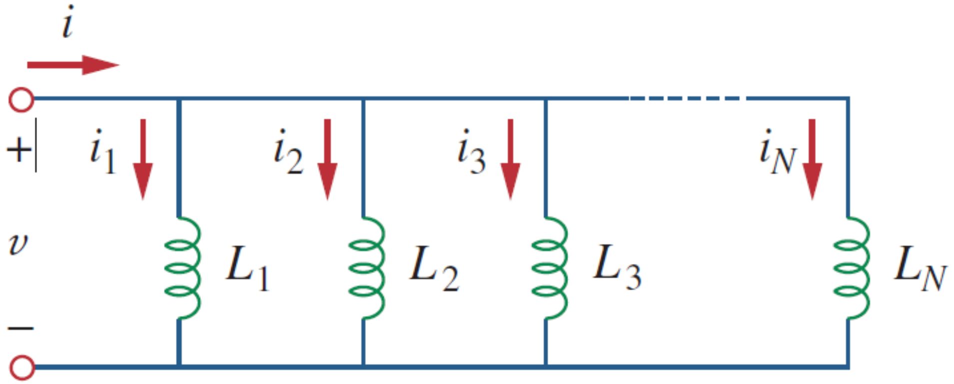
$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

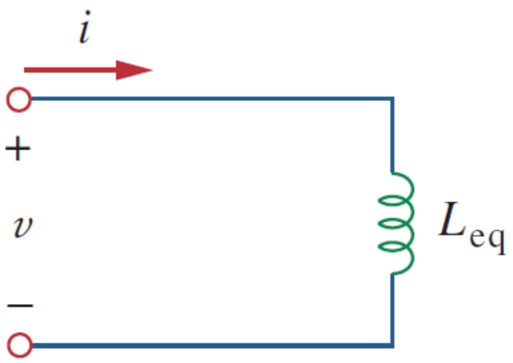
Where

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N}$$

Thus, The equivalent inductance of series-connected inductors is the sum of the individual inductances.



(a)



(b)

Inductors in series are combined in exactly the same way as resistors in series.

Applying KCL $\quad i = i_1 + i_2 + i_3 + \cdots + i_N$

But $\quad i_k = \dfrac{1}{L_k}\displaystyle\int_{t_0}^{t} v dt + i_k(t_0)\quad$ hence

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0)$$

$$+ \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

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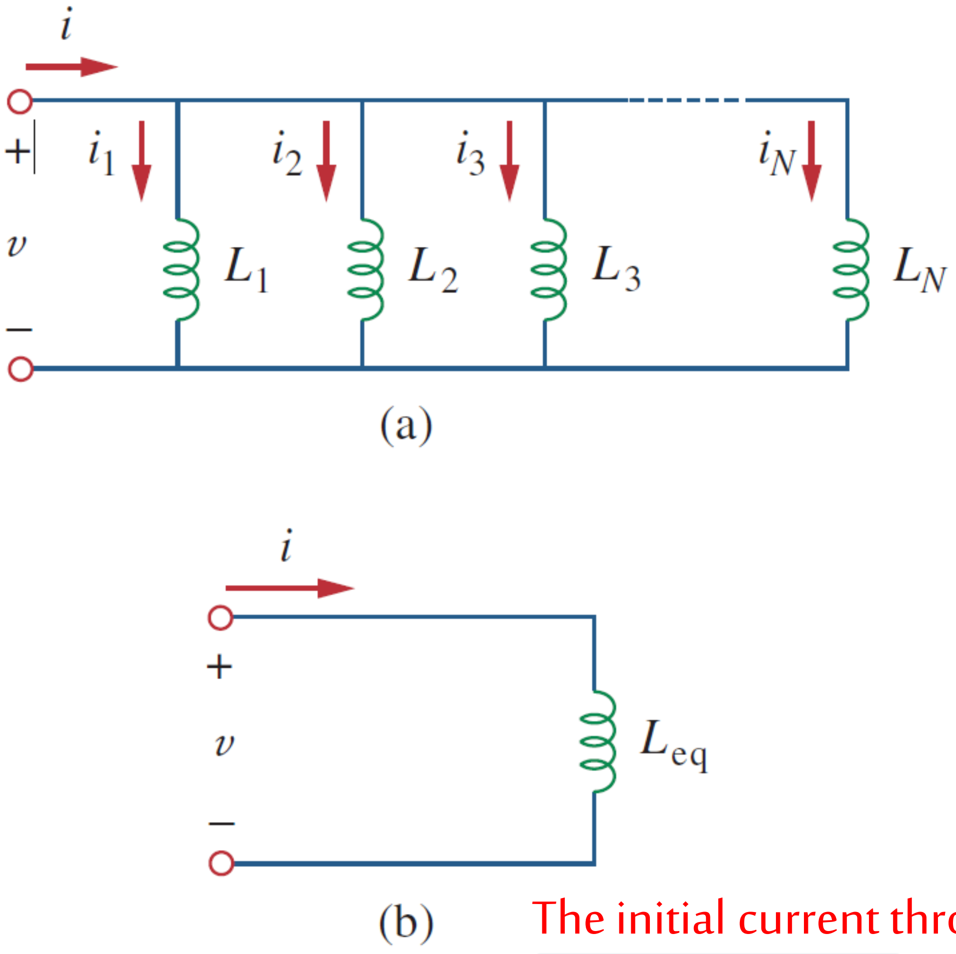


$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right) \int_{t_0}^{t} v\, dt + i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N} \frac{1}{L_k}\right) \int_{t_0}^{t} v\, dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^{t} v\, dt + i(t_0)$$

Where

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

The initial current through at is expected by KCL to be the sum of the inductor currents at Thus,

$$i(t_0) = i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

Note that the inductors in parallel are combined in the same way as resistors in parallel. For two inductors in parallel (N=2)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \qquad \text{or} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

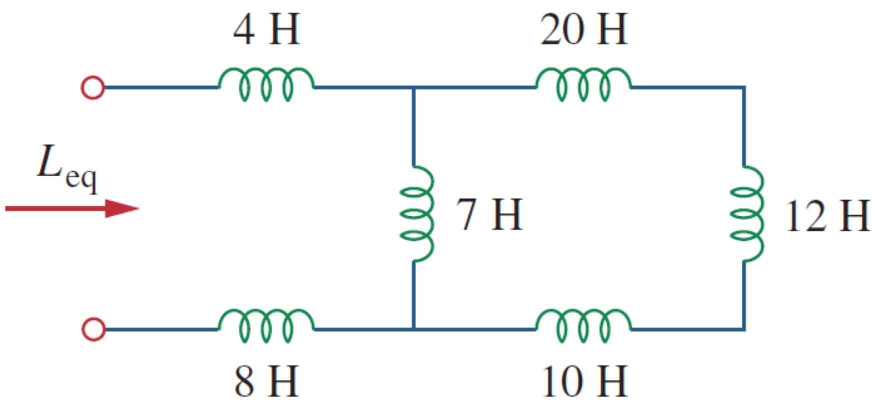
As long as all the elements are of the same type, the Δ-Y transformations for resistors ,can be extended to capacitors and inductors.

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

† *Passive sign convention is assumed.*

Find the equivalent inductance of the circuit shown in Fig.



Solution:

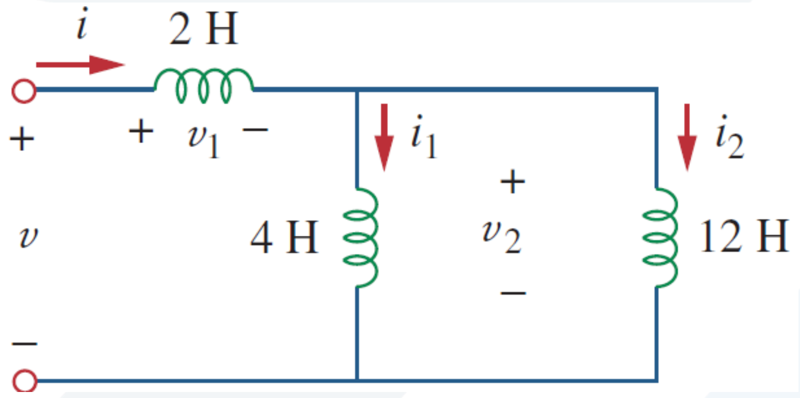
The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

$$\frac{7 \times 42}{7 + 42} = 6\text{H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 8 + 6 = 18\text{H}$$

For the circuit in Fig. $i(t) = 4(2 - e^{-10t})mA$. If $i_2(0) = 1 - mA$, find $i_1(0), v(t), v_1(t), v_2(t), i_1(t), i_2(t)$.



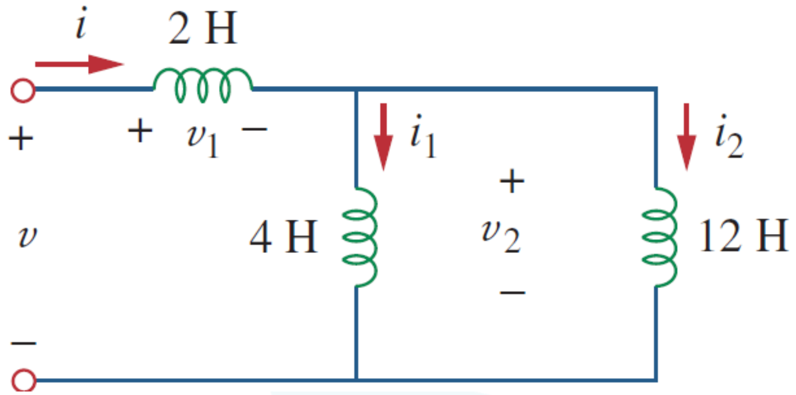
Solution:

(a) From $i(t) = 4(2 - e^{-10t})$ mA, $i(0) = 4(2 - 1) = 4$ mA. Since $i = i_1 + i_2$,

$$i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5 \text{ mA}$$

(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \,\|\, 12 = 2 + 3 = 5 \text{ H}$$



Thus,

$$v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} \text{ mV} = 200e^{-10t} \text{ mV}$$

and

$$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t} \text{ mV} = 80e^{-10t} \text{ mV}$$

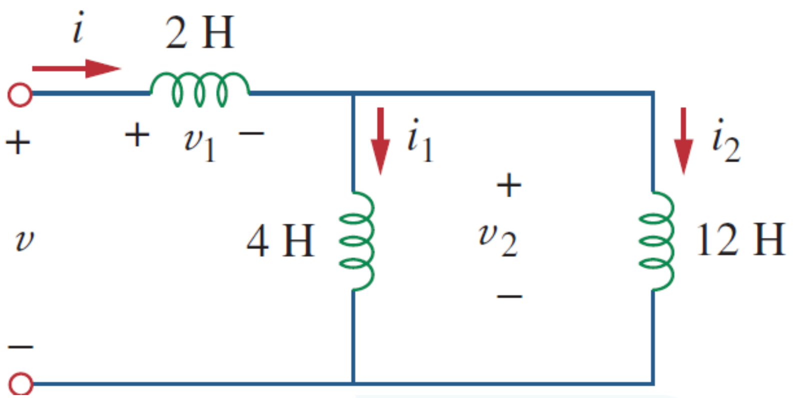
Since $v = v_1 + v_2$,

$$v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{ mV}$$

(c) The current i_1 is obtained as

$$i_1(t) = \frac{1}{4}\int_0^t v_2\, dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t}\, dt + 5 \text{ mA}$$

$$= -3e^{-10t}\Big|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA}$$



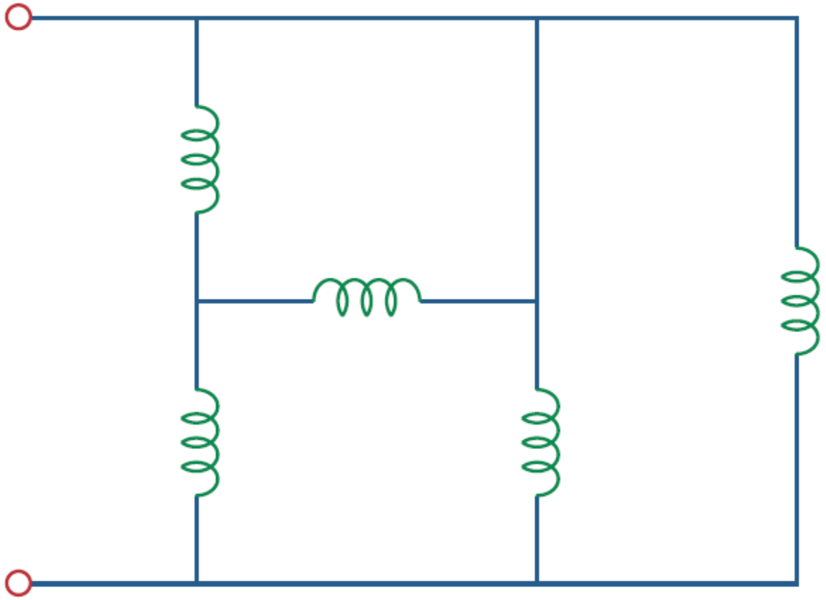
Similarly,

$$i_2(t) = \frac{1}{12}\int_0^t v_2\, dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t}\, dt - 1 \text{ mA}$$

$$= -e^{-10t}\Big|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA}$$

Note that $i_1(t) + i_2(t) = i(t)$.

Find the equivalent inductance of the circuit in Fig. Assume all inductors are 10 mH.



Find the equivalent inductance looking into the terminals of the circuit in Fig.