Structural Mechanics (1)

Week No-05

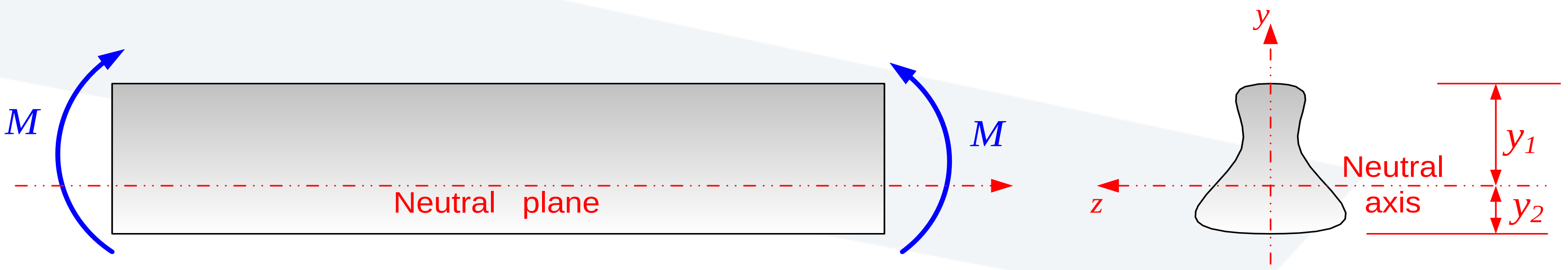
Deflection in Determinate Structures

Deflections of Trusses, Beams, & Frames: Work-Energy Methods

- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

Strain energy in a beam element

Bending Strain energy in a beam element



$$U=\iiint_V \frac{1}{2}\sigma_x\varepsilon_x dV=\iiint_V \frac{\sigma_x^2}{2E}dV=\iiint_V \frac{1}{2E}\left(\frac{M^2}{I^2}y^2\right)dxdA=\int_0^L \frac{1}{2E}dx\left(\frac{M^2}{I^2}\right)\iint_A y^2 dA=\int_0^L \frac{M^2}{2EI}dx$$

Shear Strain energy in a beam element,

$$U=\iiint_V \frac{1}{2}\tau\gamma dV=\iiint_V \frac{\tau^2}{2G}dV=k\int_0^L \frac{S^2}{2GA}dx$$

$$k=\begin{cases}1.2 & \text{for a rectangle}\\ 1.1 & \text{for a circle}\\ 1.2 & \text{for a thin circular}\end{cases}$$

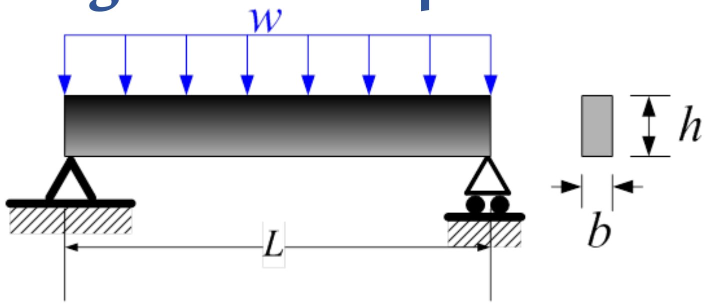
Comparison of bending and shear strain energies in a simple beam

$$U_b = \int_0^L \frac{M^2}{2EI}dx$$

$$M(x) = -\tfrac{1}{2}wx^2 + \tfrac{1}{2}wLx$$

$$U_S = 1.2\int_0^L \frac{S^2}{2GA}dx$$

$$S(x) = -wx + \tfrac{1}{2}wL$$



$$I = bh^3/12 \quad \& \quad A = bh$$

$$U_b = \int_0^L \frac{M^2}{2EI}dx = \frac{6}{Ebh^3}\int_0^L \left(-\tfrac{1}{2}wx^2 + \tfrac{1}{2}wLx\right)^2 dx$$

$$= \frac{6w^2}{Ebh^3}\int_0^L \left(\tfrac{1}{4}x^4 - \tfrac{1}{2}Lx^3 + \tfrac{1}{4}L^2x^2\right)dx$$

$$= \frac{6w^2}{Ebh^3}\left[\tfrac{1}{20}x^5 - \tfrac{1}{8}Lx^4 + \tfrac{1}{12}L^2x^3\right]_0^L = \frac{0.05w^2L^5}{Ebh^3}$$

$$U_S = 1.2\int_0^L \frac{S^2}{2GA}dx = \frac{0.6}{Gbh}\int_0^L \left(wx - \tfrac{1}{2}wL\right)^2 dx$$

$$= \frac{0.6w^2}{Gbh}\int_0^L \left(x^2 - Lx + \tfrac{1}{4}L^2\right)dx$$

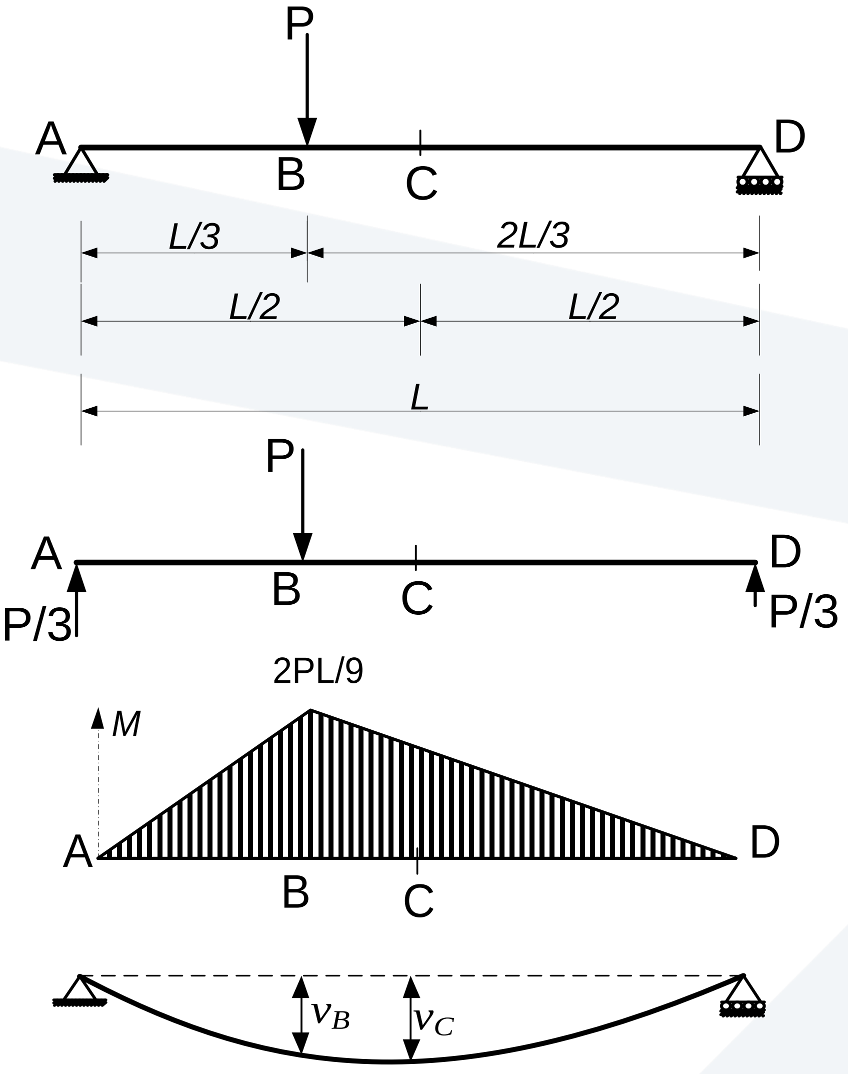
$$= \frac{0.6w^2}{Gbh}\left[\tfrac{1}{3}x^3 - \tfrac{1}{2}Lx^2 + \tfrac{1}{4}L^2x\right]_0^L = \frac{0.05w^2L^3}{Gbh}$$

$$U_S/U_b = \left(\frac{0.05w^2L^3}{Gbh}\right) / \left(\frac{0.05w^2L^5}{Ebh^3}\right)$$

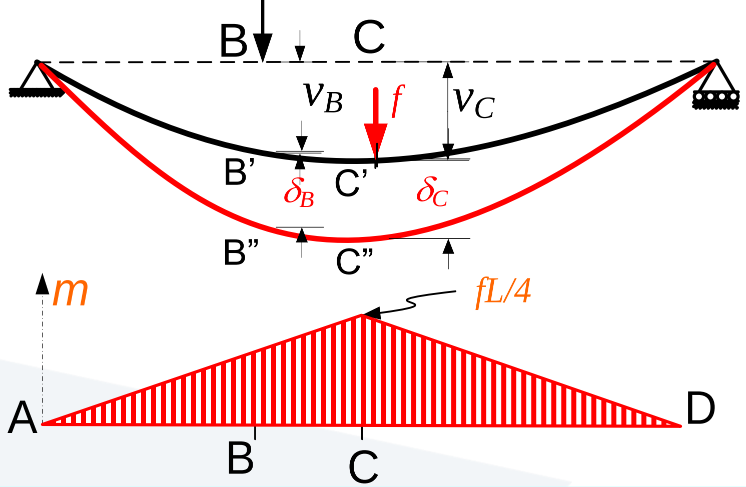
$$= (E/G)\left(h^2/L^2\right) \approx 2h^2/L^2 \ll 1.$$

Shear and Axial Strain energies are negligible in comparison with the bending moment energy

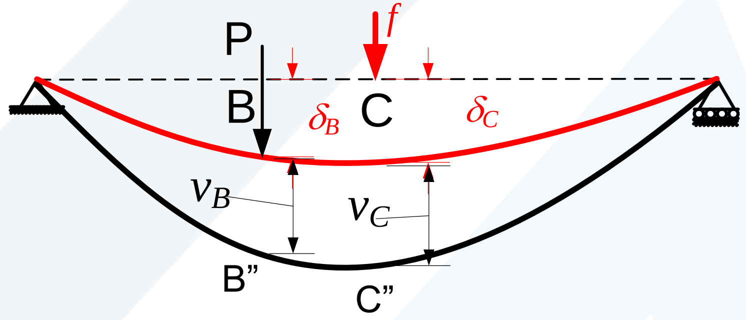
DEFLECTIONS OF BEAMS BY THE V. W. M.



$$\frac{1}{2}Pv_B=\int_0^L \frac{M^2}{2EI}\,dx$$



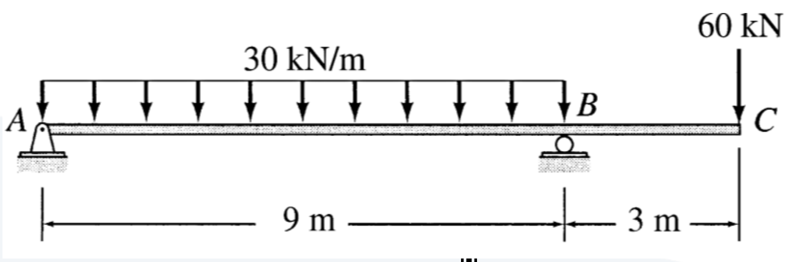
$$\frac{1}{2}Pv_B+P\delta_B+\frac{1}{2}f\,\delta_C=\int_0^L \frac{(M+m)^2}{2EI}\,dx$$



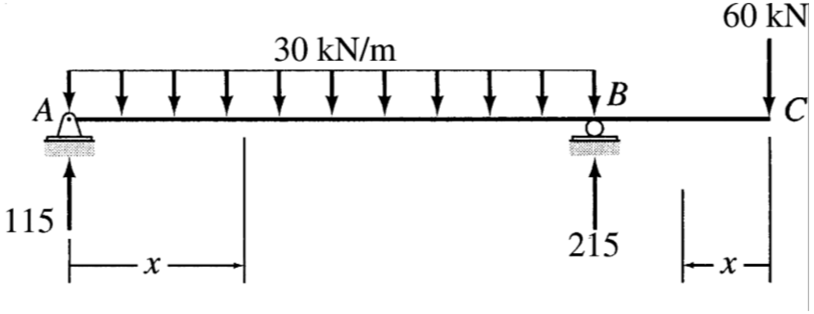
$$\frac{1}{2}f\,\delta_C+f\,v_C+\frac{1}{2}Pv_B=\int_0^L \frac{(m+M)^2}{2EI}\,dx$$

$$P\delta_B=f\,v_C=\int_0^L \frac{mM}{EI}\,dx$$

<u>Example-02</u>. Determine the deflection at point C of the beam shown in the figure, by the virtual work method. EI=const. E=200 GPa, I=800(10^6) mm^4



Real System



$Segment\ AB\colon\ 0 < x < 9, M(x) = 115x - 15x^2$

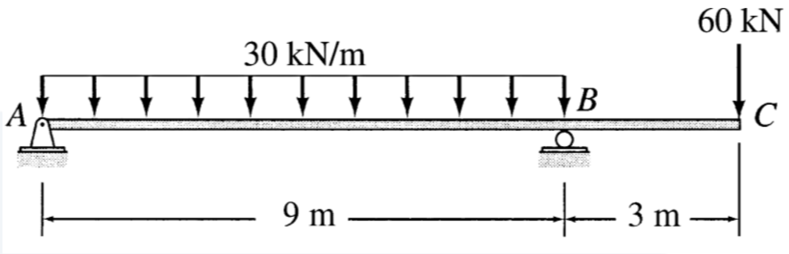
$Segment\ BC\colon\ 0 < x < 3, M(x) = -60x$

Virtual System

$Segment\ AB\colon\ 0 < x < 9, m(x) = -\frac{x}{3}$

$Segment\ BC\colon\ 0 < x < 3, m(x) = -x$

Example-02. Determine the deflection at point C of the beam shown in the figure, by the virtual work method. EI=const. E=200 GPa, I=800(10^6) mm^4



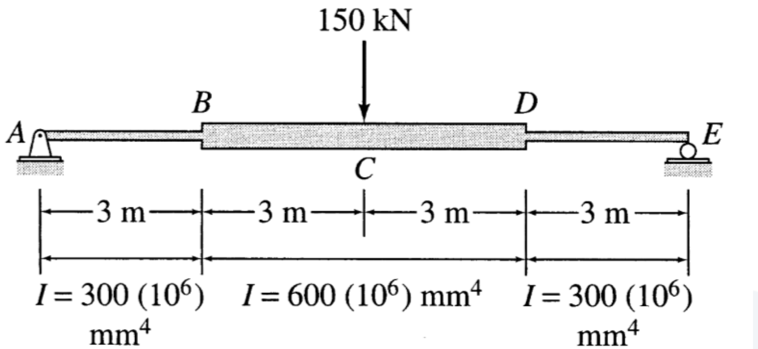
	x Coordinate			
Segment	Origin	Limits (m)	M (kN-m)	M_v (kN-m)
AB	A	0–9	$(115x - 15x^2)$	$-\frac{x}{3}$
CB	C	0–3	$-60x$	$-x$

$$\Delta_C = \int_0^L \frac{M(x).m(x)}{EI}dx = \frac{1}{EI}\int_0^9 (115x - 15x^2)\left(-\frac{x}{3}\right)dx + \frac{1}{EI}\int_0^3 (-60x)(-x)dx = -\frac{933.75}{EI}$$

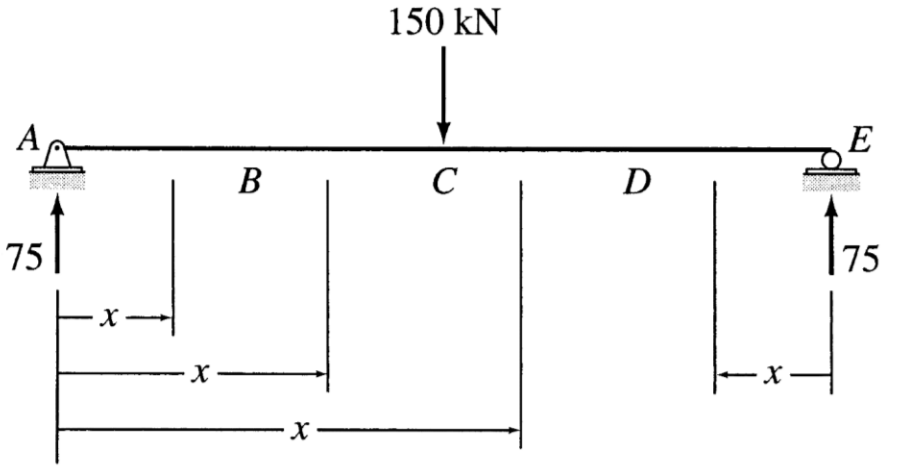
$$\Delta_C = -\frac{933.75}{EI} = -\frac{933.75}{200\,(10^6)800\,(10^{-6})} = -0.005836\,m$$

$$\boldsymbol{\Delta_C = 5.836\,mm}\ \uparrow$$

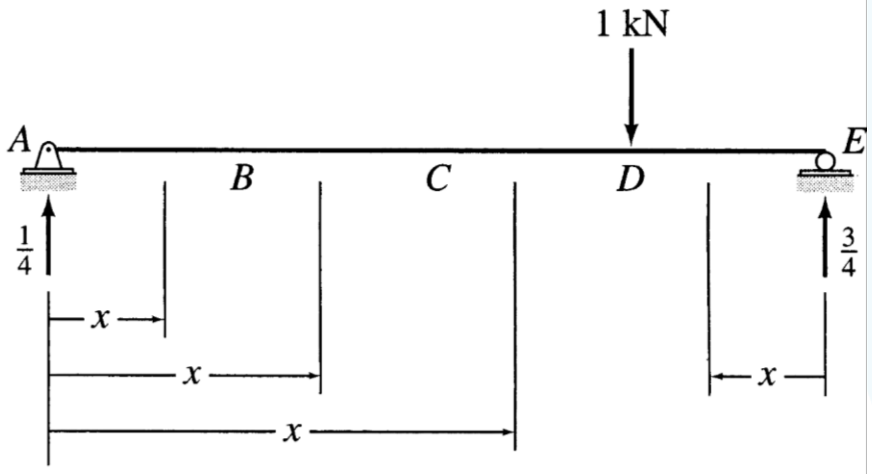
Example-03. Determine the deflection at point D of the beam shown in the figure, by the virtual work method. E=200 GPa.



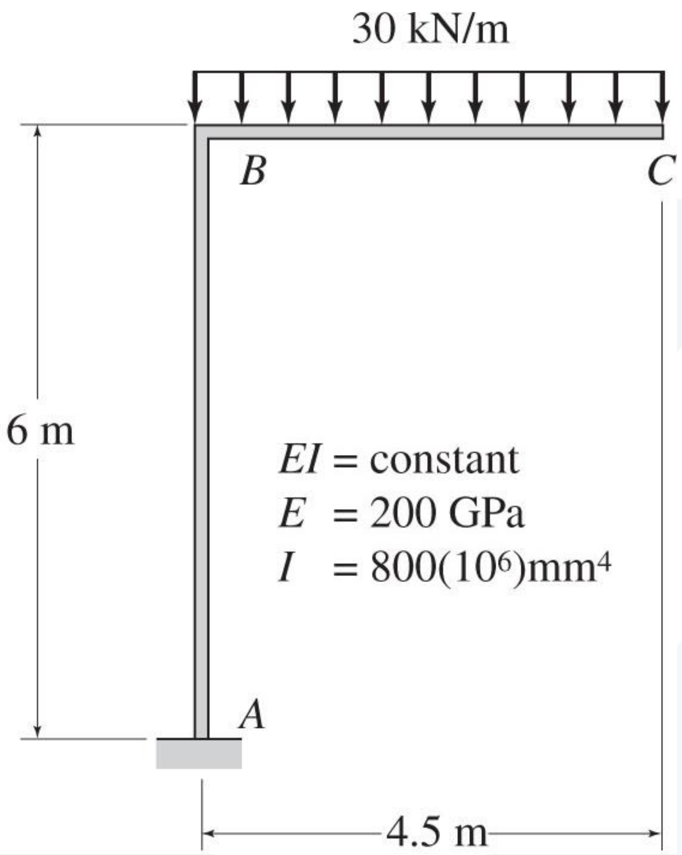
Real System



Virtual System

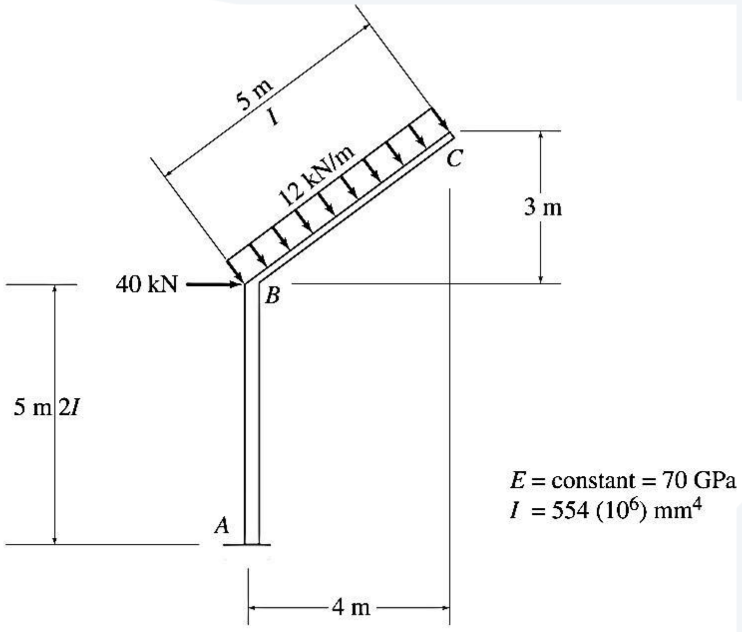


Example-04. Use the virtual work method to determine the deflection at joint C of the following frame.

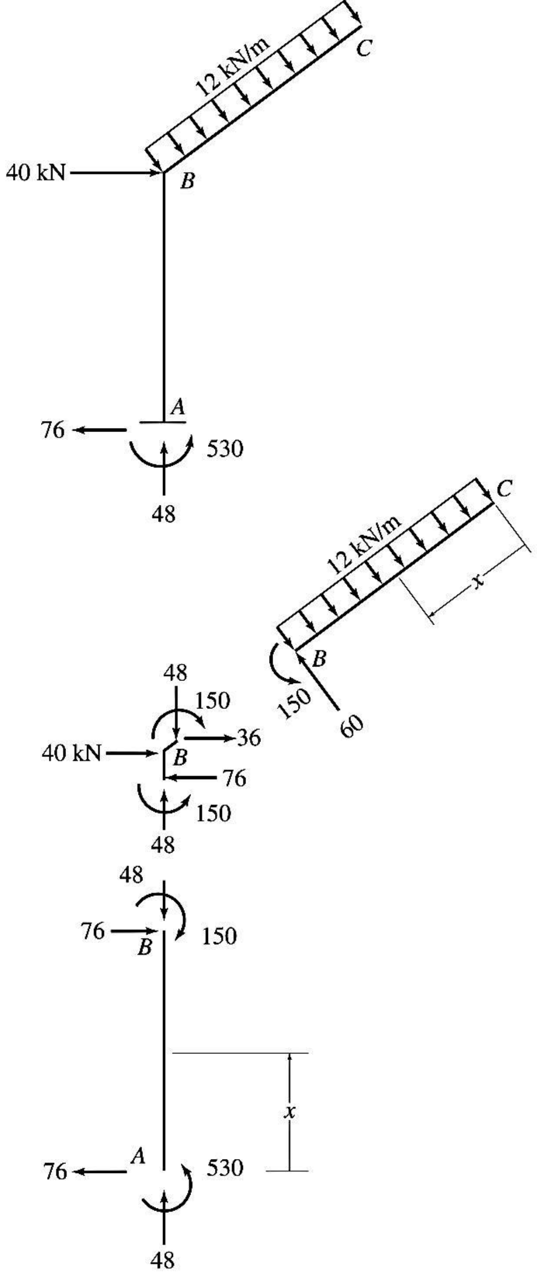


$$\Delta_C = 60.9\ mm\ \downarrow$$

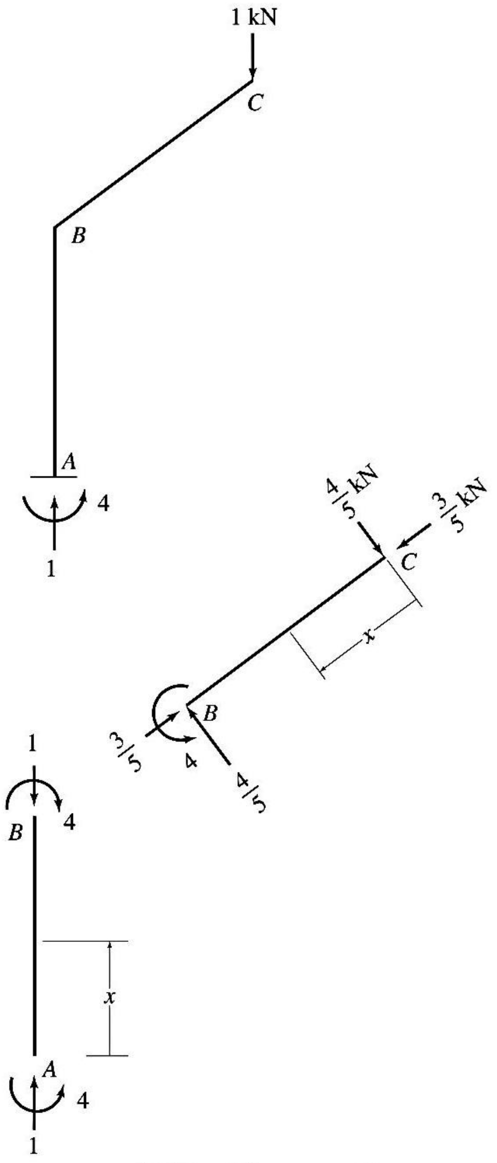
Ex.6. Compute the vertical deflection at joint C of the shown frame



(a)



(b) Real System — M



(c) Virtual System — M_v