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Structural Mechanics (2)

Lecture No-05

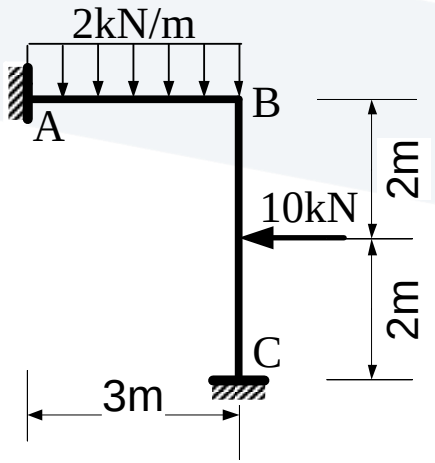
Part-01

Displacement Method for Beams and Frames or Slope-Deflection Method

Displacement Method for Beams and Frames (Slope – Deflection Method)

- Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.
- Analysis of Continuous Beams.
- Analysis of Frames without Sidesway.
- **Analysis of Frames with Sidesway.**

Ex.3: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams. EI is constant.



Solution

1) 1 DKI. The unknown is θ_B .

2) Slope-Deflection Equations:

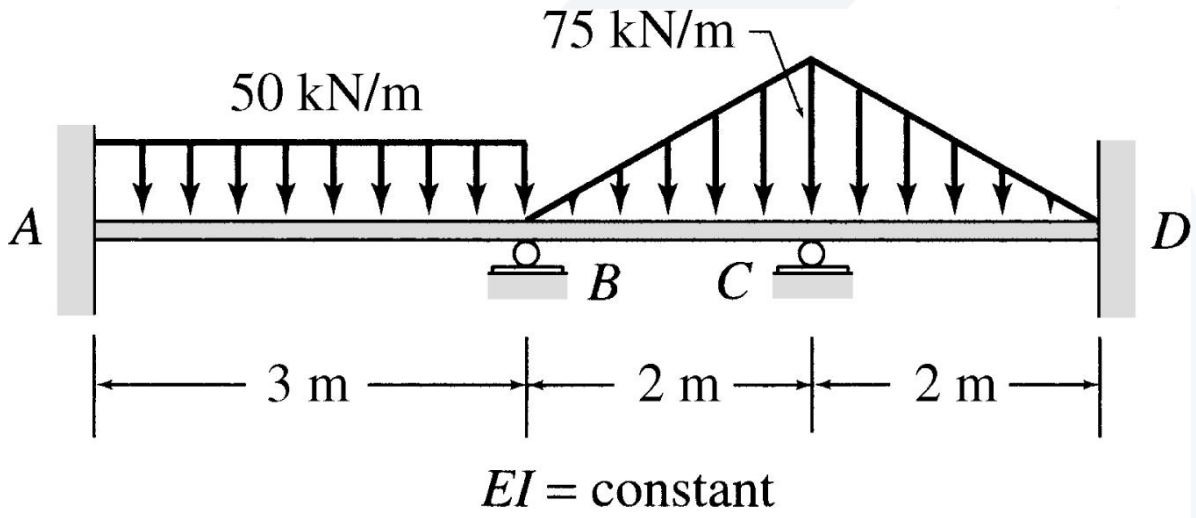
3) Equilibrium Equation:

4) End moments:

5) End shears:

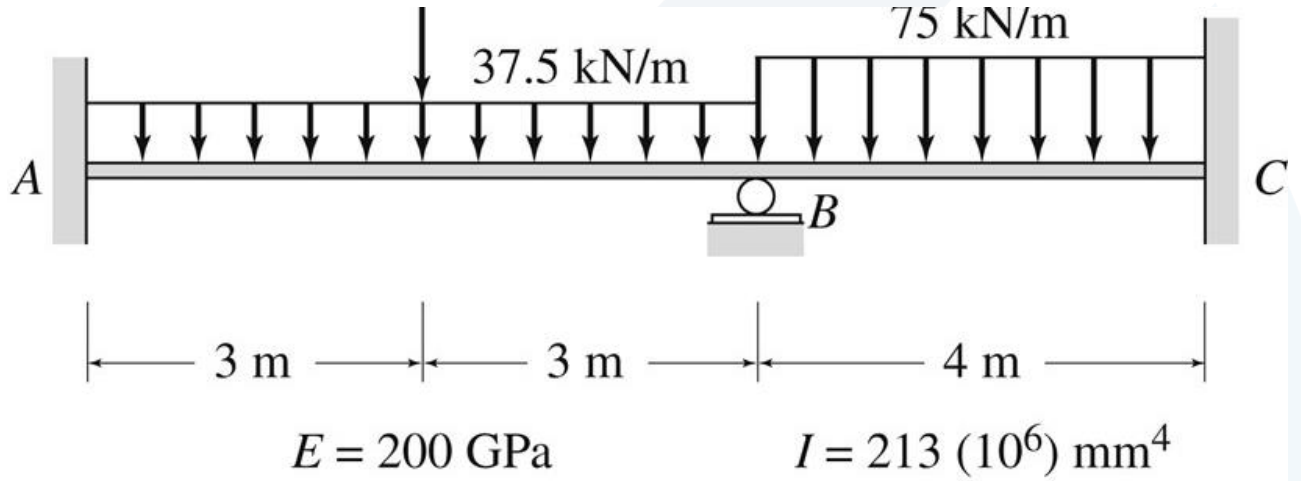
Homework-01

Pr-01: Determine the reactions and draw the shear and bending moment diagrams for the beam shown using slope deflection method.



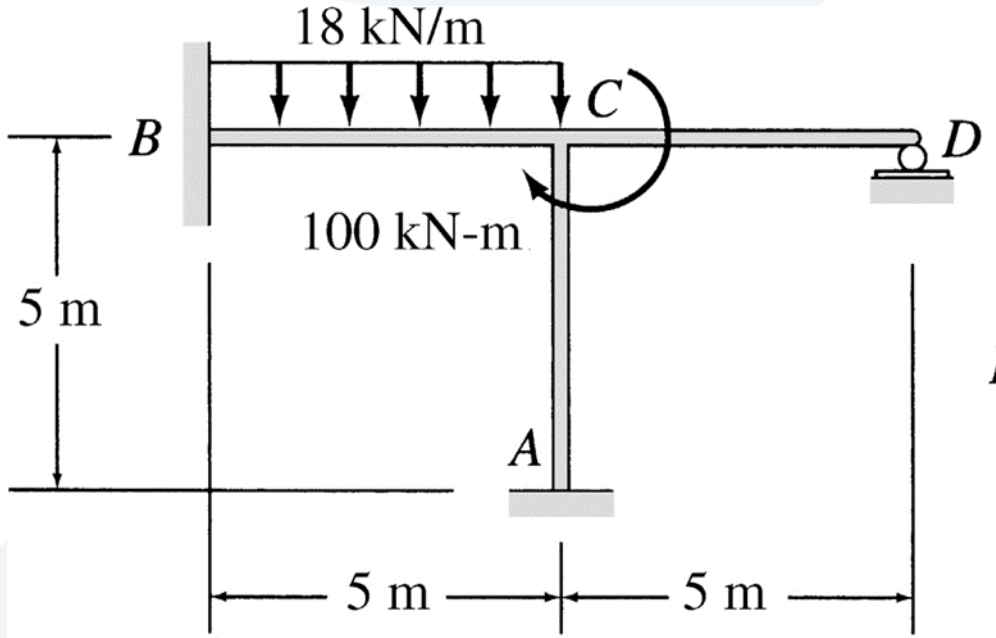
Homework-01

Pr-02: Using slope deflection method, determine the member end moments and the reactions for the beam shown due to the external loading and a settlement of 8 mm at support B, then draw the shear and bending moment diagrams.



Homework-01

Pr-03: Determine the member end moments and the reactions for the frame shown using slope deflection method.



EI = constant
$E = 200$ GPa
$I = 1350 \times 10^6$ mm^4

Structural Mechanics (2)

Lecture No-05

Part-02

Displacement Method for Beams and Frames or Slope-Deflection Method

Displacement Method for Beams and Frames (Slope – Deflection Method)

- Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.
- Analysis of Continuous Beams.
- Analysis of Frames without Sidesway.
- Analysis of Frames with Sidesway.

Beam Displacements

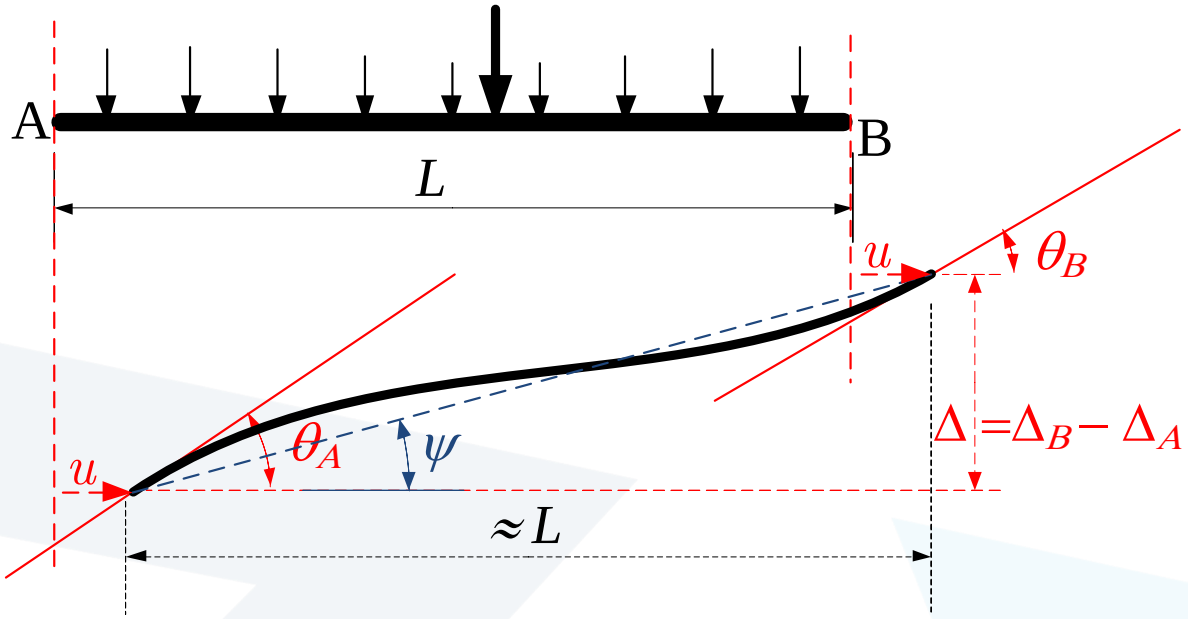
Δ Deflection

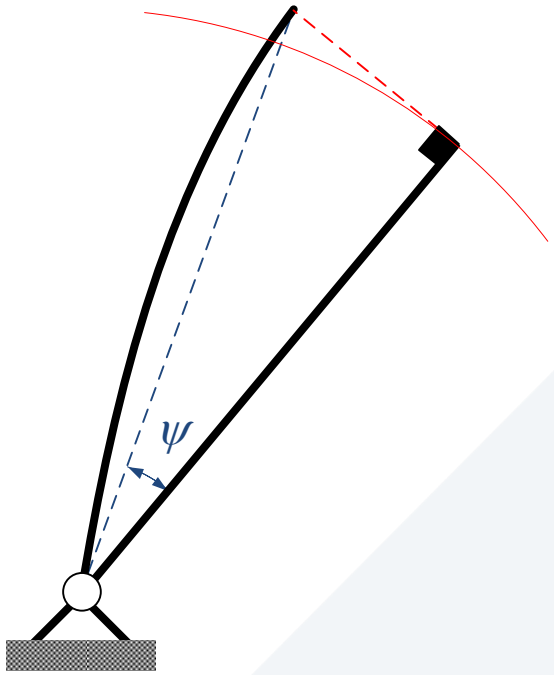
u Axial displacement

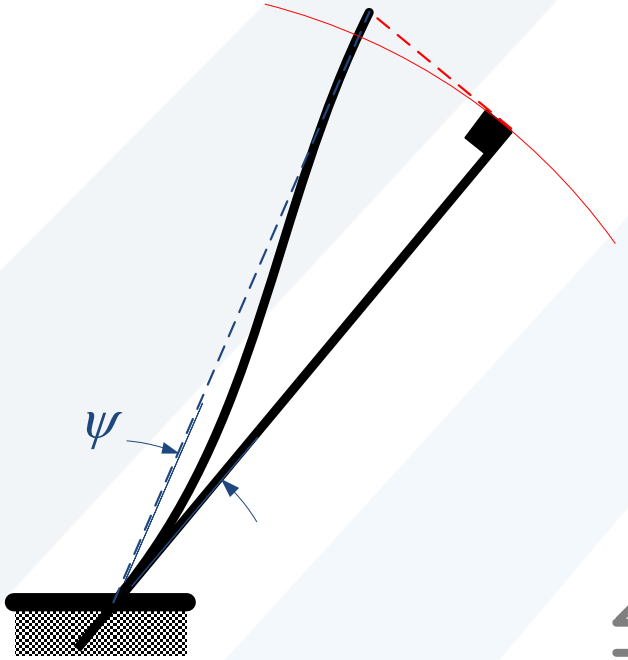
$\theta \ll 1, \sin\theta \approx \theta, \cos\theta \approx 1$

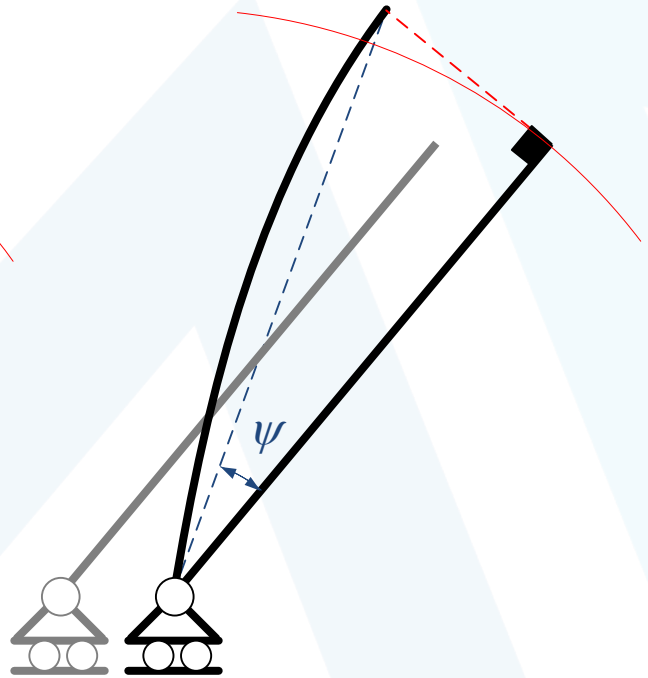
$\tan\psi \approx \sin\psi \approx \Delta/L$

$\psi \ll 1, \sin\psi \approx \psi, \cos\psi \approx 1$

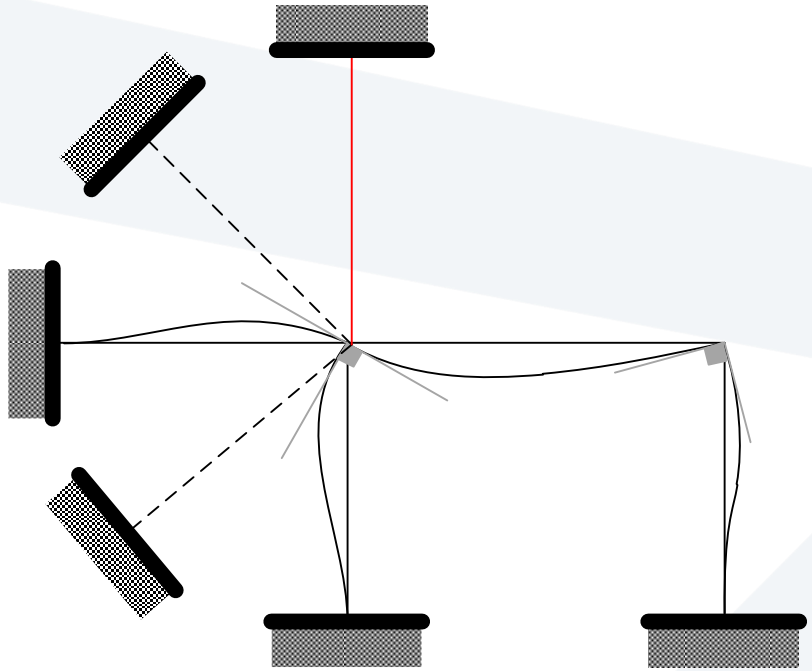




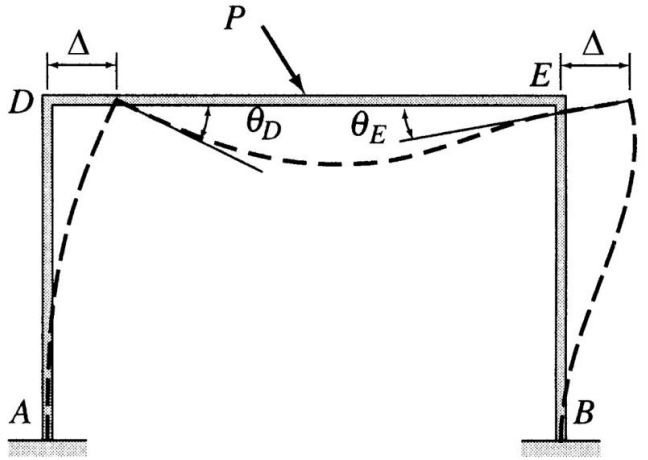




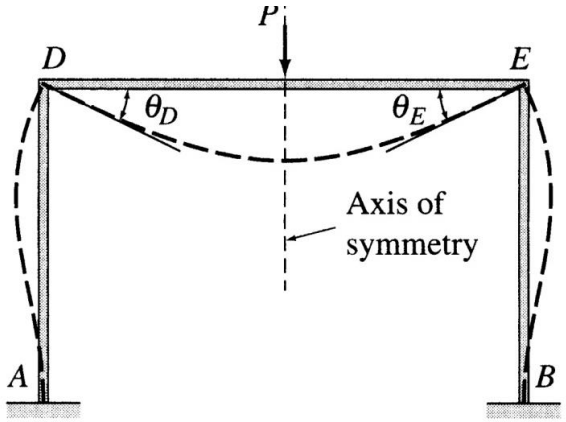
Analysis of Frames with Sidesway



(a) Frame without Sidesway



(b) Frame with Sidesway



EI = constant

(c) Symmetric Frame Subjected to Symmetric Loading — No Sidesway

$$N_{ss} = 2j - [2(f+h) + r + m]$$

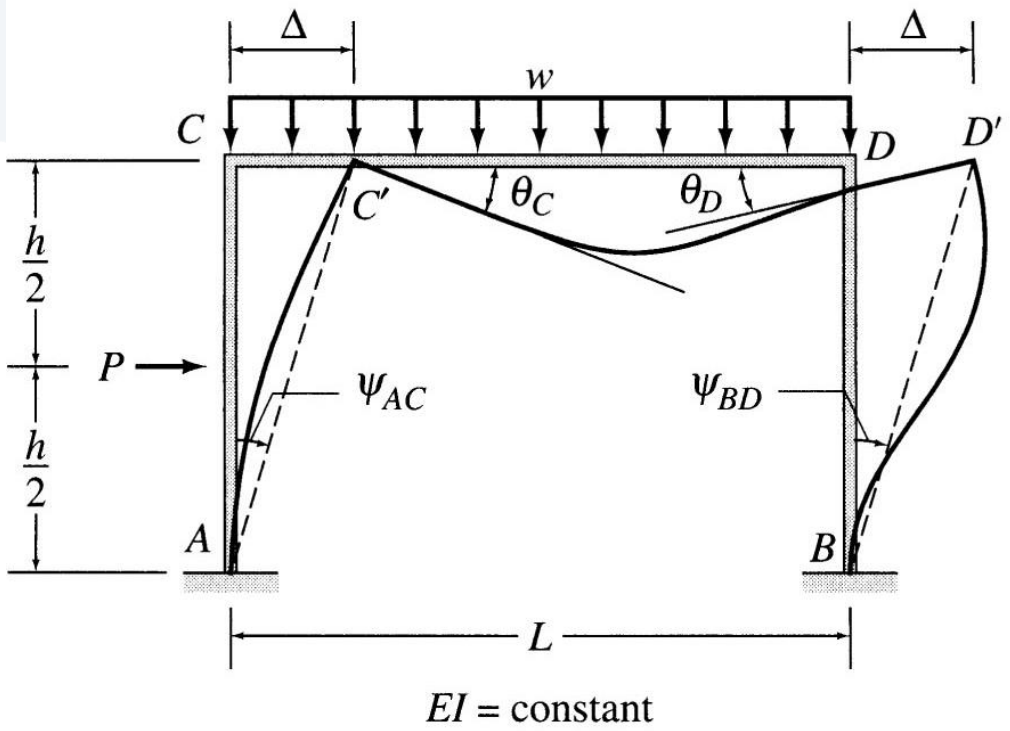
j: Number of joints *f*: Number of fixed supports

h: Number of hinged supports

r: Number of roller supports

m: Number of effective inextensible members

Analysis of Frames with Sidesway



3 kinematic unknowns

θ_C, θ_D & Δ.

$$M_{AC} = \frac{2EI}{h}\left(\theta_C + \frac{3\Delta}{h}\right) + FEM_{AC}$$

$$M_{CA} = \frac{2EI}{h}\left(2\theta_C + \frac{3\Delta}{h}\right) + FEM_{CA}$$

$$M_{BD} = \frac{2EI}{h}\left(\theta_D + \frac{3\Delta}{h}\right)$$

$$M_{DB} = \frac{2EI}{h}\left(2\theta_D + \frac{3\Delta}{h}\right)$$

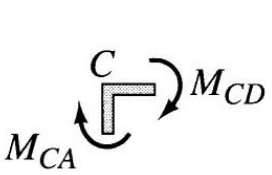
$$M_{CD} = \frac{2EI}{L}\left(2\theta_C + \theta_D\right) + FEM_{CD}$$

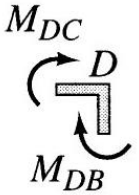
$$M_{DC} = \frac{2EI}{L}\left(2\theta_D + \theta_C\right) + FEM_{DC}$$

Analysis of Frames with Sidesway

$$M_{CA} + M_{CD} = 0 \qquad (1)$$

$$M_{DB} + M_{DC} = 0 \qquad (2)$$





$$P - S_{AC} - S_{BD} = 0$$

$$\downarrow\uparrow + \sum M_C^{AC} = 0$$

$$M_{AC} - S_{AC}(h) + P(h/2) + M_{CA} = 0$$

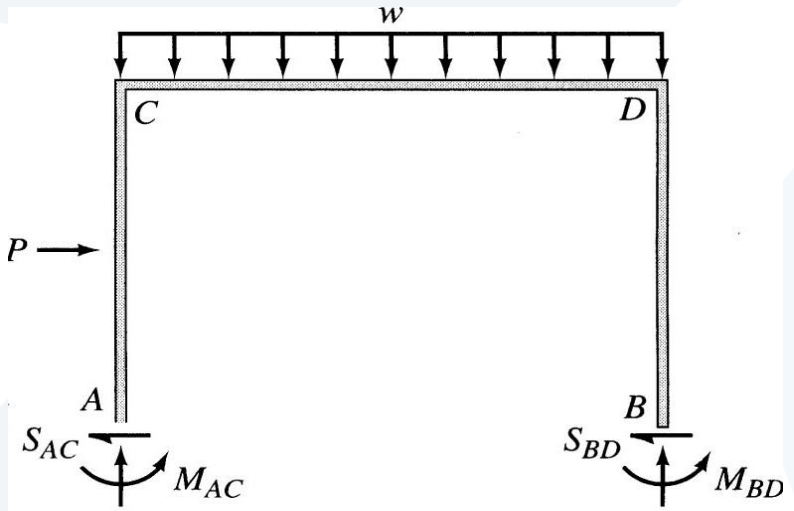
$$S_{AC} = \frac{M_{AC} + M_{CA}}{h} + \frac{P}{2}$$

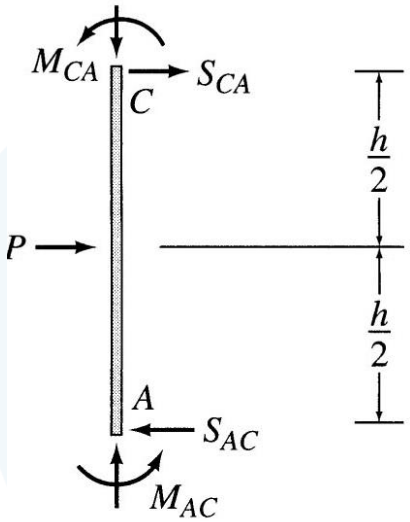
$$\downarrow\uparrow + \sum M_D^{BD} = 0$$

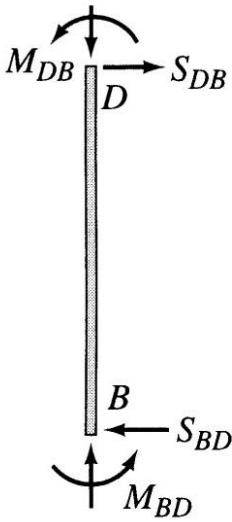
$$M_{BD} - S_{BD}(h) + M_{DB} = 0$$

$$S_{BD} = \frac{M_{BD} + M_{DB}}{h}$$

$$M_{AC} + M_{CA} + M_{BD} + M_{DB} = ph/2 \qquad (3)$$

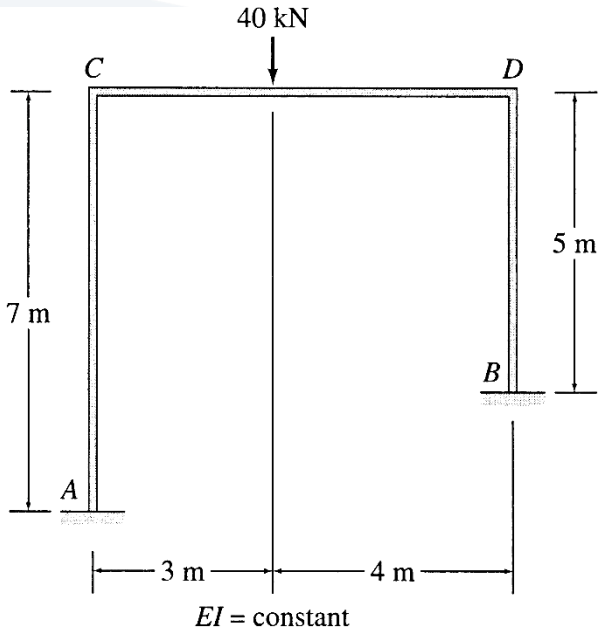




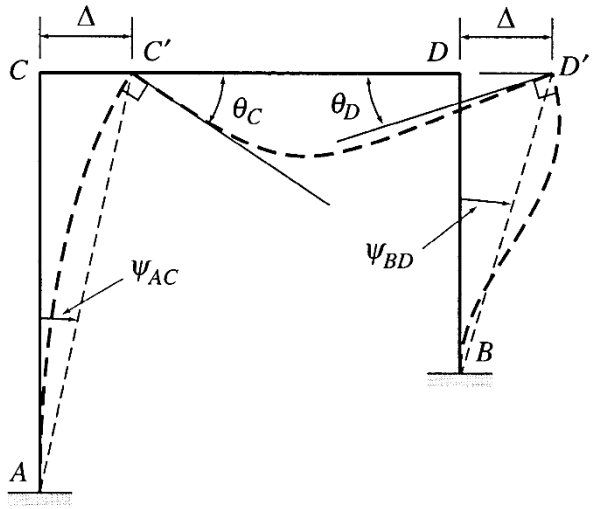


(1), (2) & (3) three equations with three unknowns θ_C, θ_D & Δ.

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



(a) Frame



(b) Qualitative Deflected Shape of the Frame

Fixed-End Moments:

$$FEM_{AC} = FEM_{CA} = FEM_{DB} = FEM_{BD} = 0$$

$$FEM_{CD} = \frac{40(3)(4)^2}{(7)^2} = 39.2\ kN.m \curvearrowleft or + 39.2\ kN.m$$

$$FEM_{DC} = -\frac{40(3)^2(4)}{(7)^2} = 29.4\ kN.m \curvearrowright or - 29.4\ kN.m$$

Degrees of freedom:

$\theta_A = 0$ & $\theta_B = 0$

$\theta_C \neq 0$, $\theta_D \neq 0$, $\Delta \neq 0$

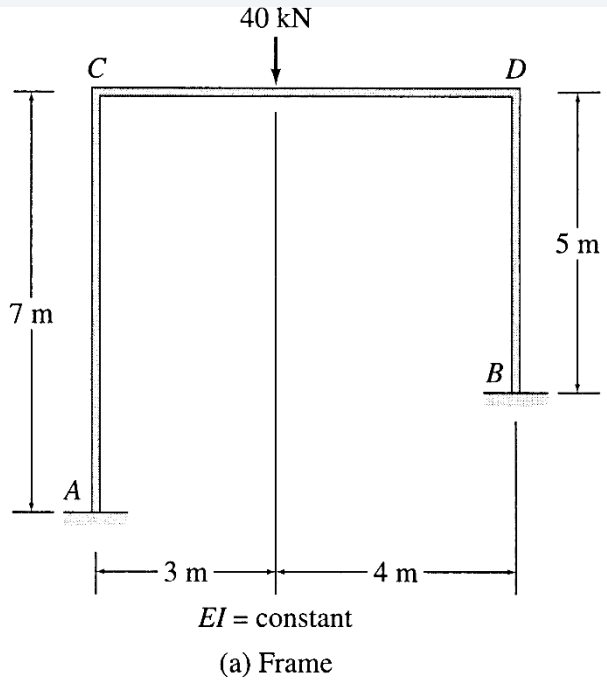
Chord rotations:

$$\psi_{AC} = -\frac{\Delta}{7} \qquad \psi_{BD} = -\frac{\Delta}{5}$$

$$\psi_{BD} = -\frac{\Delta}{5}$$

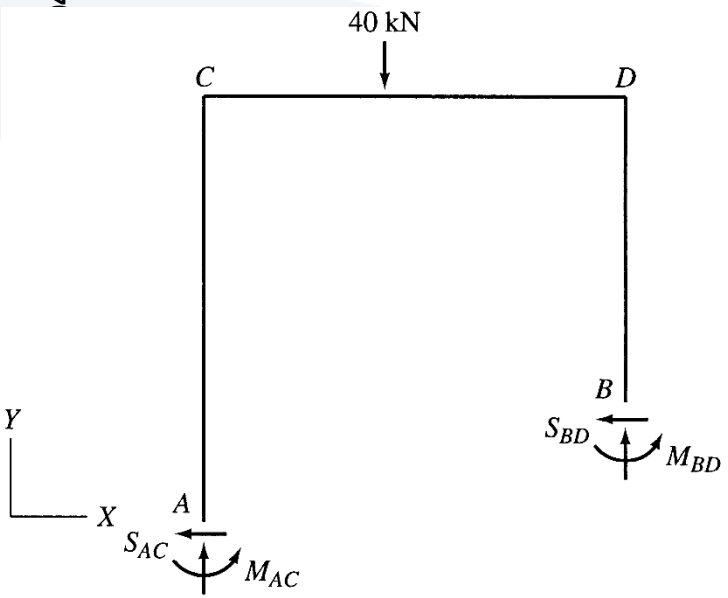
Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method

Slope Deflection Equations:

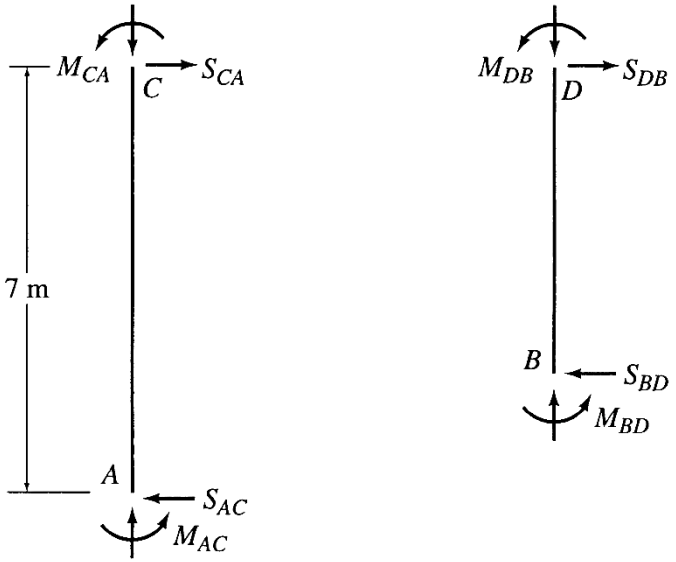


(a) Frame

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



(c) Free-Body Diagram of the Entire Frame



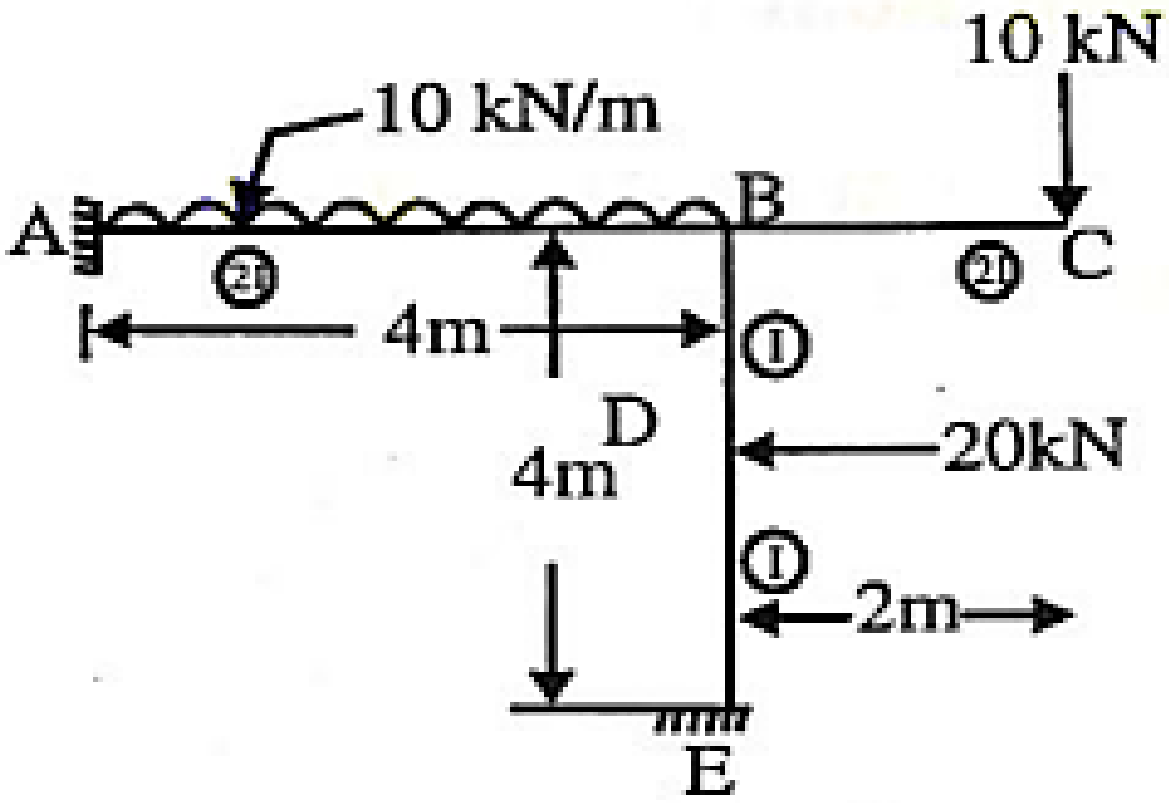
(d) Free-Body Diagrams of Columns *AC* and *BD*

Equilibrium Equations:

Joint Displacements:

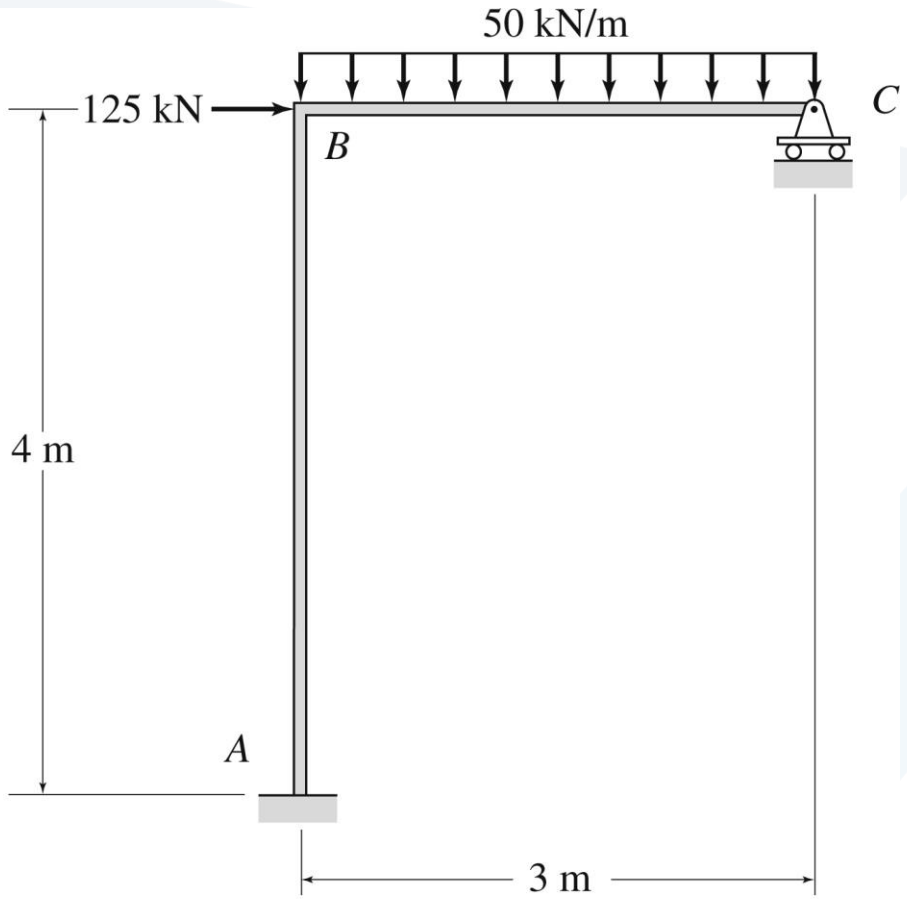
Homework

Pr-01: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams.



Homework

Pr-02: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams.



EI = constant

Homework

Pr-03: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams. (EI=const.)