B. Haidar

Structural Mechanics (1)

Week No-06

Part-01

Analysis of Indeterminate Structures - Force Method

- Determinate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)

STRUCTURES

are classified from the analysis point of view to

DETERMINATE

INDETERMINATE

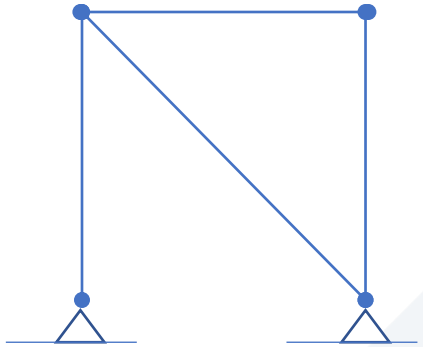
To predict the performance of a structure, its response elements such like sup. reactions, internal forces, stresses, deflections, strain, to external actions Loads, sup. settlement, temp. changes & fabric. errors, must be determined.

Response elements are separable to	Response elements are not separable
Forces: support reactions & internal forces, then stresses, are determined by equilibrium equations	support reactions & internal forces number is greater than the available equilibrium equations
Deformations: deflections & Strains, are determined after knowing the first group	additional relationships based on the geometry of deformation of structures, are needed

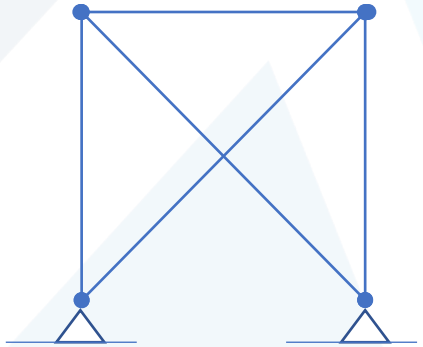
INDETERMINATE STRUCTURES

Advantages	Disadvantages
greater overall factor of safety	more sensitive to secondary effects
1. Smaller Stresses 2. Greater Stiffness 3. Redundancies	1. Fabrication errors 2. Temperature changes 3. Support settlements

Statically Indeterminate Structures - Disadvantages



Statically Determinate



Statically Indeterminate

Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

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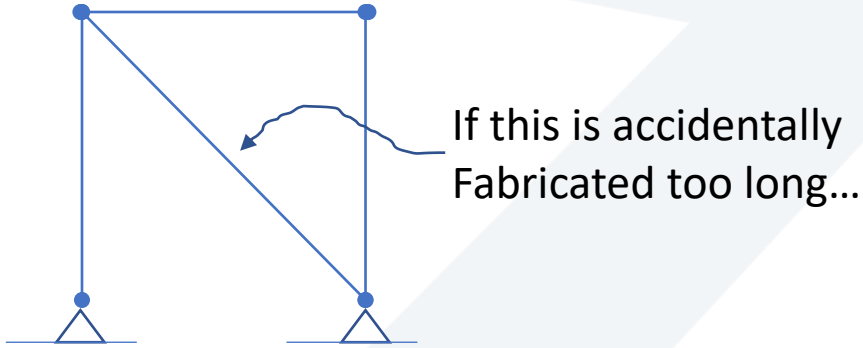

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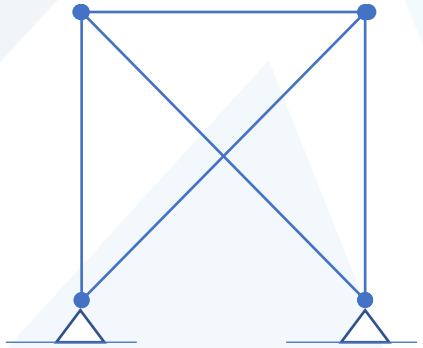

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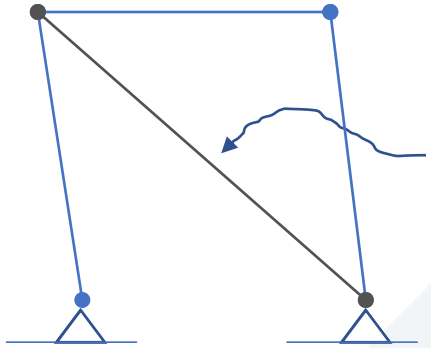
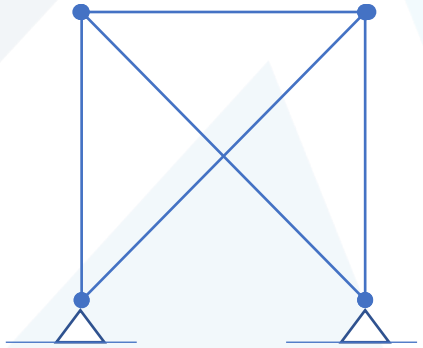


If this is accidentally
Fabricated too long…
All joints just move to
New positions



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Statically Indeterminate Structures - Disadvantages

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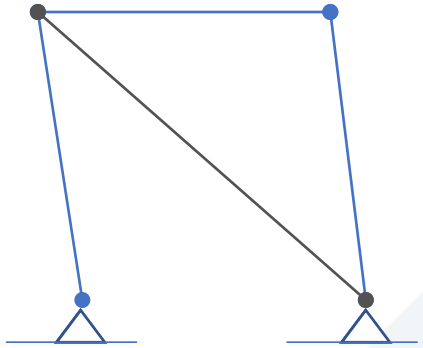

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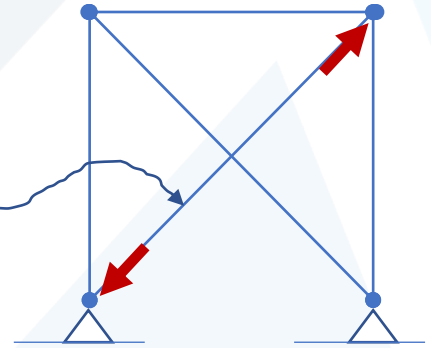

Statically Determinate

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Statically Indeterminate Structures - Disadvantages

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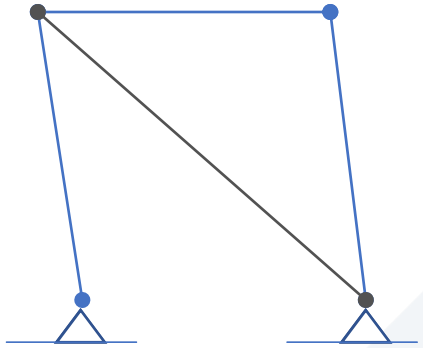

If this is accidentally
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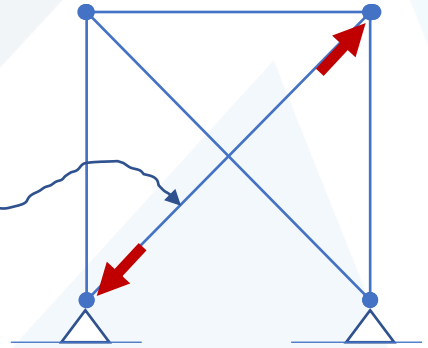
Then it must be
Compressed into place



Statically Determinate

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Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

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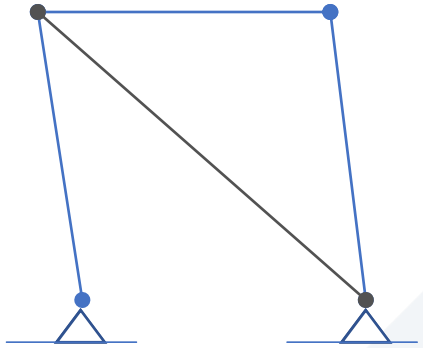

Statically Determinate

If this is accidentally
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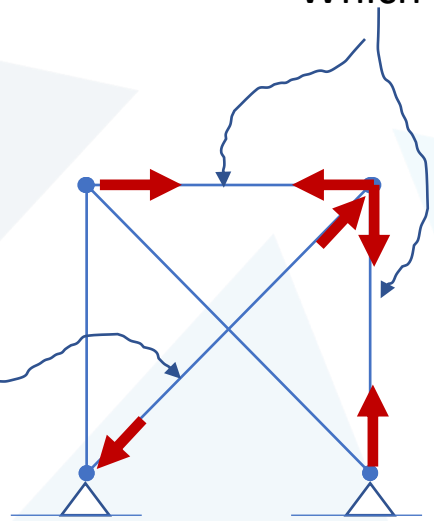
Compressed into place

Which puts these in tension



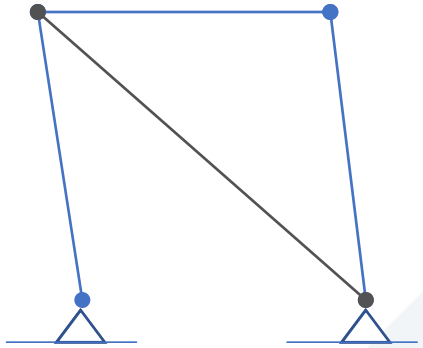
Statically Indeterminate

Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

1. Fabrication errors:

Which puts forces on all members



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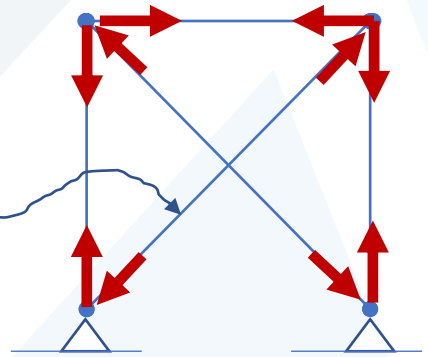

Statically Determinate

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Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

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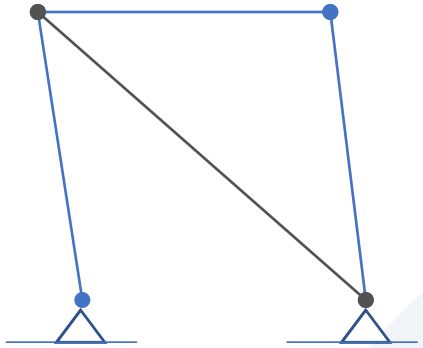
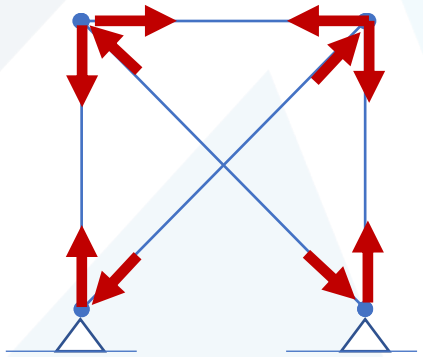


No Stresses. Members go together.

Statically Determinate



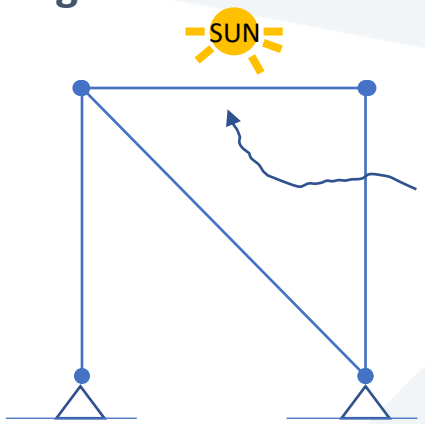
Members must be forced to fit.

Statically Indeterminate

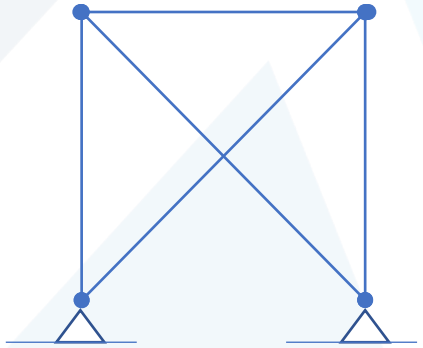
Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

2. Temperature Changes:



Statically Determinate



Statically Indeterminate

Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

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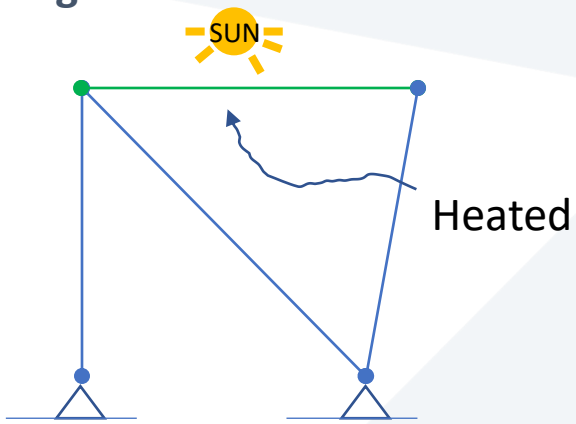
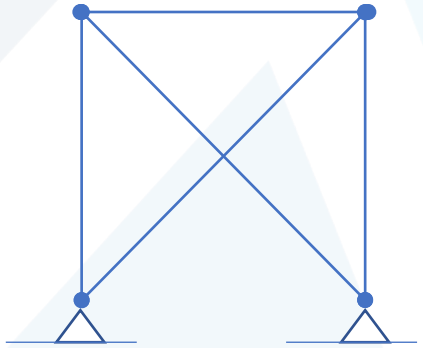


No Stresses. Positions just change

Statically Determinate



Statically Indeterminate

Statically Indeterminate Structures - Disadvantages

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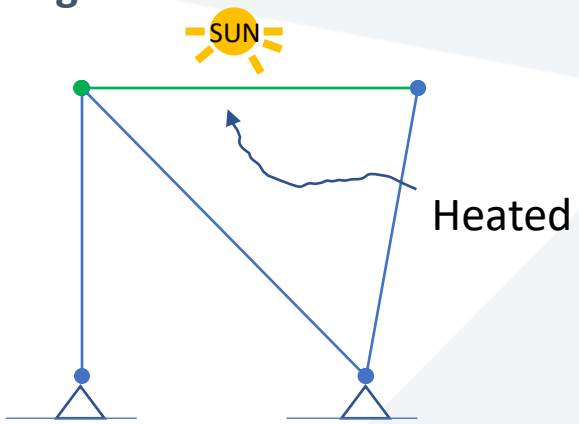
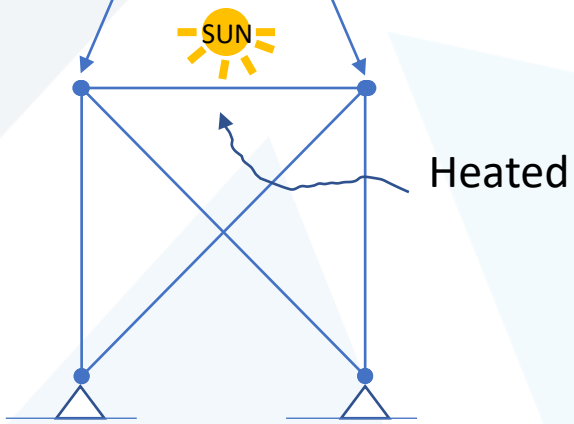


No Stresses. Positions just change

Statically Determinate

Joints restricted by truss-action



Statically Indeterminate

Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

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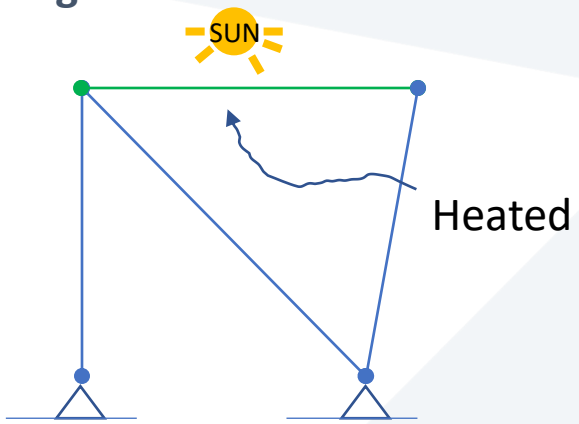

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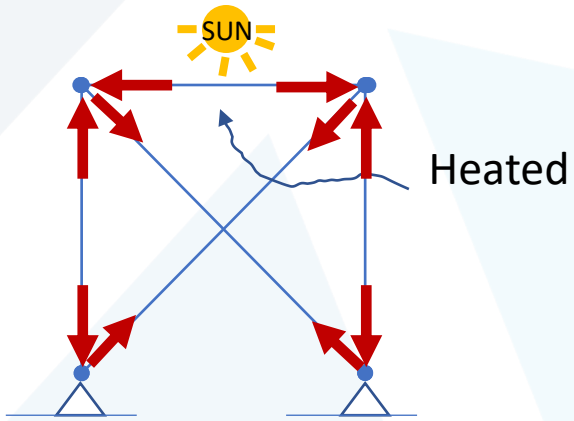


Thermal causes stresses

Statically Indeterminate

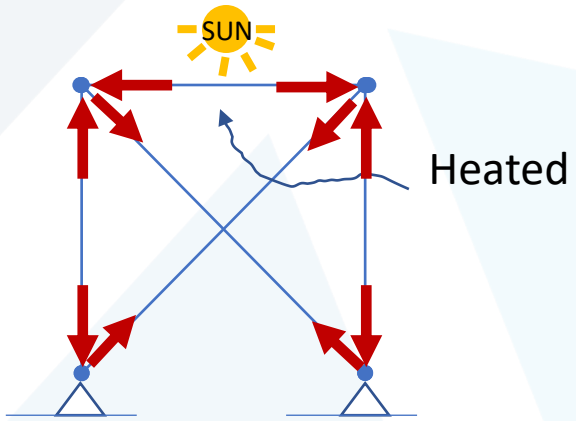
Statically Indeterminate Structures - Disadvantages

2.

If Expansion Joint does not
Allow expansion, this duct
Is statically indeterminate

d

e



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Statically Indeterminate Structures - Disadvantages

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d

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Statically Indeterminate Structures - Disadvantages

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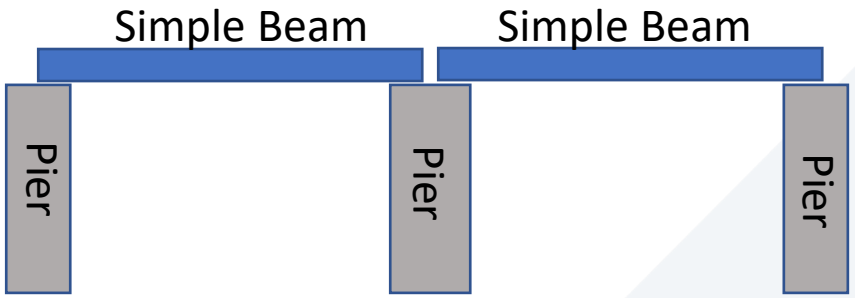

Statically Determinate

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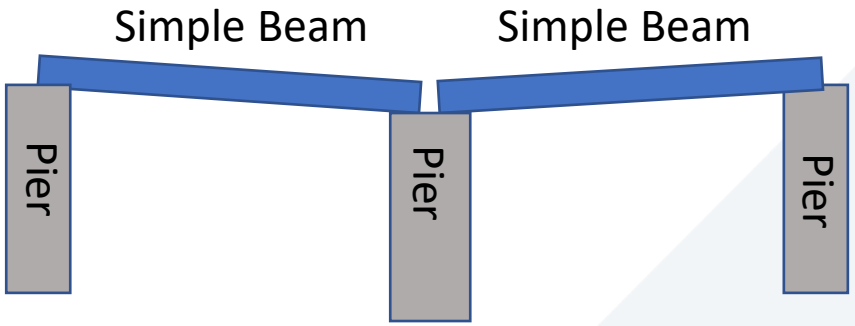
Statically Indeterminate Structures - Disadvantages

Geometric changes cause indirect stresses

3. Foundation Settlement:



Any Settlement: No Curvature. No moment

Statically Determinate

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Statically Indeterminate Structures - Disadvantages

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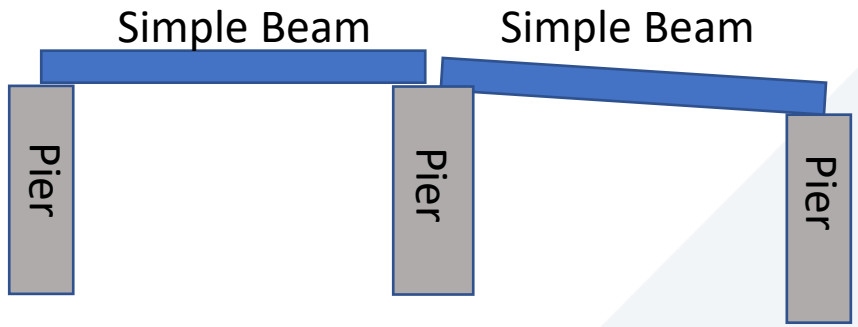

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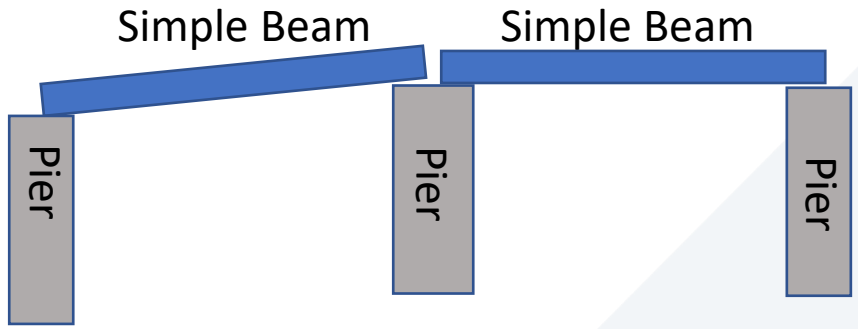

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Statically Indeterminate Structures - Disadvantages

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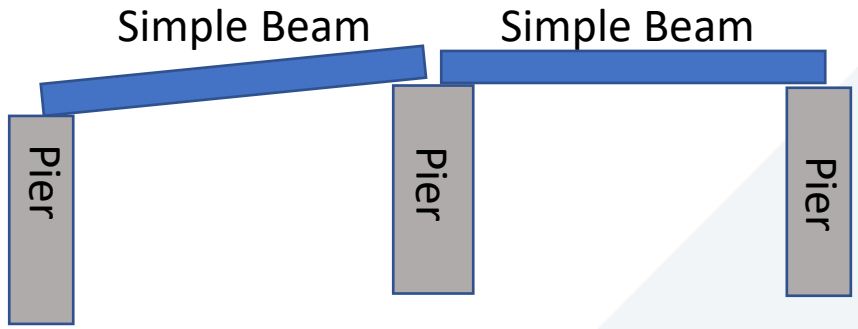

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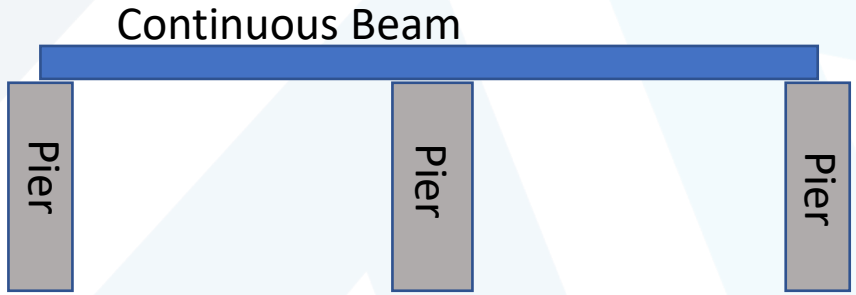

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Statically Indeterminate Structures - Disadvantages

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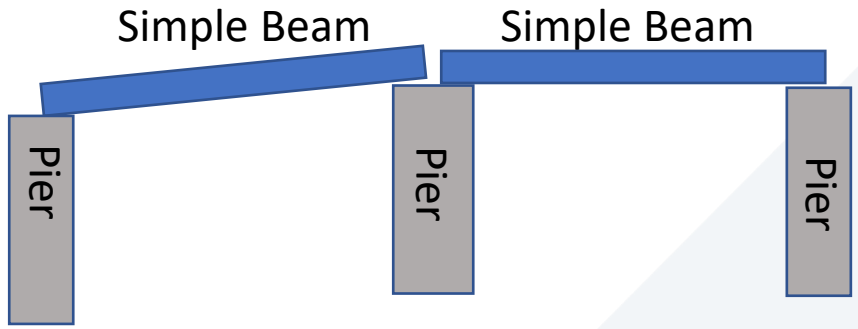
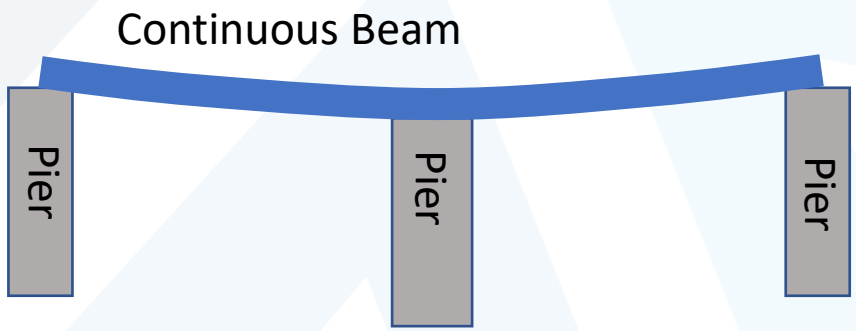


Any Settlement: : No Curvature. No moment

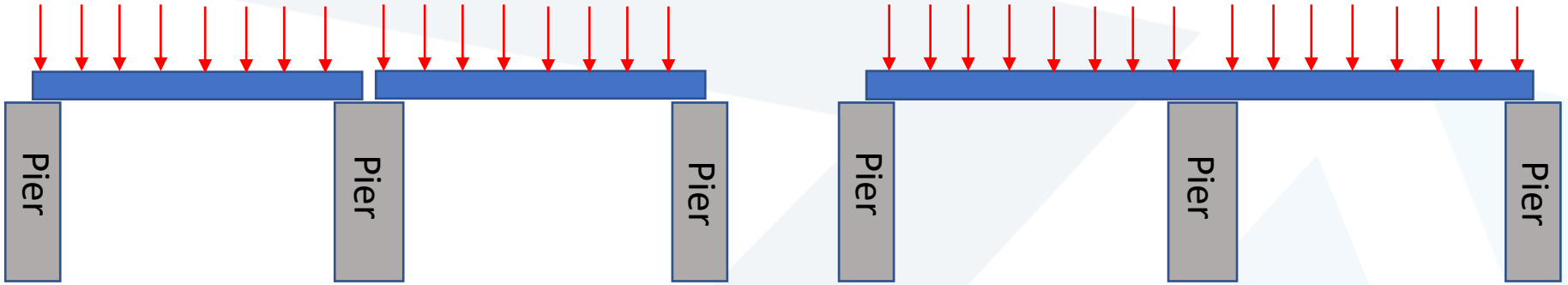
Statically Determinate



Any Settlement: : Curvature! Moment!

Statically Indeterminate

Statically Indeterminate Structures - Advantages



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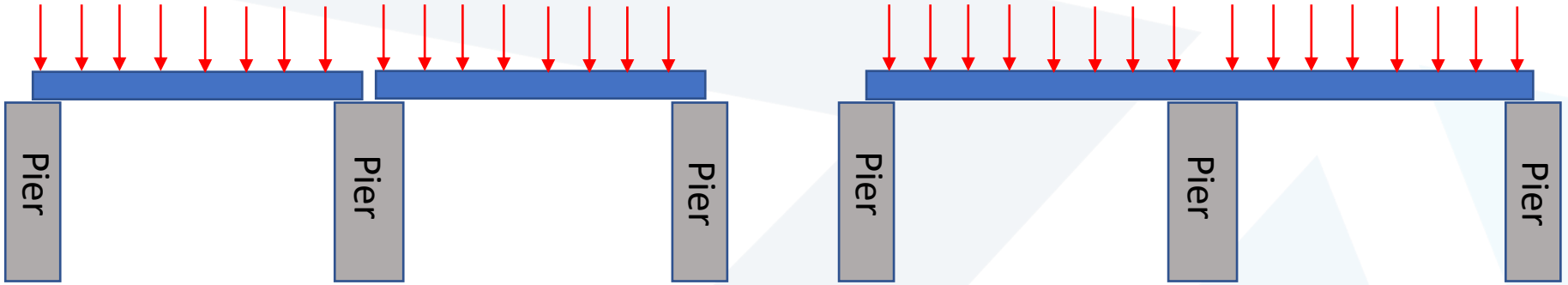
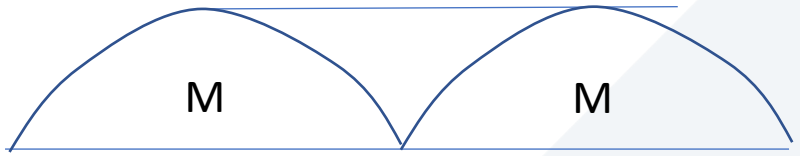


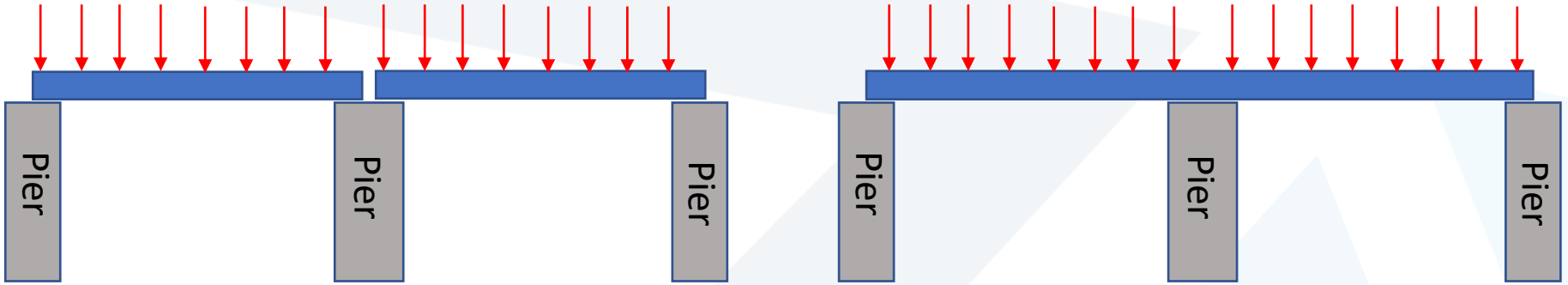


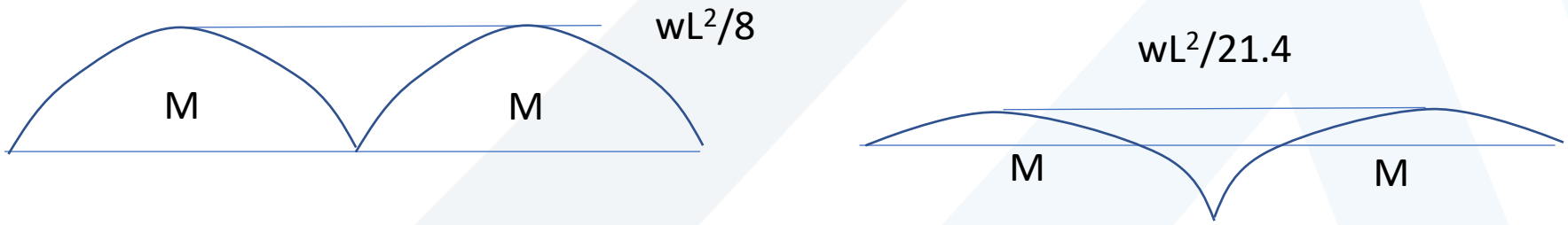
Statically Determinate

Statically Indeterminate

Statically Indeterminate Structures - Advantages

1. Lower Stresses:





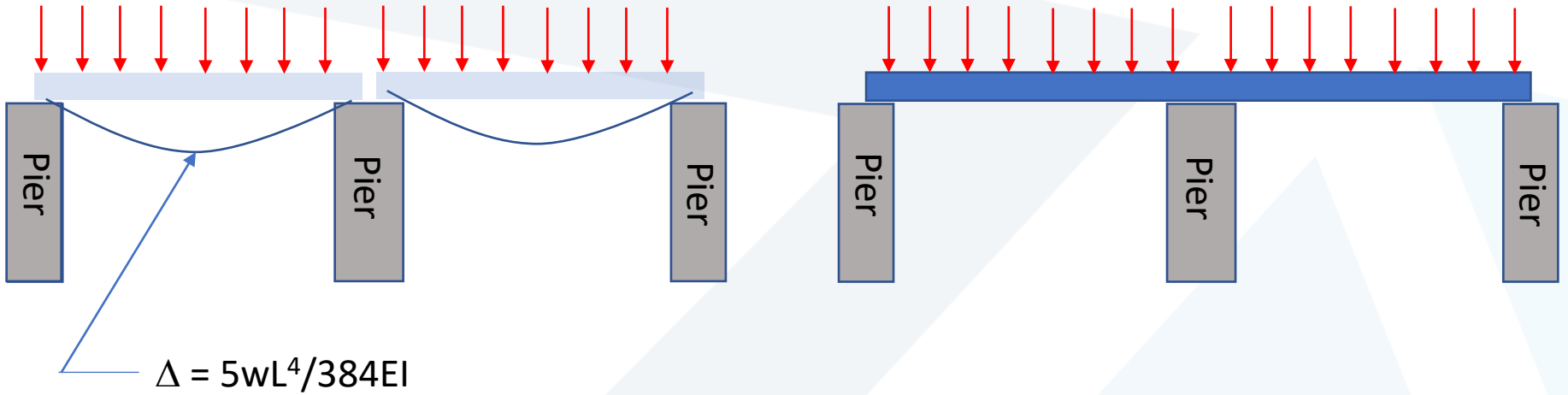
Statically Determinate

Statically Indeterminate

Statically Indeterminate Structures - Advantages

B. Haidar

2. Greater Stiffness:



Statically Determinate

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Statically Indeterminate Structures - Advantages

B. Haidar

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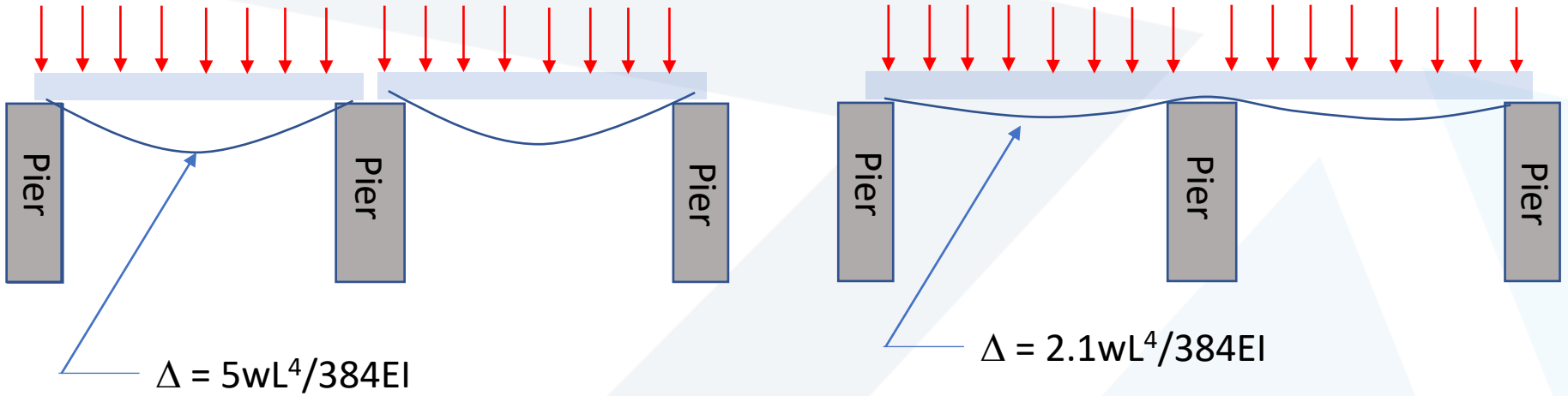

Statically Determinate

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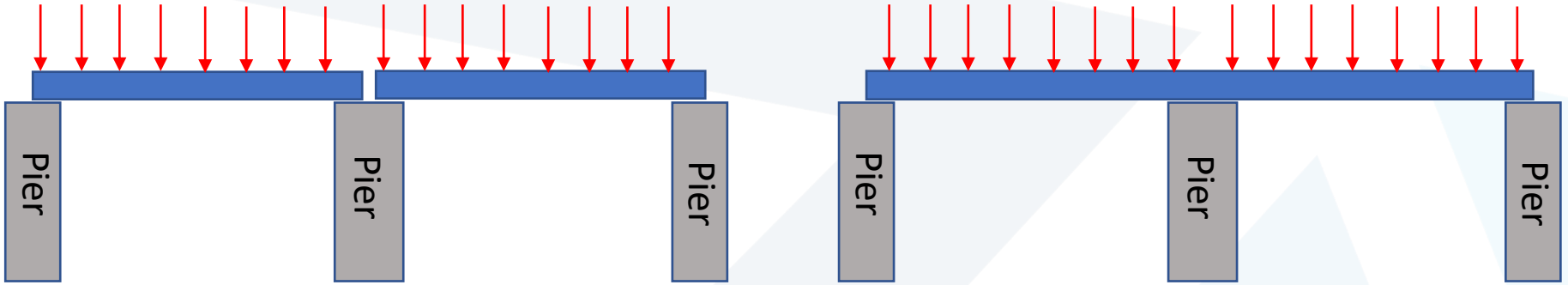
Statically Indeterminate Structures - Advantages



A catastrophic event eliminates the middle pier

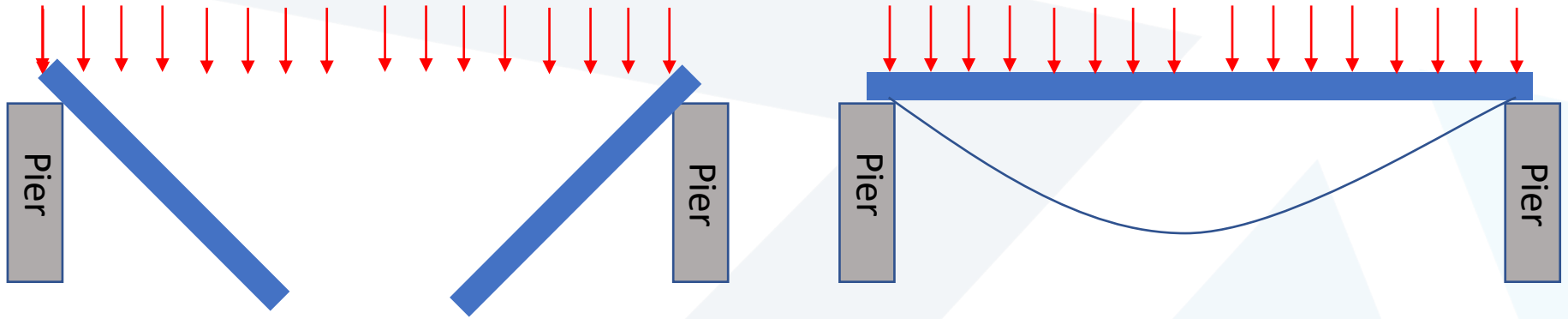
Statically Determinate: $DOI_s = 0$

Statically Indeterminate: $DOI_s = 1$

Statically Indeterminate Structures - Advantages

B. Haidar

2. Greater Robustness: – May survive redundant loss of massive overload



Will NOT Survive
(zero redundancy)

MAY Survive
(if the beam can take a
Substantial increase in moment)

A catastrophic event eliminates the middle pier

Statically Determinate: $DOI_s = 0$

Statically Indeterminate: $DOI_s = 1$

ANALYSIS OF INDETERMINATE STRUCTURES

Three Pillars of Mechanics

Equilibrium Equations	Constitutive Equations or Behavior Lows	Compatibility Equations

the response elements
(support reactions, internal forces, stresses, deflections, strain)
are classified into primary & secondary unknowns

Force methods

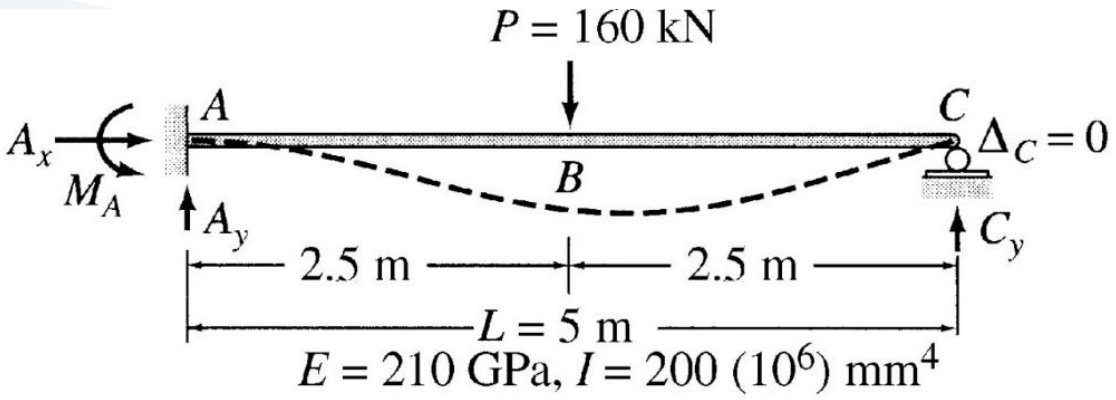
Primary unknowns are reactions & Internal forces
Secondary unknowns are displacements: deflections & rotations (slopes)

Displacement methods

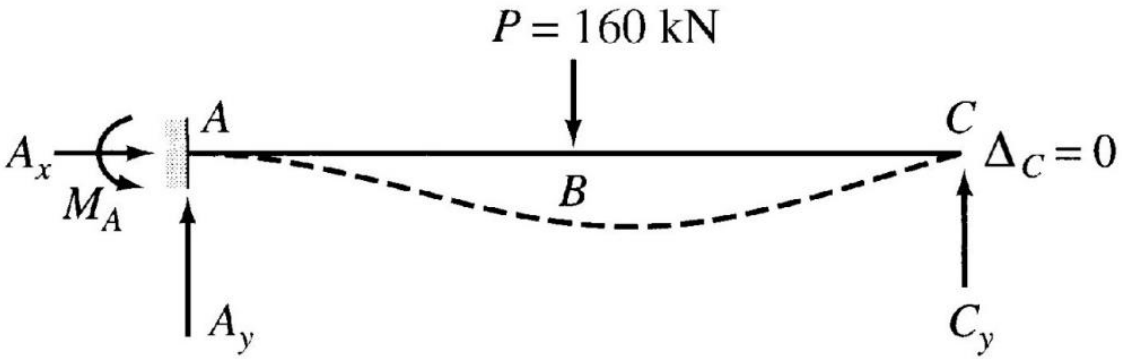
Primary unknowns are displacements: deflections & rotations (slopes)
Secondary unknowns are reactions & Internal forces

Force method: Beam with a single Degree of Indeterminacy

illustrative example



(a) Indeterminate Beam

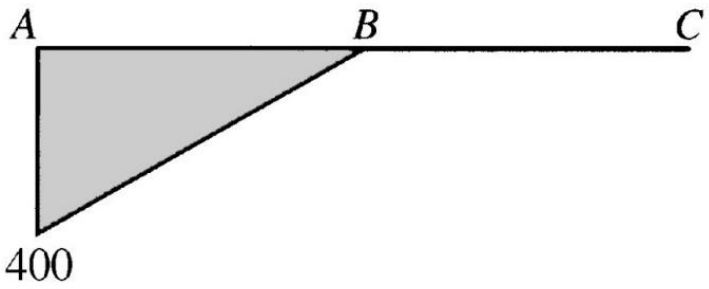


(b) Primary Beam Subjected to External Loading and Redundant C_y

Force method: Beam with a single Degree of Indeterminacy

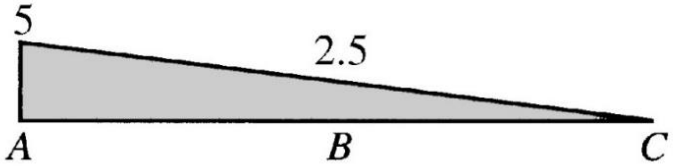
Compatibility Equation

$$\Delta_C = \Delta_{CO} + f_{CC} C_y = 0$$

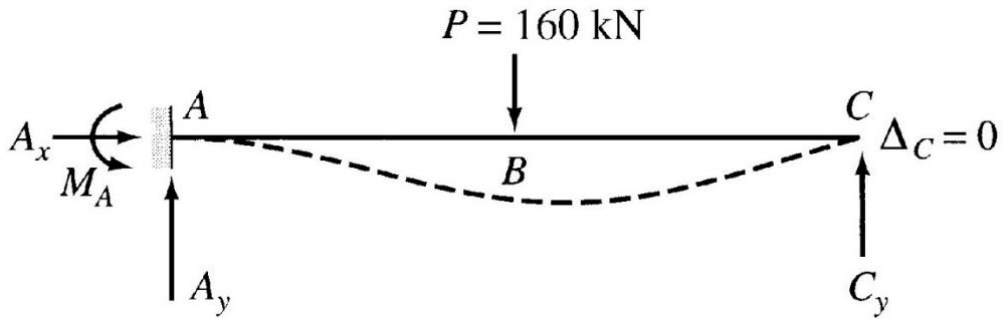


Bending Moment Diagram for Primary Beam Due to External Loading (kN-m)

+



Bending Moment Diagram for Primary Beam Due to Unit Value of C_y (kN-m/kN



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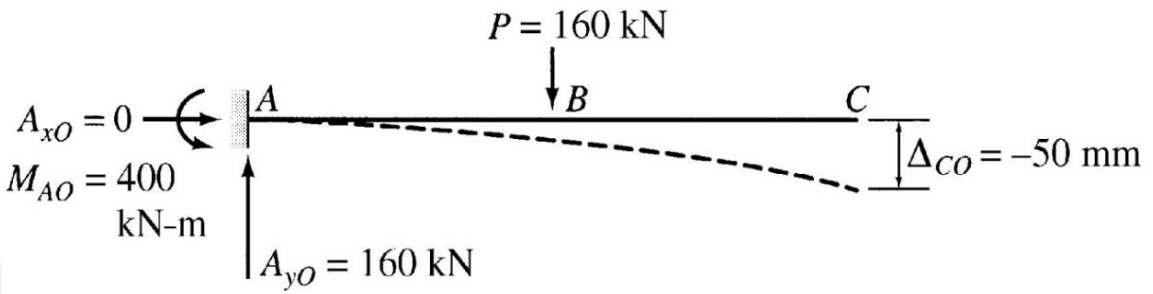
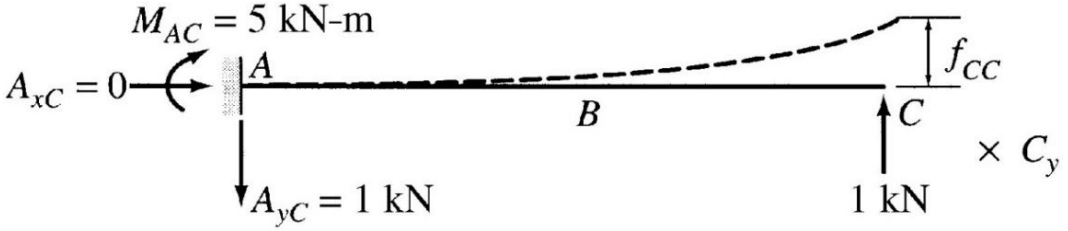


(c) Primary Beam Subjected to External Loading

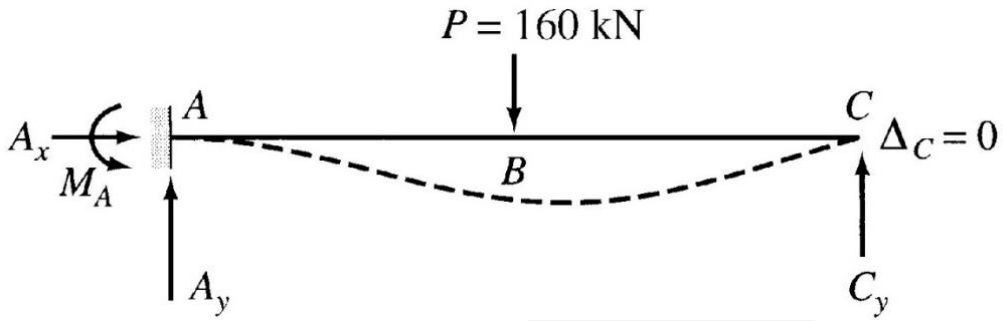
+



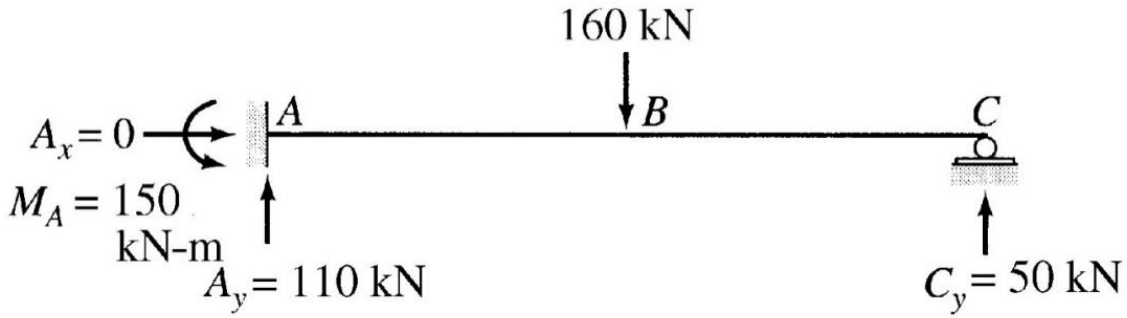
(d) Primary Beam Loaded with Redundant C_y

Force method: Beam with a single Degree of Indeterminacy

Compatibility Equation



$$C_y = -\Delta_{CO} / f_{CC} = 5P/16 = 50 \text{ kN}$$

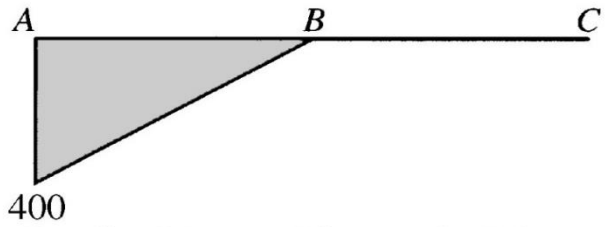


(e) Support Reactions for Indeterminate Beam

By the method of section & using the equilibrium equations, the BM & SF diagrams can be found

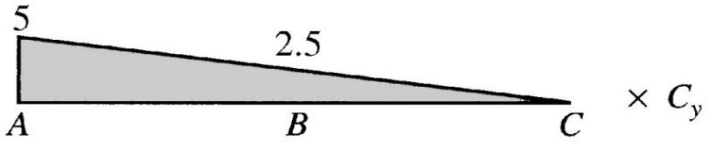
Force method: Beam with a single Degree of Indeterminacy

Or, the BM diagram can be found using the superposition principle as



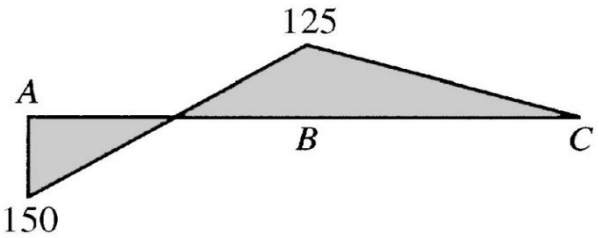
Bending Moment Diagram for Primary Beam Due to External Loading (kN-m)

+



Bending Moment Diagram for Primary Beam Due to Unit Value of C_y (kN-m/kN)

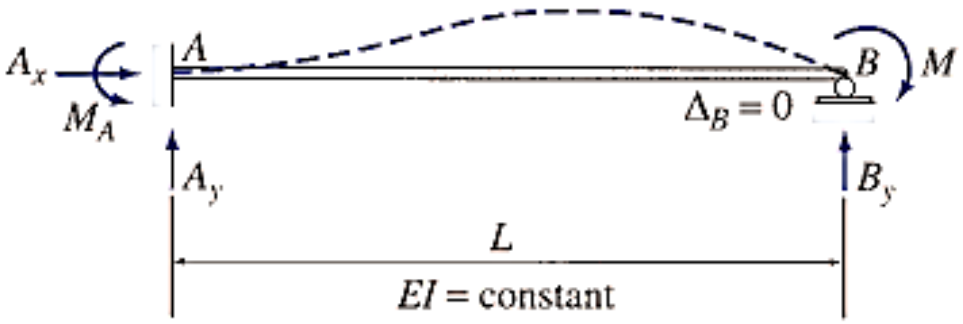
=



(f) Bending Moment Diagram for Indeterminate Beam (kN-m)

Force method: Beam with a single Degree of Indeterminacy

Example-01: Compute the support then draw the BM & SF diagrams for the following beam.

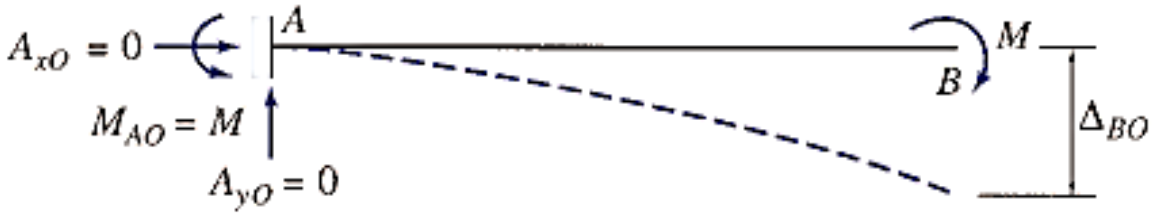


(a) Indeterminate Beam

SOLUTION: The beam is statically indeterminate to degree one. Select B_y as the redundant. Draw the two determinate frames (S_0) & (S_1)

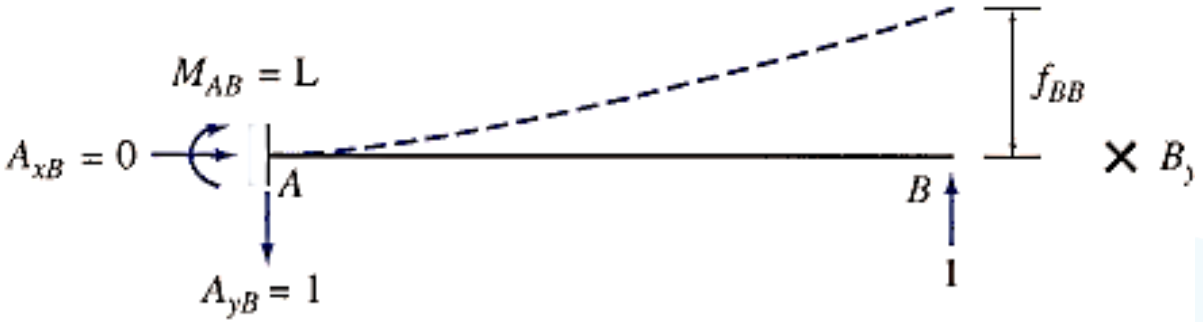
Force method: Beam with a single Degree of Indeterminacy

Example-01: Compute the support then draw the BM & SF diagrams for the following beam.



(b) Primary Beam Subjected to External Moment M

S_0



(c) Primary Beam Loaded with Redundant B_y

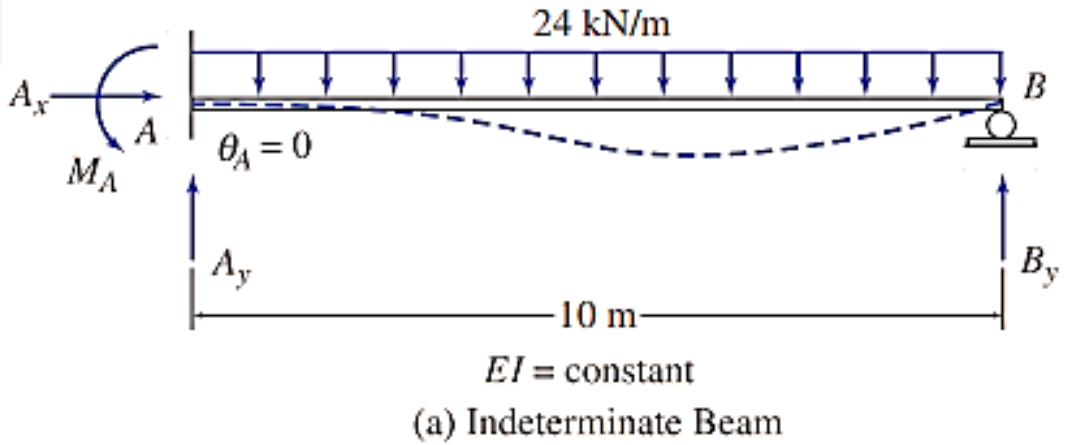
S_1

The compatibility equation is

$$\Delta_{0B} + B_y f_{BB} = 0$$

Force method: Beam with a single Degree of Indeterminacy

Example-02: Compute the support then draw the BM & SF diagrams for the following beam.



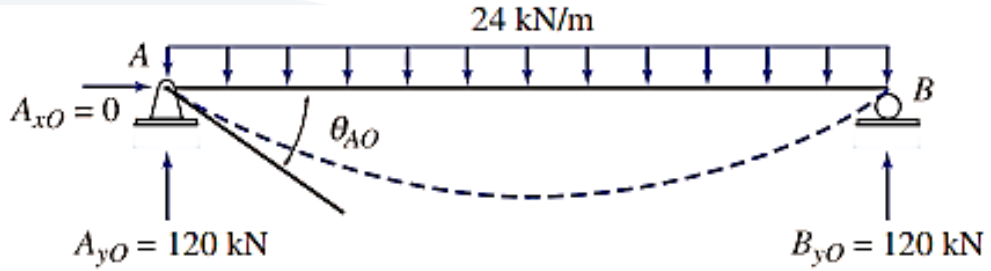
(a) Indeterminate Beam

SOLUTION: The beam is statically indeterminate to degree one. Select M_A as the redundant. Draw the two determinate frames (S_0) & (S_1)

Force method: Beam with a single Degree of Indeterminacy

Example-02: Compute the support then draw the BM & SF diagrams for the following beam.

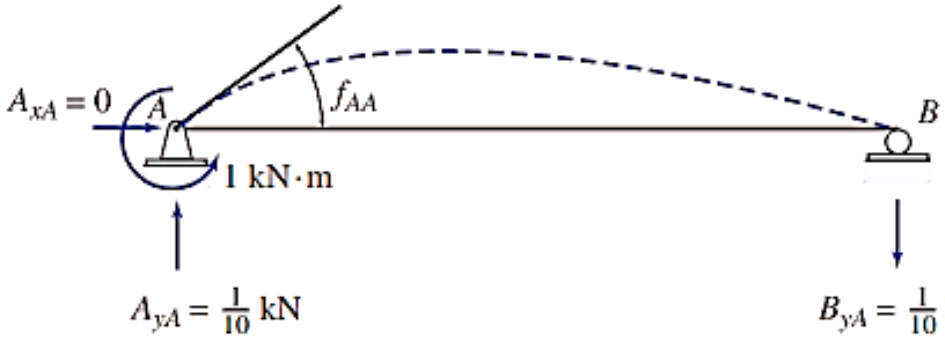
S_0



(b) Primary Beam Subjected to External Loading

+

S_1

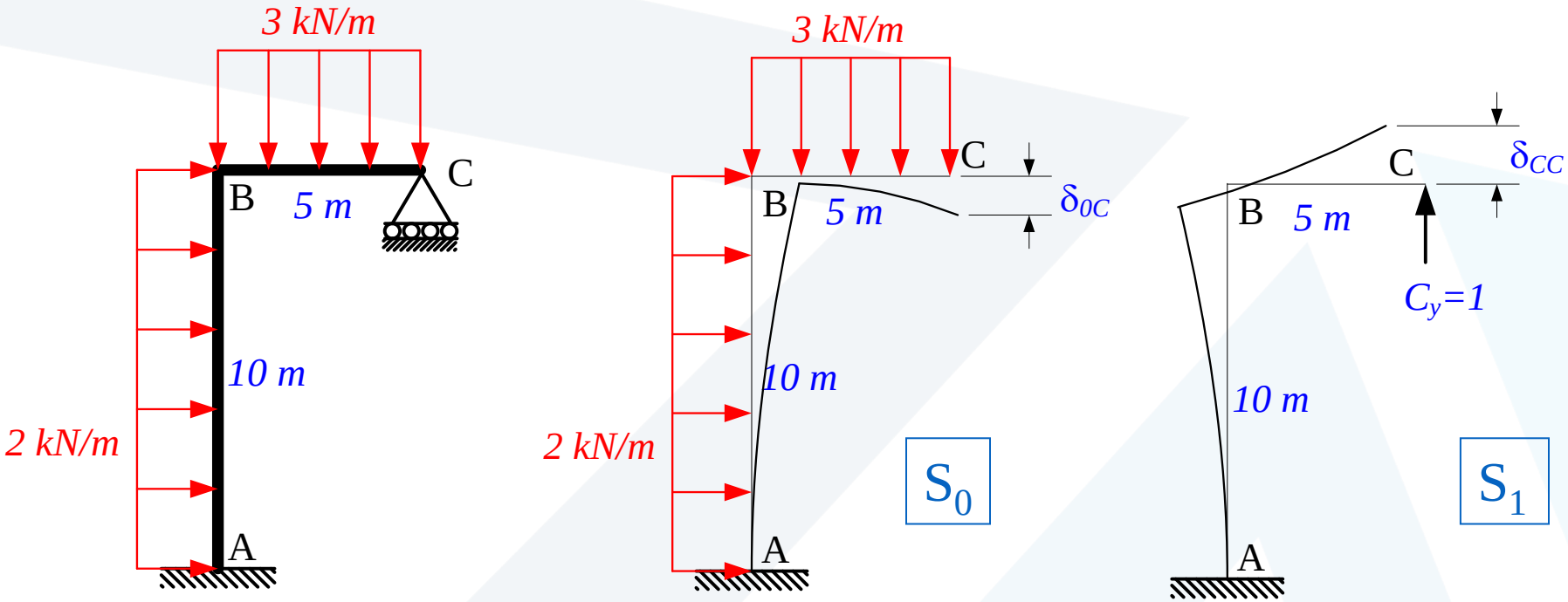


× M_A

(c) Primary Beam Loaded with Redundant M_A

Force method: Frame with a single Degree of Indeterminacy

Compute the support reactions in the frame. E= 200 GPa, I_c= 10^6 mm^4 for the column & I_b=$2I_c$, for the beam. Then draw the BM & SF diagrams

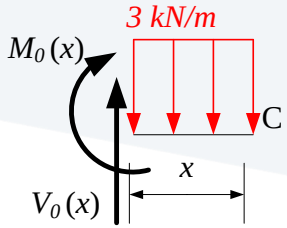
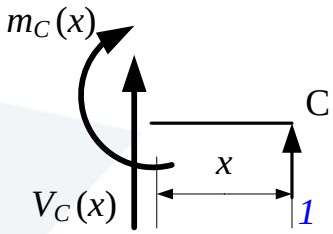
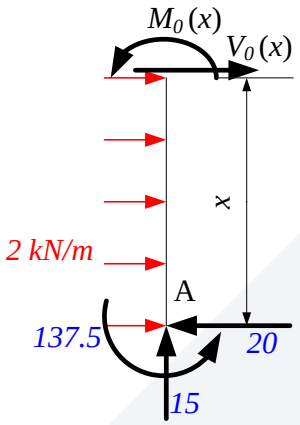
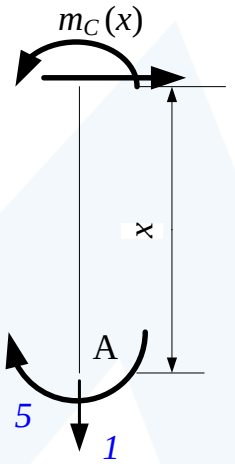


SOLUTION: The frame is statically indeterminate to degree one. Select C_y as the redundant. Draw the two determinate frames (S_0) & (S_1)

The compatibility equation is

$$\delta_{0C} + C_y \delta_{CC} = 0$$

Force method: Frame with a single Degree of Indeterminacy

Segment	FBD in S_0	$M_0(x)$	FBD in S_1	$m_C(x)$
CB $0 \le x \le 5$ $2EI$ is constant		$-3x^2/2$		x
AB $0 \le x \le 10$ EI is constant		$-x^2+20x-137.5$		5

$$\delta_{0C} = \frac{1}{2EI}\int_0^5 \left(-\frac{3x^2}{2}\right)(x)dx + \frac{1}{EI}\int_0^{10}(-x^2+20x-137.5)(5)dx = -\frac{234.375}{2EI} - \frac{3541.67}{EI} = -\frac{3658.85}{EI}$$

$$\delta_{CC} = \frac{1}{2EI}\int_0^5 x^2 dx + \frac{1}{EI}\int_0^{10} 25dx = \frac{270.833}{EI} \qquad \Rightarrow C_y = 13.5\,\text{kN}$$

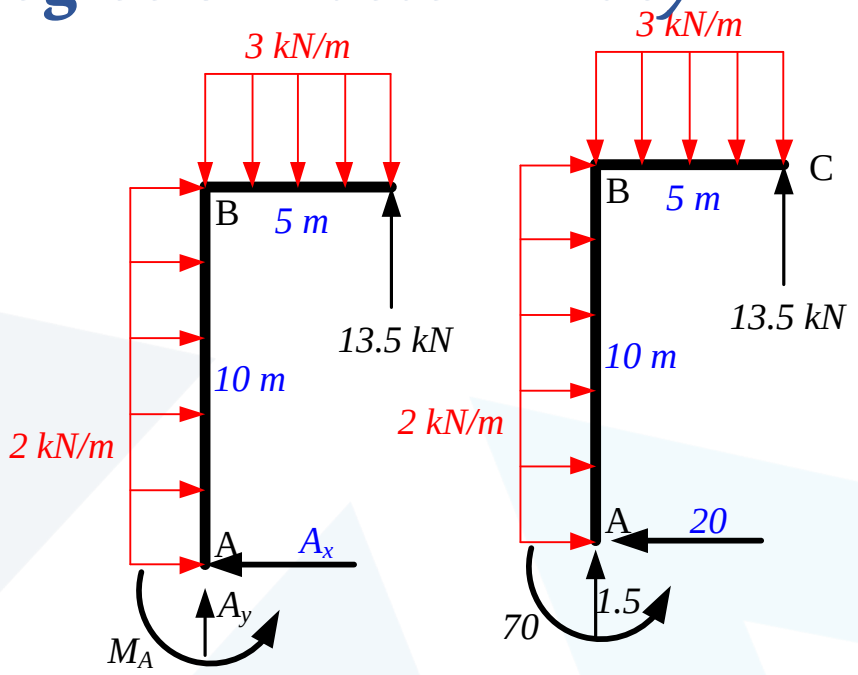
Force method: Frame with a single Degree of Indeterminacy

Computing the reactions

$$\uparrow + \sum F_y = 0 \Rightarrow A_y + 13.5 - 15 = 0 \Rightarrow A_y = 1.5kN$$

$$\overset{\rightarrow +}{\sum} F_x = 0 \Rightarrow A_x + 20 = 0 \Rightarrow A_x = -20kN = 20kN(\leftarrow)$$

$$(\downarrow\uparrow +) \sum M_A = 0 \Rightarrow M_A - (5)20 - (2.5)15 + (5)13.5 = 0 \Rightarrow M_A = 70\text{kN.m}$$



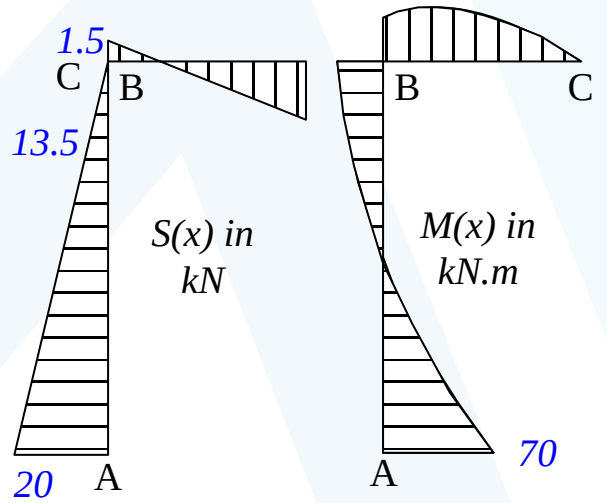
Drawing the BM & SF diagrams

In the segment AB: $0 < x < 10$,

$$M(x) = -x^2 + 20x - 70 \& S(x) = -2x + 20$$

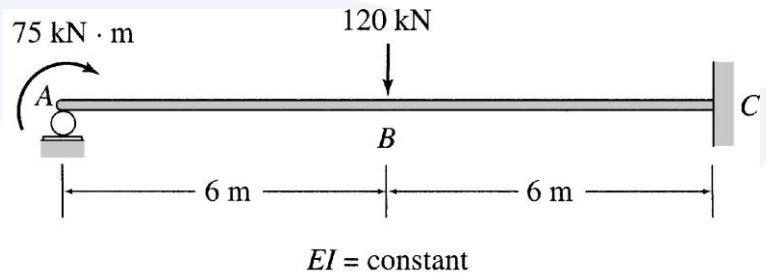
In Segment CB: $0 < x < 5$,

$$M(x) = -(3/2)x^2 + (27/2)x \& S(x) = 3x - 13.5$$



Homework

Pr.01: Determine the reactions and draw the shear and bending moment diagrams for the beam shown using force method. Select the reaction at the roller support to be the redundant.



Pr.02: Determine the reactions and draw the shear and bending moment diagrams for the frame shown using force method. Select the reaction at the roller support to be the redundant.