جامعة المنارة
كلية الهندسة

مقرر الرياضيات المتقطعة

جلسة العملي الرابعة

Sets

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Recall that **R** denotes the set of all real numbers, **Z** the set of all integers, and $\mathbf{Z}^+$ the set of all positive integers. Describe each of the following sets.

a. $\{x \in \mathbf{R} \mid -2 < x < 5\}$

b. $\{x \in \mathbf{Z} \mid -2 < x < 5\}$

c. $\{x \in \mathbf{Z}^+ \mid -2 < x < 5\}$

Solution:

a.]-2,5[

b. $\{-1,0,1,2,3,4\}$

c. {1,2,3,4}

Sets

4. Indicate the elements in each set defined in (a)–(f).
   a. $S = \{n \in \mathbf{Z} \mid n = (-1)^k, \text{ for some integer } k\}$.
   b. $T = \{m \in \mathbf{Z} \mid m = 1 + (-1)^i, \text{ for some integer } i\}$.
   c. $U = \{r \in \mathbf{Z} \mid 2 \leq r \leq -2\}$
   d. $V = \{s \in \mathbf{Z} \mid s > 2 \text{ or } s < 3\}$
   e. $W = \{t \in \mathbf{Z} \mid -1 < t < -3\}$
   f. $X = \{u \in \mathbf{Z} \mid u \leq 4 \text{ or } u \geq 1\}$

Solution:

a. $S=\{1,-1\}$

b. $T=\{2,0\}$

c. $U=\{\}=\emptyset$

d. $V=[3,\infty[\cup]-\infty,2]=Z$

e. $W=\{\}=\emptyset$

f. $X=]-\infty,4]\cup[1,\infty[=Z$

Sets

Which of the following sets are equal?

$A = \{a, b, c, d\}$ $B = \{d, e, a, c\}$

$C = \{d, b, a, c\}$ $D = \{a, a, d, e, c, e\}$

Solution:

A=C

B=D

Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$. Answer each of the following questions. Give reasons for your answers.

a. Is $B \subseteq A$? b. Is $C \subseteq A$?

Solution:

a. False , $\exists\, j \in B$ but $j \notin A$

b. True, every element in C exist in A

Sets

List the members of these sets.

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
b) $\{x \mid x \text{ is a positive integer less than } 12\}$
c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$
d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

Solution:

a. $\{-1,1\}$
b. $\{1,2,3,4,5,6,7,8,9, 10,11\}$
c. $\{0,1,4,9,16,25,36,49,64,81\}$
d. $\emptyset$

Sets

a. Is the number 0 in $\emptyset$?
b. Is $\emptyset = \{\emptyset\}$?
c. Is $\emptyset \in \{\emptyset\}$?
d. Is $\emptyset \in \emptyset$?

Solution:

a. False.
b. False
c. True
d. False

Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$.

c. Is $C \subseteq C$?
d. Is C a proper subset of A?

Solution:

c. True
d. True

Sets

Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9\}$, and $C = \{2, 4, 6, 8\}$. Find each of the following:

a. $A \cup B$ **b.** $A \cap B$ **c.** $A \cup C$ **d.** $A \cap C$

e. $A - B$ f. $B - A$ g. $B \cup C$ h. $B \cap C$

Solution:

a. {1,3,5,7,9,6}

b. {3,9}

c. {1,2,3,4,5,6,7,8,9}

d. Ø

e. {1,5,7}

f. {6}

g. {3,6,9,2,4,8}

h. {6}

Sets

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Let $A = \{x, y, z, w\}$ and $B = \{a, b\}$. List the elements of each of the following sets:

a. $A \times B$ b. $B \times A$

c. $A \times A$ d. $B \times B$

Solution:

a. {(x,a), (x,b), (y,a), (y,b), (z,a), (z,b), (w,a), (w,b)}

b. {(a,x), (a,y), (a,z), (a,w), (b,x), (b,y), (b,z), (b,w)}

c. {(x,x), (x,y), (x,z), (x,w), (y,x), (y,y), (y,z), (y,w),(z,x), (z,y), (z,z), (z,w), (w,x), (w,y), (w,z), (w,w) }

d. {(a,a), (a,b), (b,a), (b,b)}

Sets

Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$

b) $C \times B \times A$

Solution:

a. A×B×C={(a,x,0), (a,x,1), (a,y,0), (a,y,1), (b,x,0), (b,x,1), (b,y,0), (b,y,1), (c,x,0,), (c,x,1), (c,y,0), (c,y,1)}

b. C×B×A={(0,x,a), (0,x,b), (0,x,c), (0,y,a), (0,y,b), (0,y,c), (1,x,a), (1,x,b), (1,x,c), (1,y,a), (1,y,b), (1,y,c),}

Relations

Relations

Let $C = \{2, 3, 4, 5\}$ and $D = \{3, 4\}$ and define a binary relation S from C to D as follows:

$$\text{For all } (x, y) \in C \times D, \quad (x, y) \in S \quad \Leftrightarrow \quad x \geq y.$$

a. Is $2\ S\ 4$? Is $4\ S\ 3$? Is $(4, 4) \in S$? Is $(3, 2) \in S$?
b. Write S as a set of ordered pairs.

Solution:

a. $2\ S\ 4$:False

$4\ S\ 3$:True

$(4,4) \in S$

$(3,2) \notin S$

a. $S=\{(3,3),(4,3),(4,4),(5,3),(5,4)\}$

Relations

Define a binary relation R from $\mathbf{R}$ to $\mathbf{R}$ as follows:

$$\text{For all } (x, y) \in \mathbf{R} \times \mathbf{R}, \quad x\,R\,y \quad \Leftrightarrow \quad y = x^2.$$

a. Is $(2, 4) \in R$? Is $(4, 2) \in R$? Is $(-3)\,R\,9$? Is $9\,R\,(-3)$?

Solution:

a. $(2,4) \in R$

$(4,2) \notin R$

$(-3)\ R\ 9$:True

$9\ R\ (-3)$:False

Relations

Let S be the set of all strings of a's and b's of length 4. Define a relation R on S as follows:

For all $s, t \in S$,

$$s \; R \; t \quad \Leftrightarrow \quad s \text{ has the same first two characters as } t.$$

a. Is $abaa \; R \; abba$?

b. Is $aabb \; R \; bbaa$?

c. Is $aaaa \; R \; aaab$?

Solution:

a. True

b. False

c. True

Relations

- **Let A={1,2,3,4} , Define a binary relation *S* from A to A as follows:**

  S ={ (a,b) :a divides b}

  Write *S* as a set of ordered pairs.

Solution:

S={(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)}

Relations

Consider the following relations on $\{1, 2, 3, 4\}$:

$$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\},$$
$$R_2 = \{(1, 1), (1, 2), (2, 1)\},$$
$$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\},$$
$$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\},$$
$$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\},$$

	reflexive	irreflexive	symmetric	*antisymmetric*	asymmetric	transitive
R1						
R2			×			
R3	×		×			
R4		×		×	×	×
R5	×			×		×

Properties of Relations

Consider these relations on the set of integers:

$$R_1 = \{(a, b) \mid a \leq b\},$$
$$R_2 = \{(a, b) \mid a > b\},$$
$$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\},$$
$$R_4 = \{(a, b) \mid a = b\},$$
$$R_5 = \{(a, b) \mid a = b + 1\},$$
$$R_6 = \{(a, b) \mid a + b \leq 3\}.$$

Determine the properties of these relations

	reflexive	irreflexive	symmetric	*antisymmetric*	asymmetric	transitive
R1	×			×		×
R2		×		×	×	×
R3	×		×			×
R4	×		×	×		×
R5		×		×	×	
R6			×			

Note :

- R5 not transitive because ∃(2,1)∈R3 and (1,0) ∈R3 but (2,0)∉ R3
- R6 not transitive because ∃(2,0) ∈ R6:2+0<=3 and (0,3) ∈ R6 :0+3<=3 but (2,3)∉ R6: 2+3>=3

Represent relations using matrices

Represent each of these relations on {1, 2, 3} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 1), (1, 2), (1, 3)}
b) {(1, 2), (2, 1), (2, 2), (3, 3)}
c) {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}
d) {(1, 3), (3, 1)}

Solution:

a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Representing relations using matrices

Represent each of these relations on {1, 2, 3, 4} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
b) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}
c) {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}
d) {(2, 4), (3, 1), (3, 2), (3, 4)}

Solution:

a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Representing relations using matrices

List the ordered pairs in the relations on {1, 2, 3} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Solution:

a) {(1,1),(1,3),(2,2),(3,1),(3,3)}

b) {(1,2),(2,2),(3,2)}

c) {(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)}

Combining relations using matrices

Suppose that the relations R1 and R2 on a set A={1,2,3} ,

R1={(1,1),(1,3), (2,1),(3,2)}

R2={(1,1),(1,3),(2,2),(2,3),(3,1)}

Find R1∪ R2 , R1∩ R2

Solution:

$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ ⇒ R1∪ R2 ={(1,1),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2)}

$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ⇒ R1∩ R2 = {(1,1),(1,3)}

Combining relations using matrices

Exercise: Suppose that the relations R1 and R2 on a set A={1,2,3} ,
R1={(1,1),(1,3), (2,1),(2,2)}
R2={(1,2),(2,3),(3,1),(3,3)}
Find $S \circ R$

Solution:

$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow S \circ R$={(1,1),(1,2),(1,3),(2,2),(2,3)}

Combining relations using matrices

Let R_1 and R_2 be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent

a) $R_1 \cup R_2$. b) $R_1 \cap R_2$. c) $R_2 \circ R_1$.
d) $R_1 \circ R_1$.

Solution:

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_2 \circ R_1} = M_{R_1} \odot M_{R_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{R_1 \circ R_1} = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Properties of Relations

Represent each of these relations on {1,2,3,4} with a matrix ,and determine its properties.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
c) {(2, 4), (4, 2)}
d) {(1, 2), (2, 3), (3, 4)}

Solution:

a) $\begin{bmatrix} 0&0&0&0 \\ 0&1&1&1 \\ 0&1&1&1 \\ 0&0&0&0 \end{bmatrix}$ b) $\begin{bmatrix} 1&1&0&0 \\ 1&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$ c) $\begin{bmatrix} 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&1&0&0 \end{bmatrix}$

d) $\begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \end{bmatrix}$

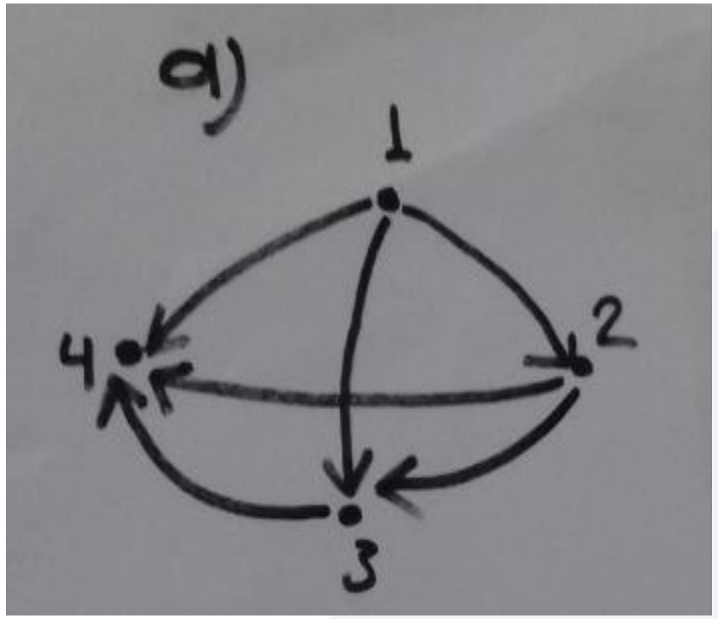
	reflexive	irreflexive	symmetric	*antisymmetric*	asymmetric	transitive
a)						×
b)	×		×			×
c)		×	×			
d)		×		×	×	

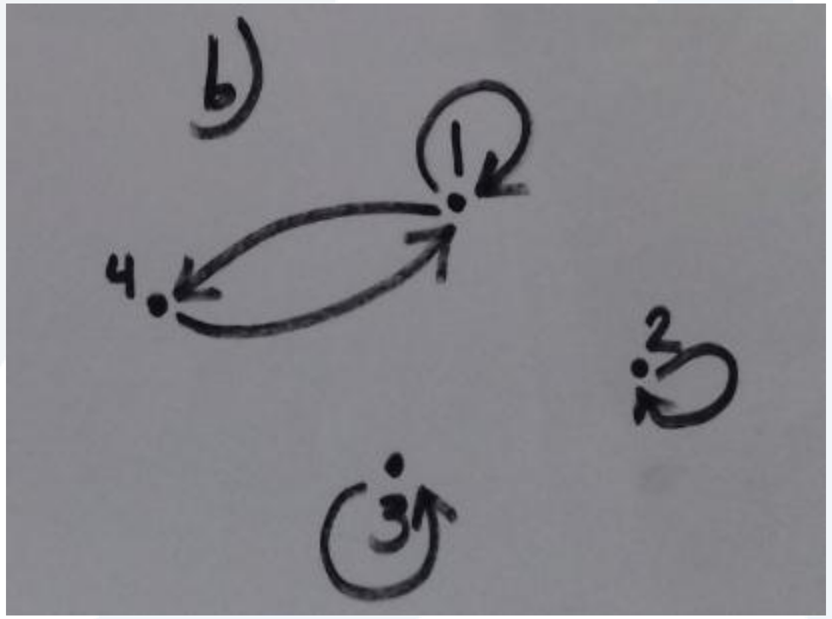
Representing relations with graph

represent these relations with graph.

a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$

Solution:





Properties of relation

Let A={-3,-1,2,4,5} and relation R defined from A to A as follows R={ (a,b): ab ≥ 0}

1- write R as a set of ordered pairs.

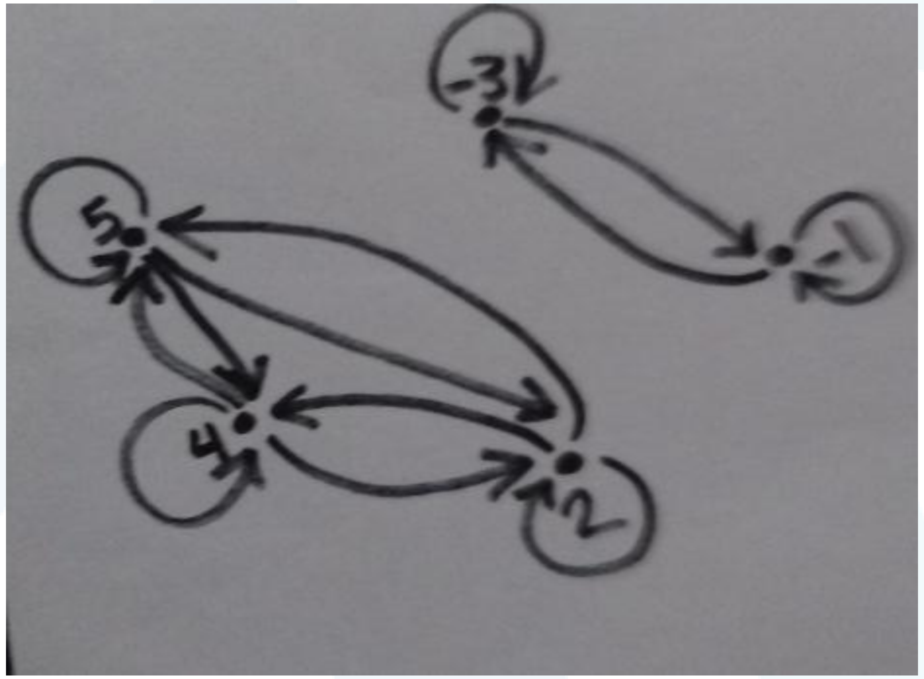
2- represent the relation with matrix.

3-represent the relation with graph.

4-determine if R is equivalence relation

Solution:

R={(-3,-3),(-3,-1),(-1,-3),(-1,-1),(2,2),(2,4),(2,5),(4,2),(4,4),(4,5),(5,2),(5,4),(5,5)}



$$M_R = \begin{bmatrix} 1\,1\,0\,0\,0 \\ 1\,1\,0\,0\,0 \\ 0\,0\,1\,1\,1 \\ 0\;\;0\,1\,1\,1 \\ 0\,0\,1\,1\,1 \end{bmatrix}$$

R is equivalence relation because it is reflexive, symmetric,transitive .

Properties of relation

Let A={-1,1,2,3,4} and relation R defined from A to A as follows R={ (x,y): x = y^2}

1- write R as a set of ordered pairs.

2- represent the relation with matrix.

3-represent the relation with graph.

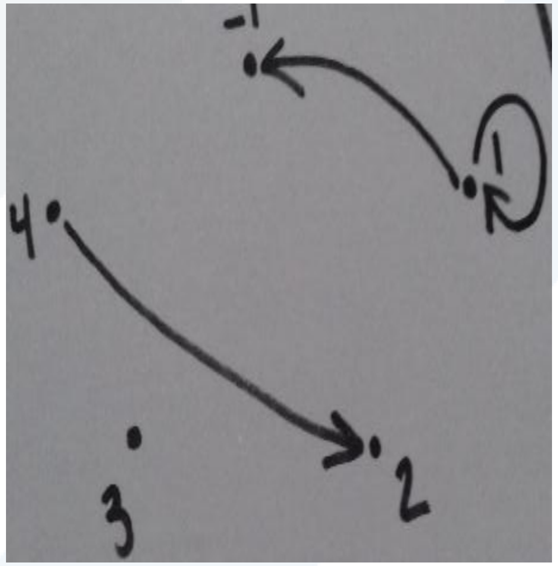
4-determine if R is equivalence relation

5-determine if R is partial ordering relation

Solution:

R={(1,-1),(1,1),(4,2)}

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$



R is not reflexive , antisymmetric , transitive

R is not equivalence relation

R is not partial ordering relation