

Example

It is known that the Laplace transform $F(s)$ has two distinct poles, one at $s = 0$, the other at $s = -1$. It also has a single zero at $s = 1$, and we

$$\text{know that } \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s) = 10$$

Find $F(s)$ and $f(t)$.

We are given that $F(s) = \frac{A(s-1)}{s(s+1)}$ and $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s) = 10$. Then,

$$\lim_{s \rightarrow 0} s \frac{A(s-1)}{s(s+1)} = A \lim_{s \rightarrow 0} \frac{(s-1)}{(s+1)} = -A = 10$$

Therefore,

$$F(s) = \frac{-10(s-1)}{s(s+1)} = \frac{r_1}{s} + \frac{r_2}{s+1} = \frac{10}{s} - \frac{20}{s+1} \Leftrightarrow (10 - 20e^{-t})$$

that is,

$$f(t) = (10 - 20e^{-t})$$

and we observe that

$$\lim_{t \rightarrow \infty} f(t) = 10$$