Denavit-hartenberg method

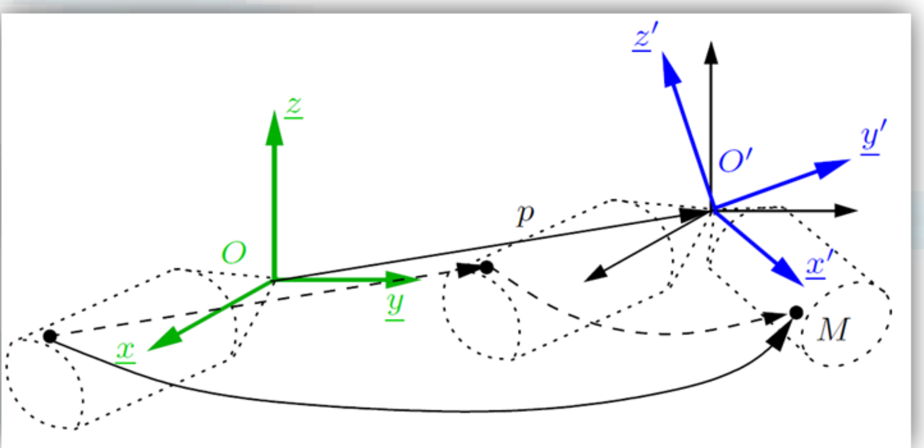
Homogenous transformation

Homogeneous Transformation

$$Transformation = Rotation + Translation$$

$$T = R + P$$

$$\begin{pmatrix} m \\ 1 \end{pmatrix} = \begin{pmatrix} R & P \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m' \\ 1 \end{pmatrix} \Leftrightarrow \overline{m} = T.\overline{m}'$$



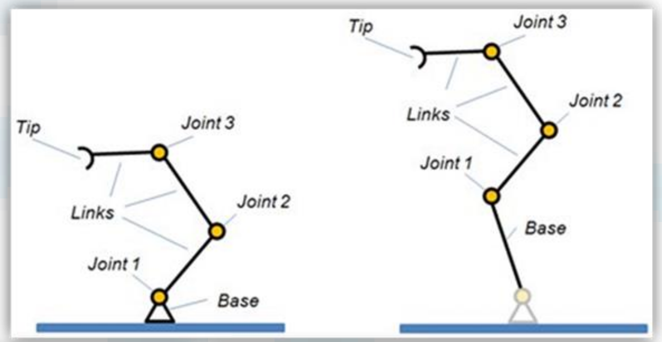
$$\left. \begin{array}{l} T_1 = \begin{pmatrix} R_1 & P_1 \\ 0 & 1 \end{pmatrix} \\ T_2 = \begin{pmatrix} R_2 & P_2 \\ 0 & 1 \end{pmatrix} \end{array} \right\} \Rightarrow T_1 \times T_2 = \begin{pmatrix} R_1 R_2 & R_1 P_2 + P_1 \\ 0 & 1 \end{pmatrix}$$

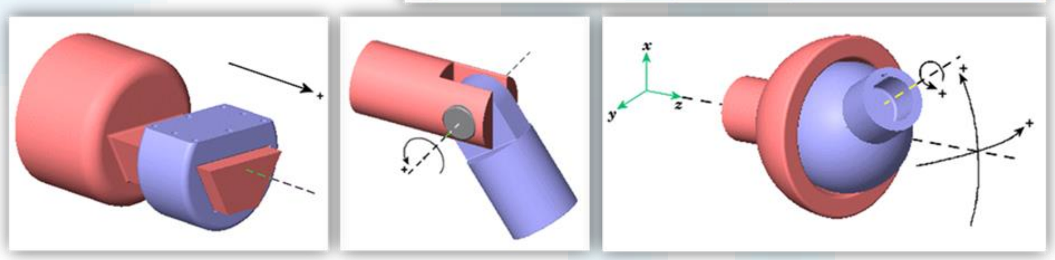
$$T = \begin{pmatrix} R & P \\ 0 & 1 \end{pmatrix} \Rightarrow T^{-1} = \begin{pmatrix} R^T & -R^T P \\ 0 & 1 \end{pmatrix}$$

Kinematic chains

- Kinematic chain is a set of segments and joints.
- Serial robot is a set of kinematic chains
- Joints
  - Simple joint (1 DOF)
  - Composed joint (2 or 3 DOF)

$$Robot\begin{cases} n \text{ joint } \left(\text{link i - 1} \xrightarrow{\text{joint i}} \textit{link} \text{ i}\right) \\ n+1 \text{ link } \left(\text{joint i actuates link i}\right) \end{cases}$$





Serial Robot Studying

Denavit-Hartenberg

Frame definition → Distance and Rotation Between Frames → Transformations → Kinematic Models

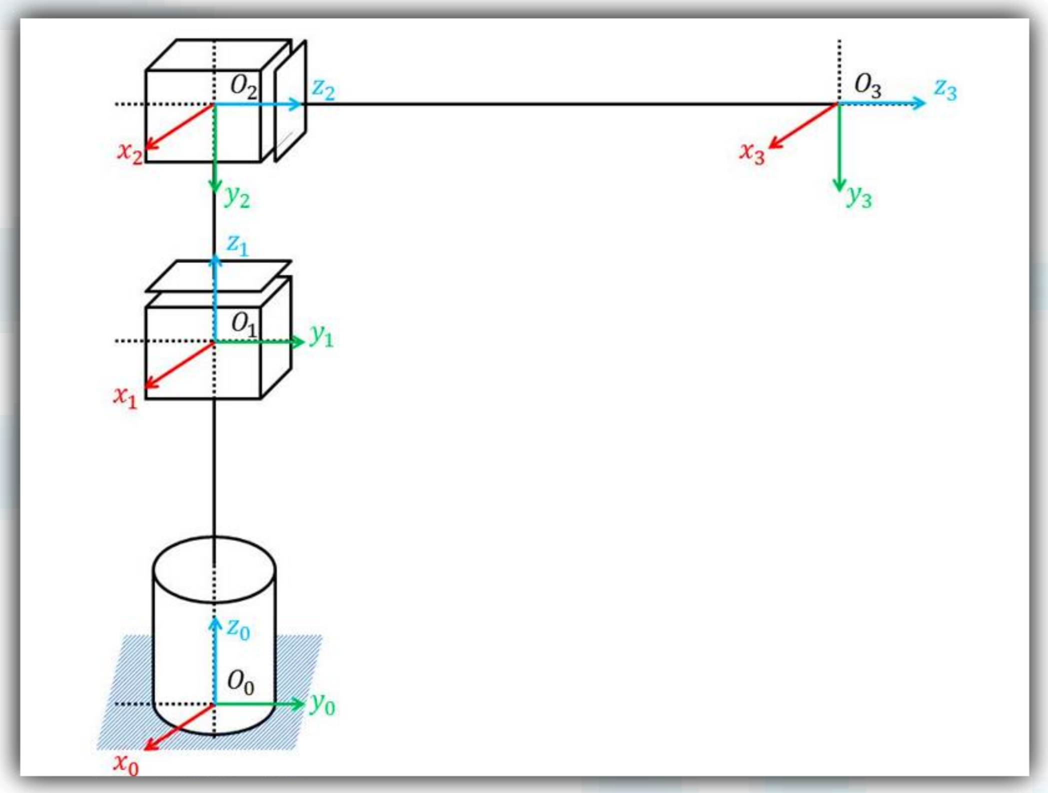
Denavit-hartenberg method

For every joint, we have to determine:

1. DH Frames
2. DH Parameters
3. DH Transformation

DH Frames (Theorems)

- $x_i \perp z_{i-1}$
- x_i , z_{i-1} in intersection
- z_i is the action axis of joint $i+1$



Three cases

1 - z_i , z_{i-1} not in the same plan

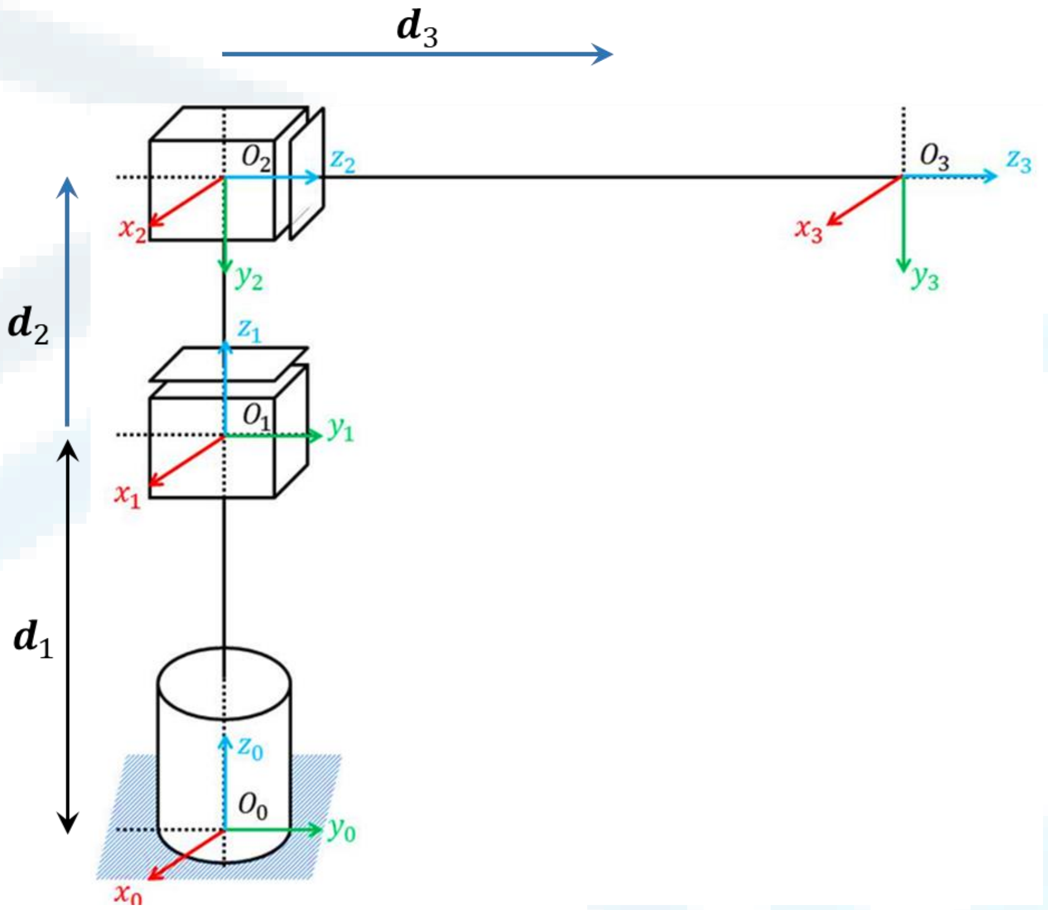
2 - z_i , z_{i-1} in intersection

3 - z_i , z_{i-1} parallel

DH Parameters

- a_i : the distance between z_{i-1} and z_i axes along the x_i axis
- α_i : the angle between z_{i-1} and z_i axis about x_i axis
- d_i : the distance between x_{i-1} and x_i axes along the z_{i-1} axis
- θ_i : the angle between x_{i-1} axis and x_i axis about the z_{i-1} axis

Joint	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	0	-90	d_2	0
3	0	0	d_3	0
....				



DH Transformation

$$T_{i-1}^{i} = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$T_{i-1}^{i} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{i-1}^{i} = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Cylindrical robot

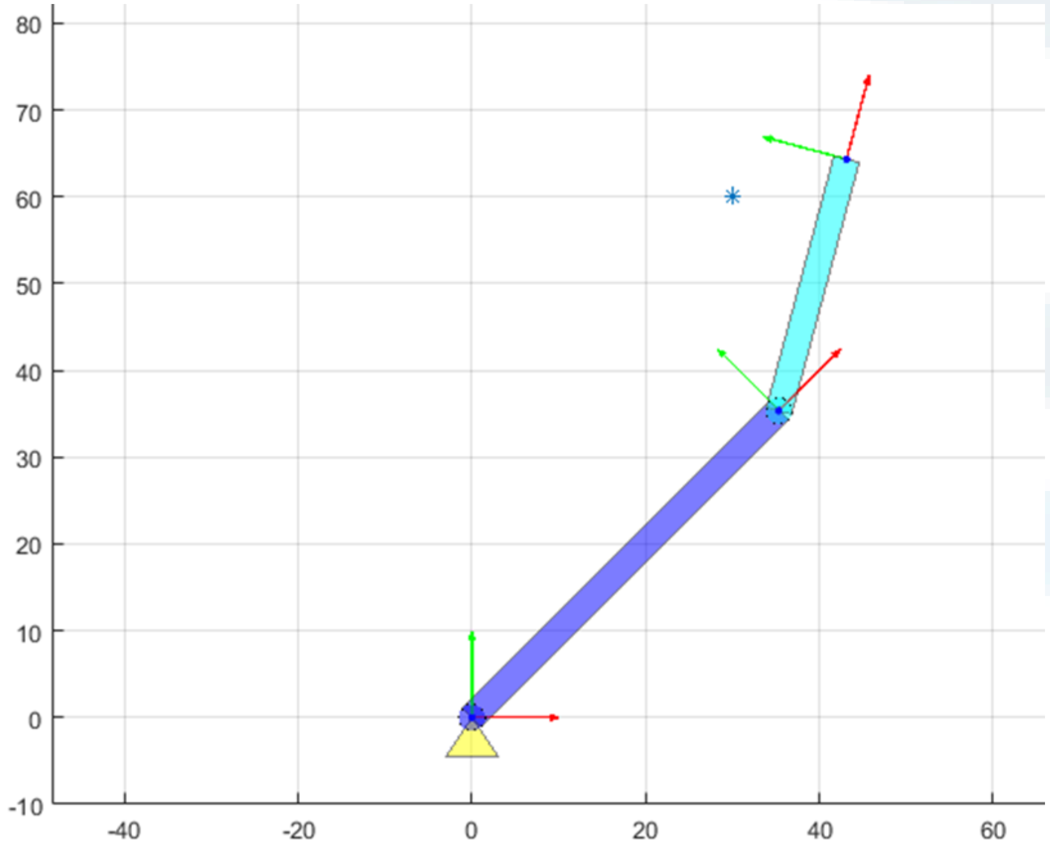
link	a_i	$\propto_i$	d_i	θ_i
1	0	0	d_1	$\theta_1{}^*$
2	0	-90	$d_2{}^*$	0
3	0	0	$d_3{}^*$	0

$$T_{i-1}^{i}=\begin{bmatrix} C_{\theta_i} & -S_{\theta_i}C_{\alpha_i} & S_{\theta_i}S_{\alpha_i} & a_iC_{\theta_i} \\ S_{\theta_i} & C_{\theta_i}C_{\alpha_i} & -C_{\theta_i}S_{\alpha_i} & a_iS_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} T_0^1=\begin{bmatrix} C_{\theta_1} & -S_{\theta_1} & 0 & 0 \\ S_{\theta_1} & C_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_1^2=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_2^3=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{cases}$$

Examples

Denavit-Hartenberg

2R Planar robot



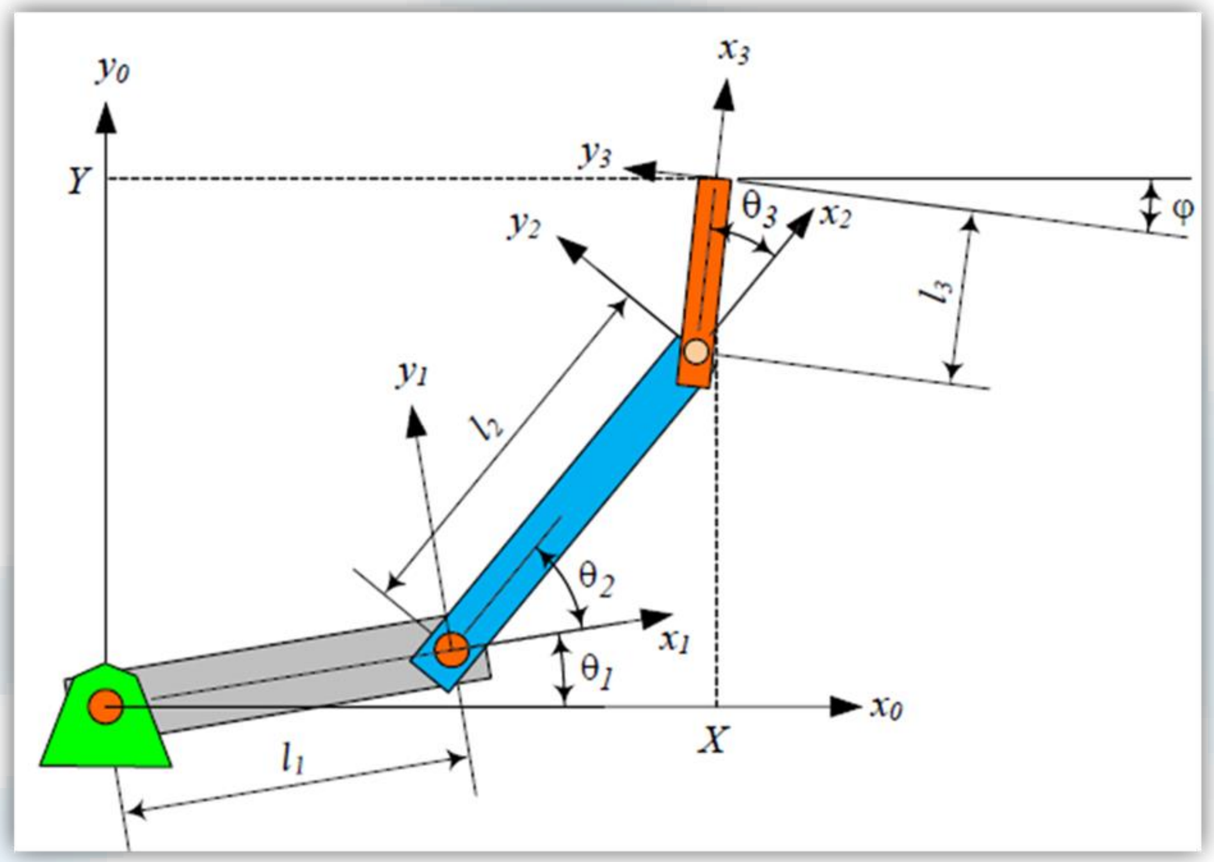
Link	a_i	α_i	d_i	θ_i
1	50	0	0	$\boldsymbol{\theta_1}$
2	30	0	0	$\boldsymbol{\theta_2}$

```
T01 =

[ cos(theta1), -sin(theta1), 0, 50*cos(theta1)]
[ sin(theta1),  cos(theta1), 0, 50*sin(theta1)]
[           0,            0, 1,              0]
[           0,            0, 0,              1]
T12 =

[ cos(theta2), -sin(theta2), 0, 30*cos(theta2)]
[ sin(theta2),  cos(theta2), 0, 30*sin(theta2)]
[           0,            0, 1,              0]
[           0,            0, 0,              1]
```

3R Planer Robot



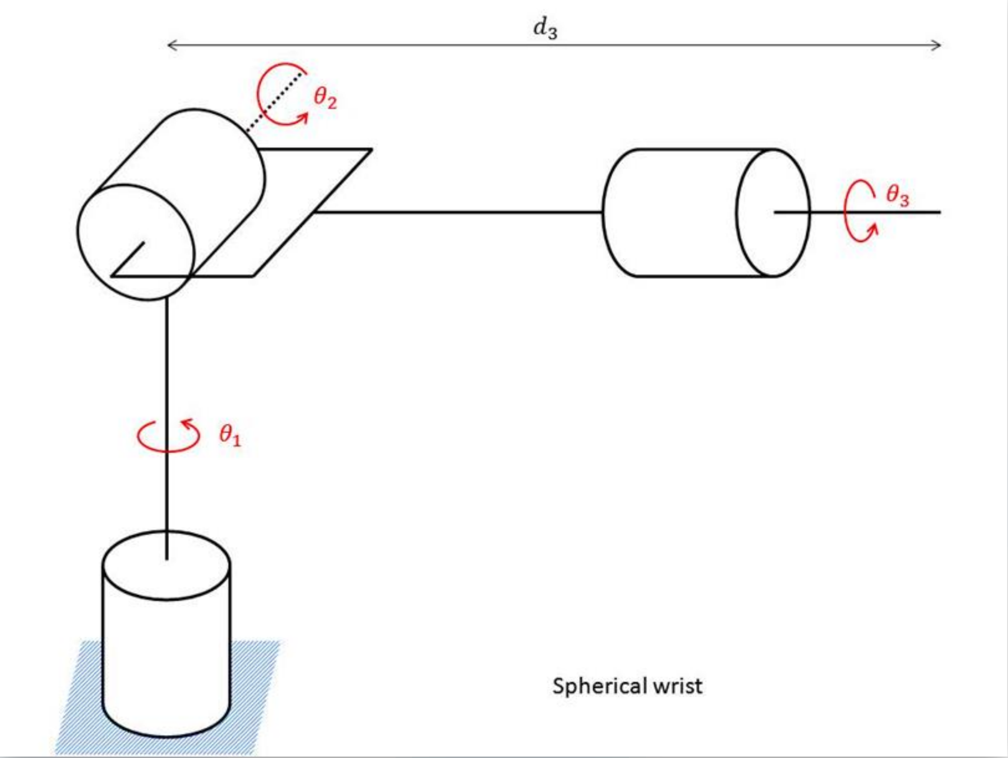
3R Planer Robot

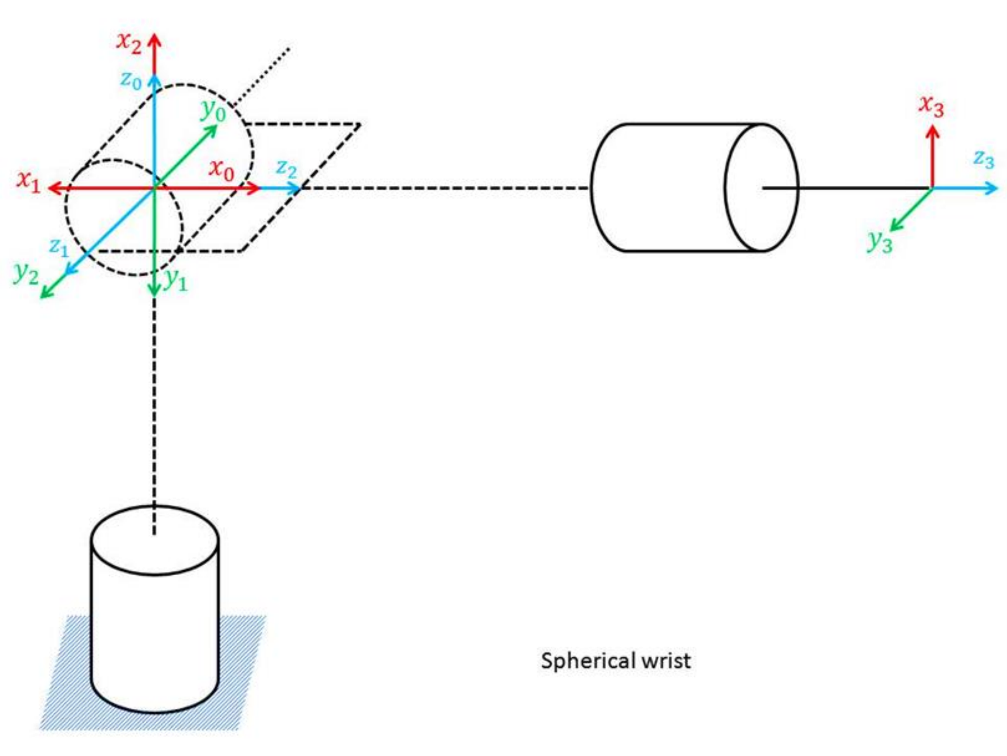
link	a_i	α_i	d_i	θ_i
1	l1	0	0	θ1
2	l2	0	0	θ2
3	l3	0	0	θ3

$$A_i = T_{i-1}^{i} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1c_1 \\ s_1 & c_1 & 0 & l_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2c_2 \\ s_2 & c_2 & 0 & l_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_2^3 = \begin{bmatrix} c_3 & -s_3 & 0 & l_3c_3 \\ s_3 & c_3 & 0 & l_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical wrist





link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\theta_1{}^*$
2	0	90	0	$\theta_2{}^*$
3	0	0	d_3	$\theta_3{}^*$

$$T_{i-1}^{i} = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} T_0^1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_1^2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ T_2^3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{cases}$$

Thanks

Denavit-Hartenberg is the first step to introduce the kinematic models........