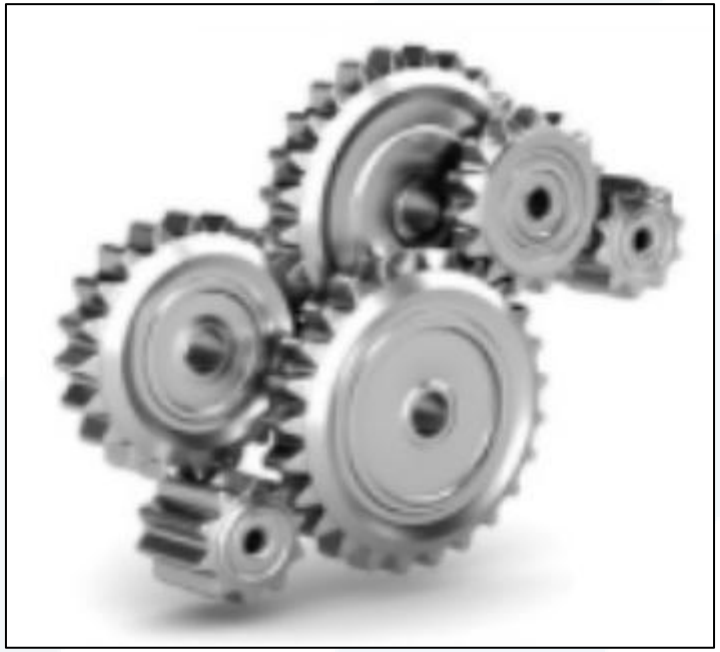
جامعة المنارة

كلية الهندسة

قسم الروبوتيك و الأنظمة الذكية

مقرر نمذجة الميكانيزمات

Vibratory Systems Applications



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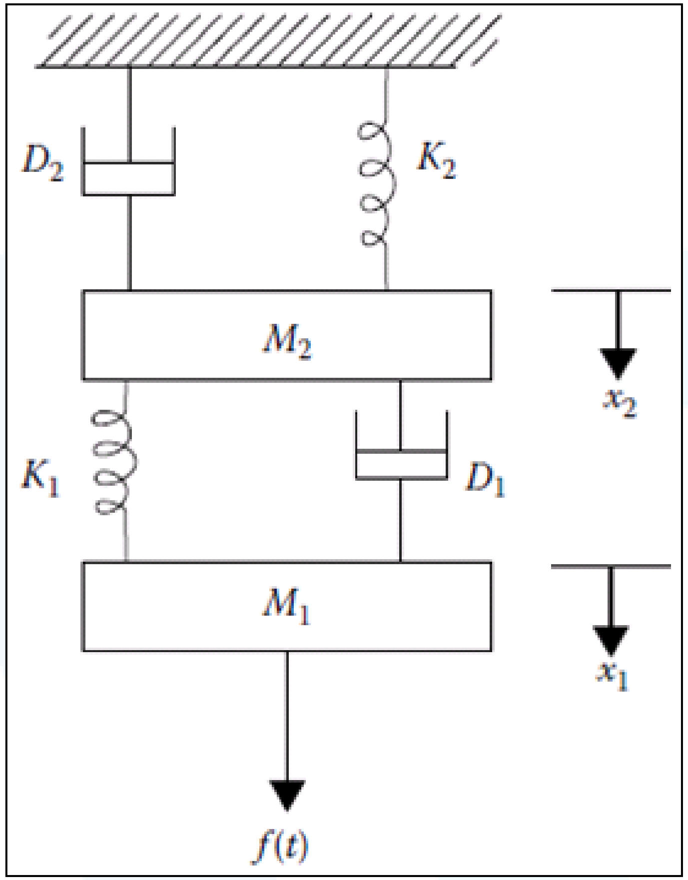
العام الدراسي 2023-2024

Contents

Model of a General Two Mass Suspension System

Determine the state model for the mechanical system shown in Figure



The state-space model for the mechanical system

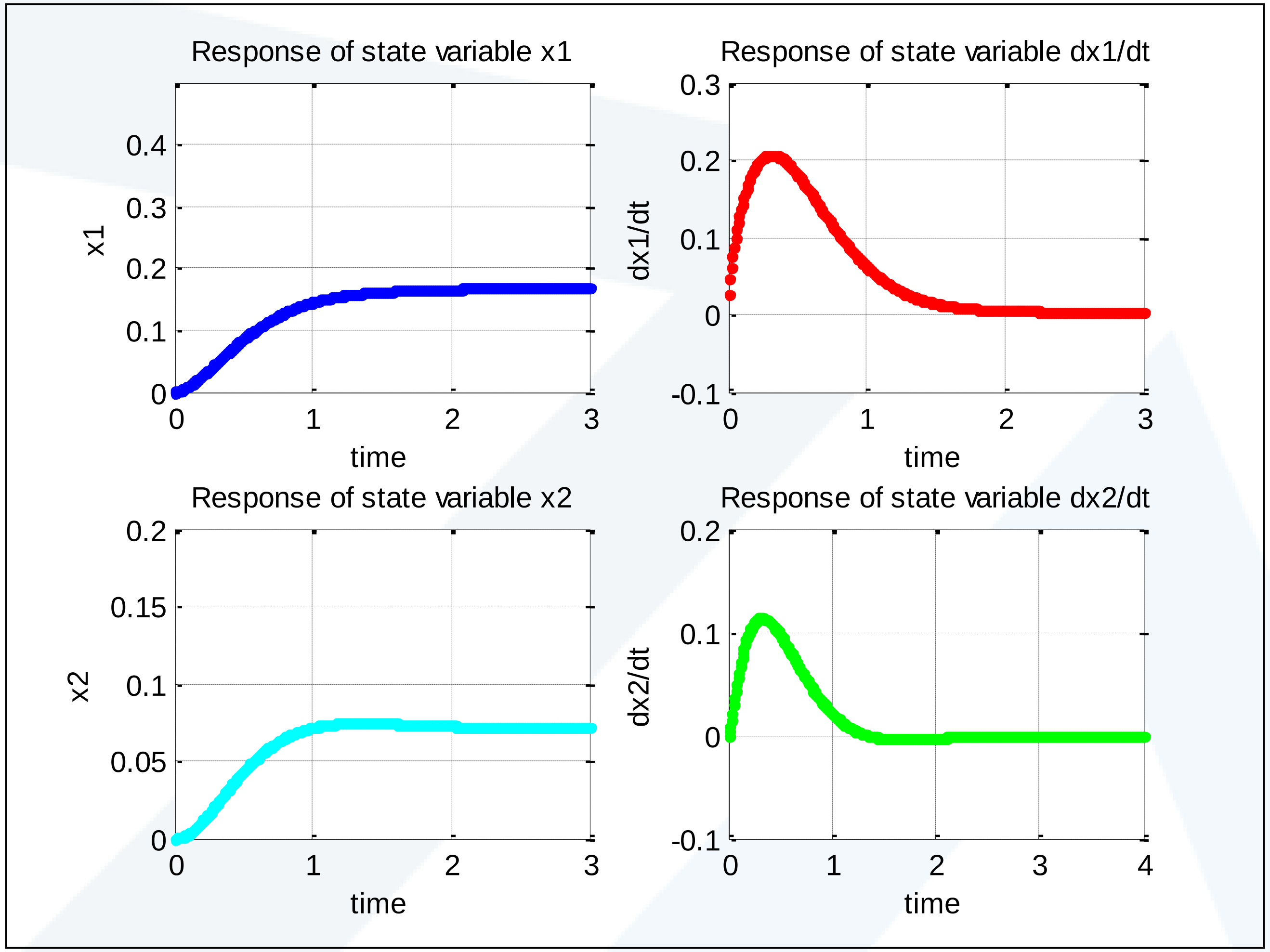
$$\frac{d}{dt}\begin{bmatrix} X_1 \\ \dot{X}_1 \\ X_2 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1/M_1 & -D_1/M_1 & k_1/M_1 & D_1/M_1 \\ 0 & 0 & 0 & 1 \\ k_1/M_2 & D_1/M_2 & -(k_1+k_2)/M_2 & -(D_1+D_2)/M_2 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ X_2 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_1 \\ 0 \\ 0 \end{bmatrix} f(t)$$

State Space Model using Matlab

k1=50 [N/m] ; k2=70 [N/m] ; m1=2 [kg]; m2=3 [kg]; d1=40 [Ns/m] ; d2=30 [Ns/m];

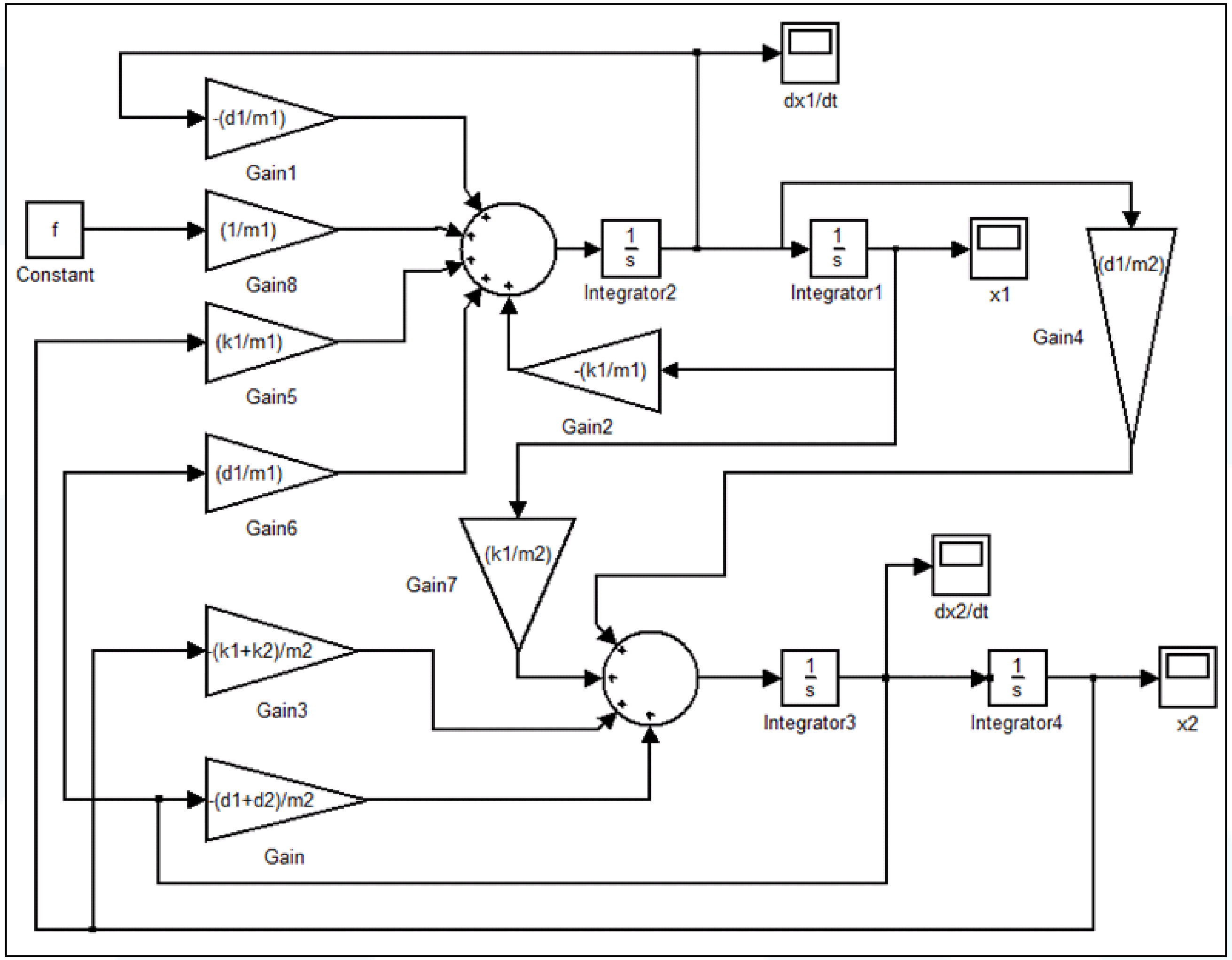
```
t=0; % Initial time
dt=0.01; % step size
tsim=10.0; % Simulation time
n=round( (tsim-t)/dt); % number of iterations
%system parameters
k1=50;
k2=70;
m1=2;
m2=3;
d1=40;
d2=30;
A=[0 1 0 0; -k1/m1 -d1/m1 k1/m1 d1/m1; 0 0 0 1; k1/m2
d1/m2 -(k1+k2)/m2 -(d1+d2)/m2];
B=[0; 1/m1; 0; 0];
X=[0 0 0 0]';
u=5;
for i=1:n;
dx=A*X+B*u;
X=X+dx*dt;
X1(i,:)=[t,X'];
t=t+dt;
end
```

```
subplot(2,2,1)
plot(X1(:,1),X1(:,2),'b.')
grid
axis([0 3 0 0.5])
xlabel('time')
ylabel('x1')
title('Response of state variable x1')
subplot(2,2,2)
plot(X1(:,1),X1(:,3),'r.')
grid
axis([0 3 -0.1 0.3])
xlabel('time')
ylabel('dx1/dt')
title('Response of state variable dx1/dt')
subplot(2,2,3)
plot(X1(:,1),X1(:,4),'c.')
grid
axis([0 3 0 0.2])
xlabel('time')
ylabel('x2')
title('Response of state variable x2')
subplot(2,2,4)
plot(X1(:,1),X1(:,5),'g.')
grid
axis([0 4 -0.1 0.2])
xlabel('time')
ylabel(' dx2/dt ')
title('Response of state variable dx2/dt')
```



Response of state variable x1
x1
time
Response of state variable dx1/dt
dx1/dt
time
Response of state variable x2
x2
time
Response of state variable dx2/dt
dx2/dt
time

Block Diagram Model using Simulink



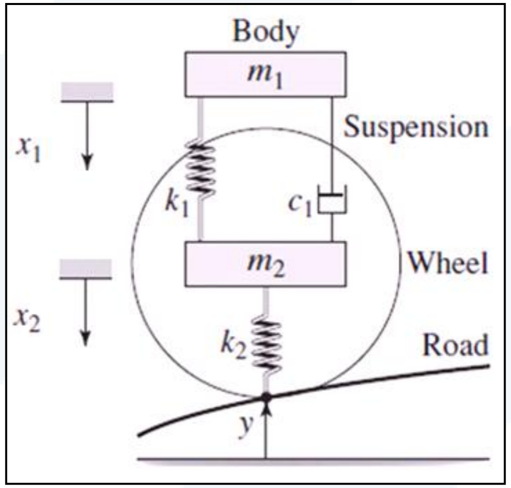
Model of a Two Mass Suspension System in a Car

The following are the equations of motion of the two-mass suspension model shown in Figure

$$m_1\ddot{x}_1 = k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1)$$

$$m_2\ddot{x}_2 = -k_1(x_2 - x_1) - c_1(\dot{x}_2 - \dot{x}_1) + k_2(y - x_2)$$

Develop a Simulink model of this system to obtain the plots of x_1 and x_2. The input $y(t)$ is a unit step function, and the initial conditions are zero. Use the following values: $m_1 = 250$ kg, $m_2 = 40$ kg, $k_1 = 1.5 \times 10^4$ N/m, $k_2 = 1.5 \times 10^5$ N/m, and $c_1 = 1917$ N · s/m.



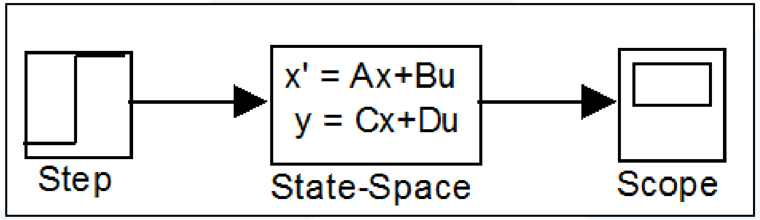
These equations are expressed in vector-matrix form as

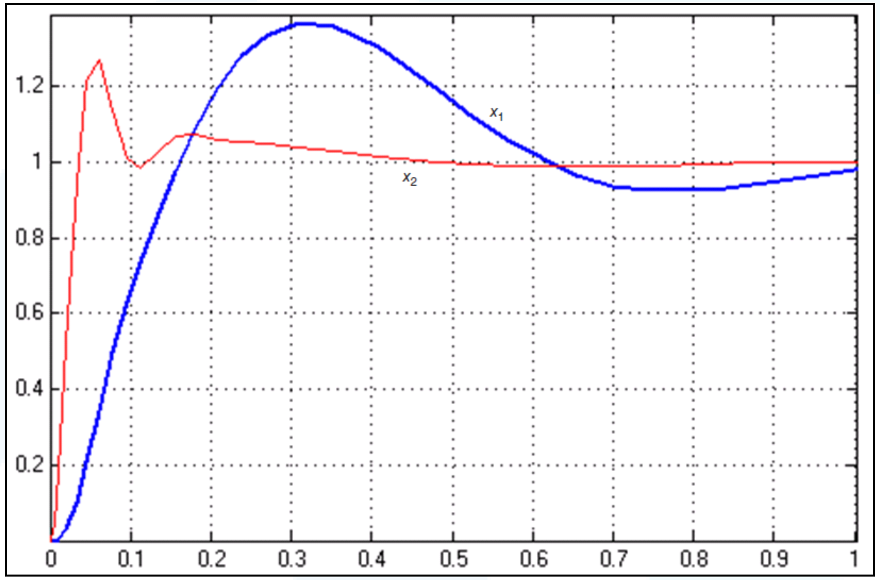
$$\dot{x} = \mathbf{A}x + \mathbf{B}y(t)$$

where

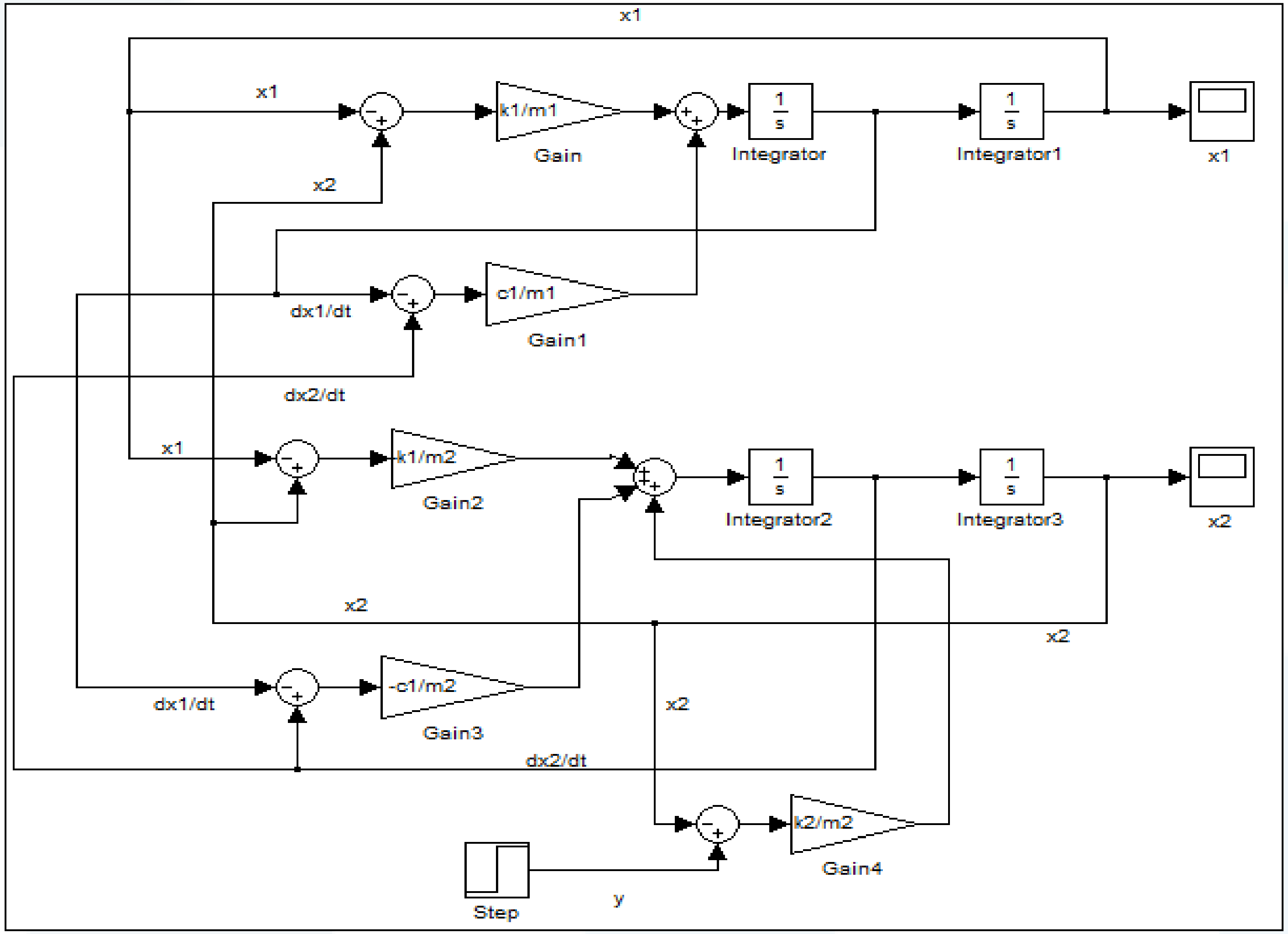
$$x = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{c_1}{m_1} & \frac{k_1}{m_1} & \frac{c_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{c_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{c_1}{m_2} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_2}{m_2} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

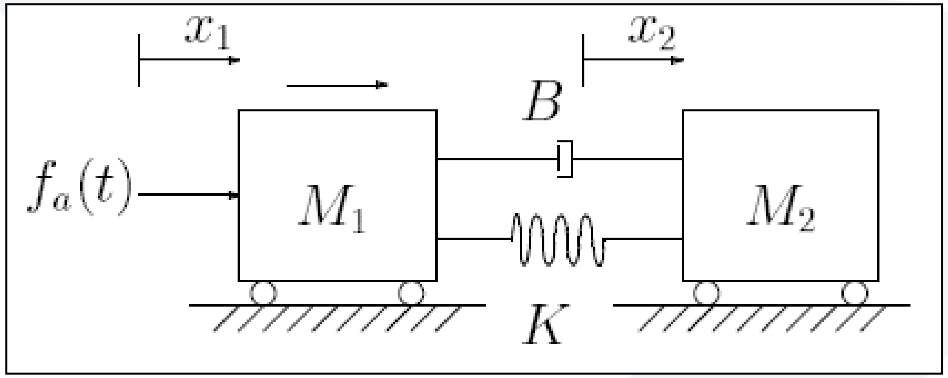




m1 = 250; m2 = 40; k1 = 1.5e+4;
k2 = 1.5e+5; c1 =1917;


x1
x1
k1/m1
Gain
1
s
Integrator
1
s
Integrator1
x1
x2
c1/m1
Gain1
dx1/dt
dx2/dt
x1
k1/m2
Gain2
1
s
Integrator2
1
s
Integrator3
x2
x2
x2
-c1/m2
Gain3
dx1/dt
x2
dx2/dt
k2/m2
Gain4
Step
y

Model of Two Carts Connected by a Spring and Dashpot



$$
\begin{aligned}
M_1\ddot{x}_1 + B\dot{x}_1 + Kx_1 &= B\dot{x}_2 + Kx_2 + f_a(t)\\
M_2\ddot{x}_2 + B\dot{x}_2 + Kx_2 &= B\dot{x}_1 + Kx_1
\end{aligned}
$$

$$
\begin{aligned}
\dot{v}_1 &= -\frac{B}{M_1}v_1 - \frac{K}{M_1}x_1 + \frac{B}{M_1}v_2 + \frac{K}{M_1}x_2 + \frac{1}{M_1}f_a(t)\\
\dot{x}_1 &= v_1\\
\dot{v}_2 &= -\frac{B}{M_2}v_2 - \frac{K}{M_2}x_2 + \frac{B}{M_2}v_1 + \frac{K}{M_2}x_1\\
\dot{x}_2 &= v_2
\end{aligned}
$$

$$
\mathbf{A} = \begin{bmatrix} -\frac{B}{M_1} & -\frac{K}{M_1} & \frac{B}{M_1} & \frac{K}{M_1}\\ 1 & 0 & 0 & 0\\ \frac{B}{M_2} & \frac{K}{M_2} & -\frac{B}{M_2} & -\frac{K}{M_2}\\ 0 & 0 & 1 & 0 \end{bmatrix},
$$

$$
\mathbf{B} = \begin{bmatrix} \frac{1}{M_1}\\ 0\\ 0\\ 0 \end{bmatrix} \quad \mathbf{C} = [0\ 0\ 1\ 0] \quad \mathbf{D} = [0]
$$

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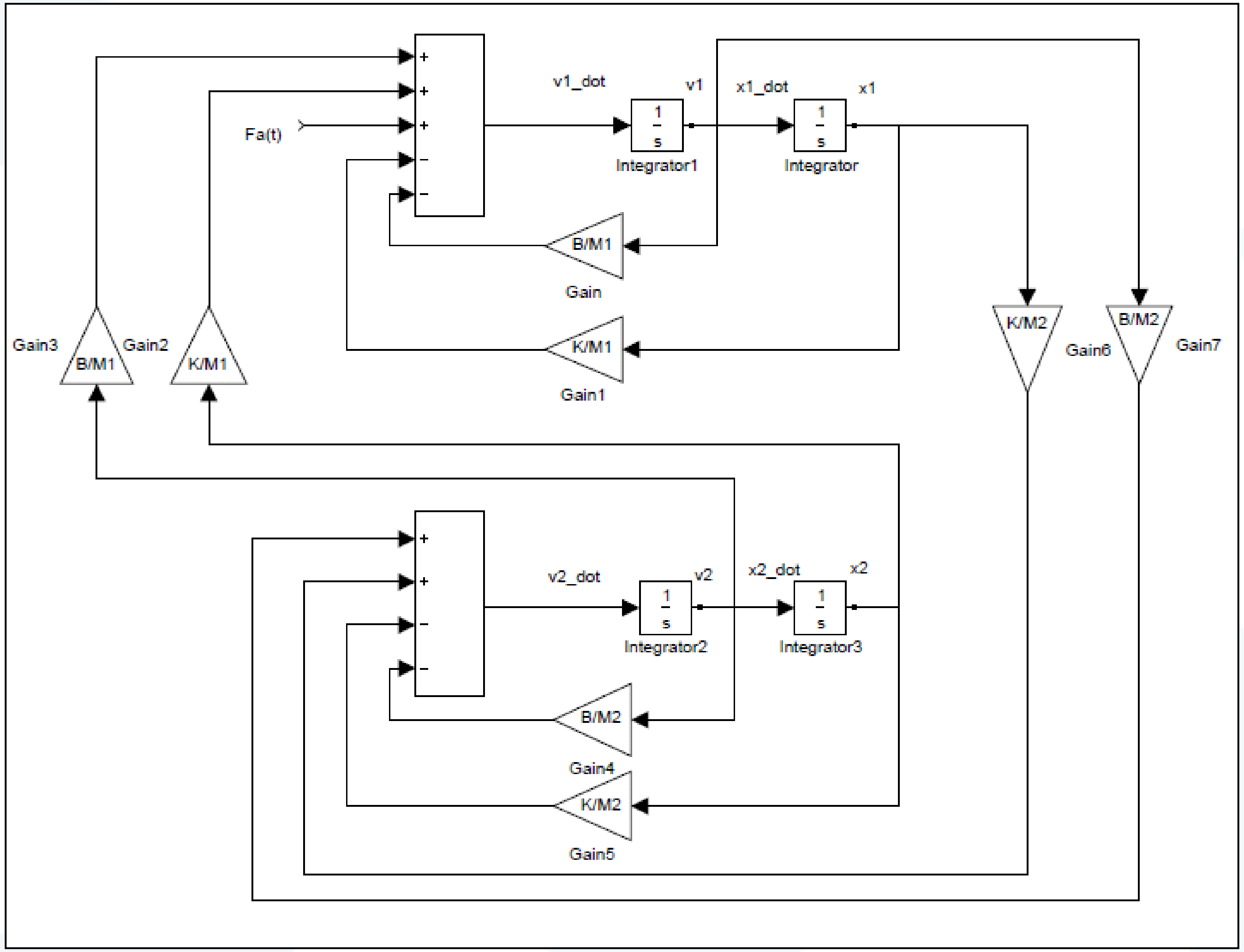

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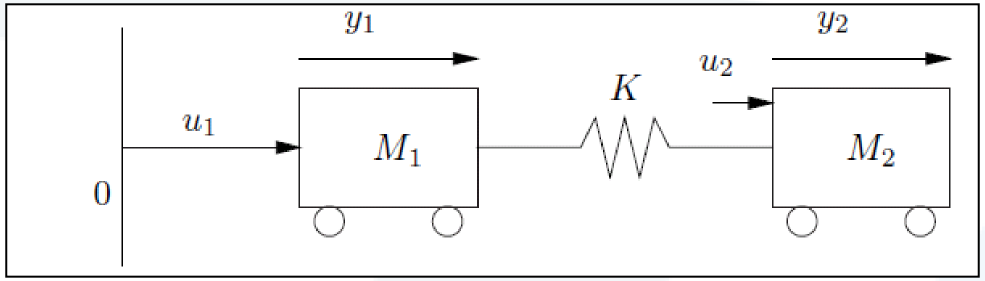
Model of Two Carts Connected by a Spring with Two Inputs

Consider the following system of 2 carts connected by a spring



$$M_1\ddot{y}_1 = -K(y_1 - y_2) + u_1$$
$$M_2\ddot{y}_2 = -K(y_2 - y_1) + u_2$$

Let us recast the equations in state space form. Define $x = \begin{bmatrix} y_1 & \dot{y}_1 & y_2 & \dot{y}_2 \end{bmatrix}^T$. Then

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K}{M_1} & 0 & \frac{K}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{M_2} & 0 & \frac{-K}{M_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{M_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

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