جامعة المنارة

كلية الهندسة

قسم الميكاترونكس

مقرر المعادلات التفاضلية و التحويلات

Introduction to Control Actions using Transfer Functions

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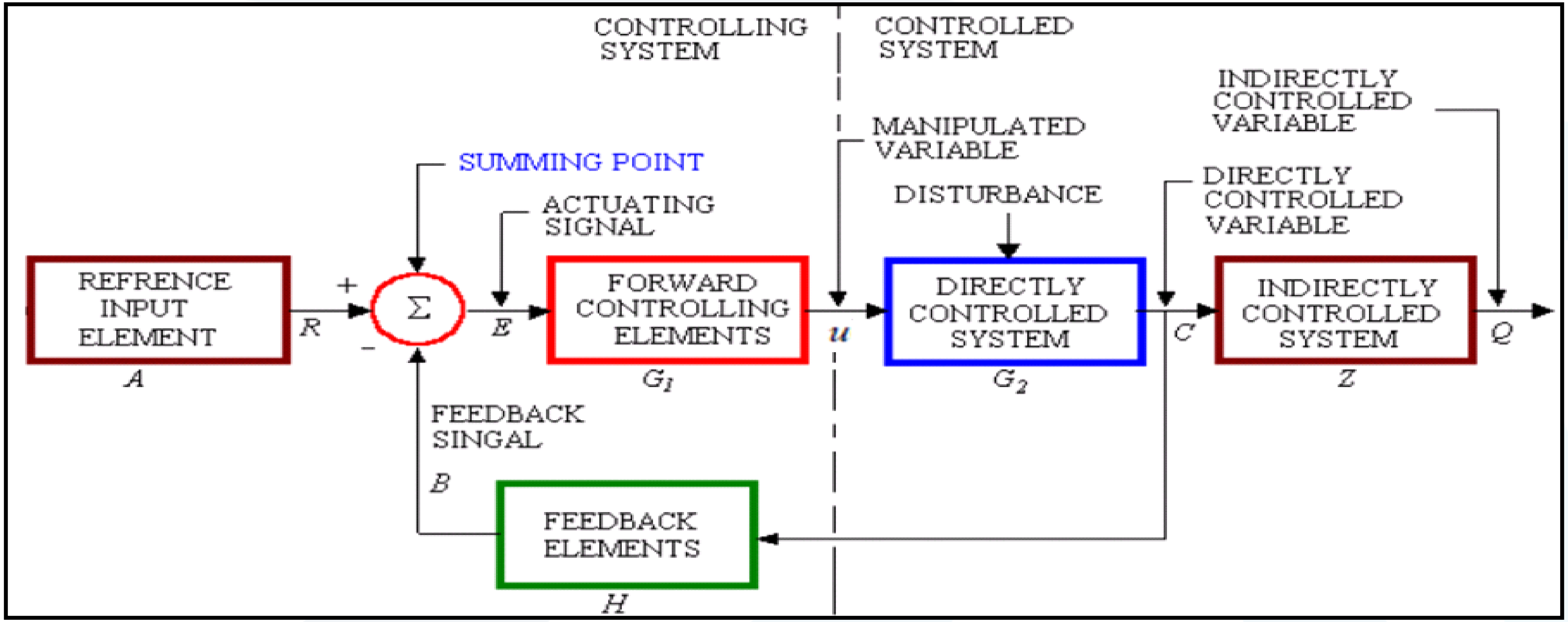
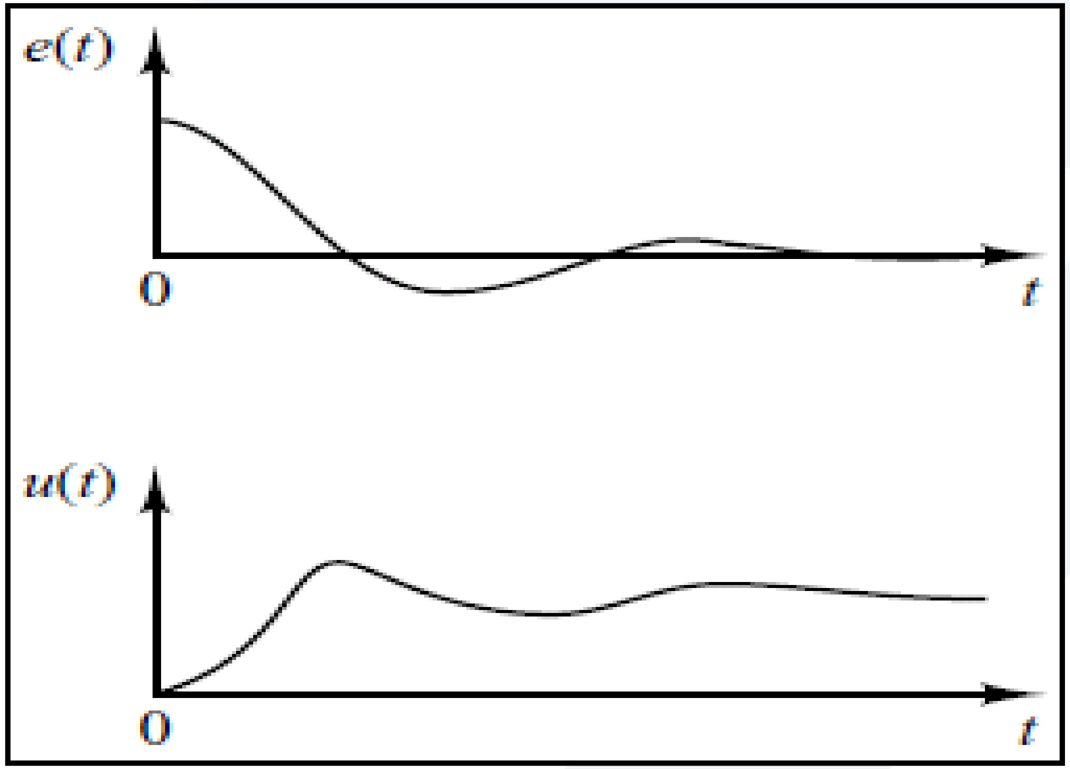
Control Actions

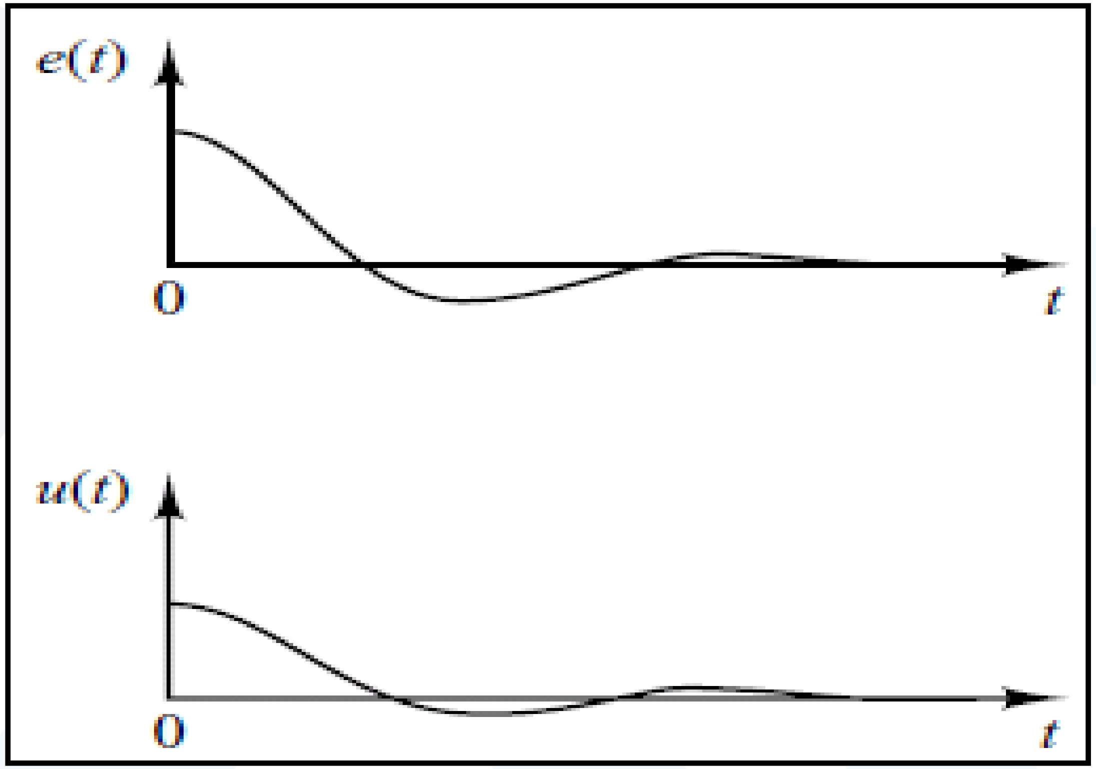
The Closed-Loop Control System



Integral vs Proportional Control Action

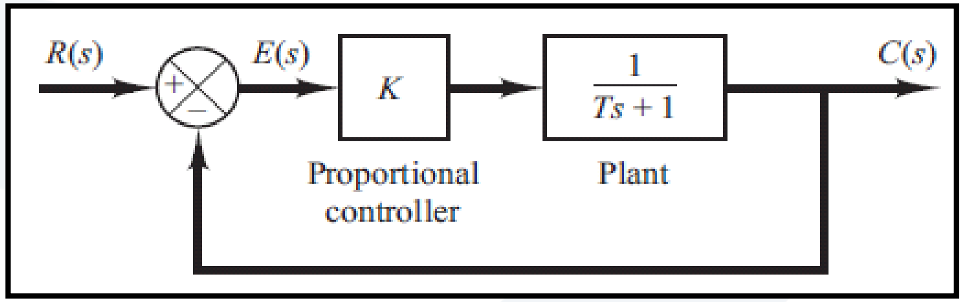


Integral Control Action



Proportional Control Action

Proportional Control of System



$$G(s) = \frac{K}{Ts + 1}$$

Since

$$\frac{E(s)}{R(s)} = \frac{R(s) - C(s)}{R(s)} = 1 - \frac{C(s)}{R(s)} = \frac{1}{1 + G(s)}$$

$$E(s) = \frac{1}{1 + G(s)} R(s) = \frac{1}{1 + \dfrac{K}{Ts + 1}} R(s)$$

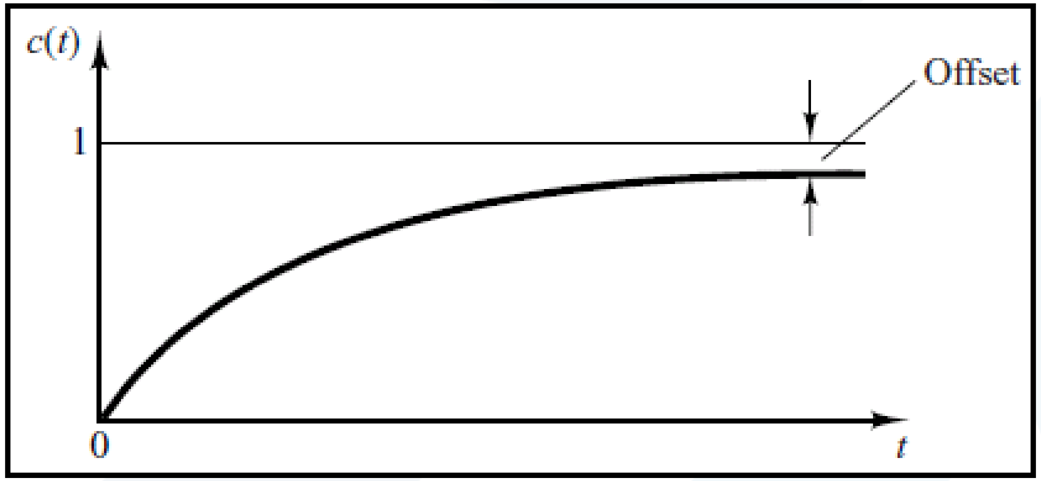
For the unit-step input $R(s) = 1/s$, we have

$$E(s) = \frac{Ts + 1}{Ts + 1 + K} \frac{1}{s}$$

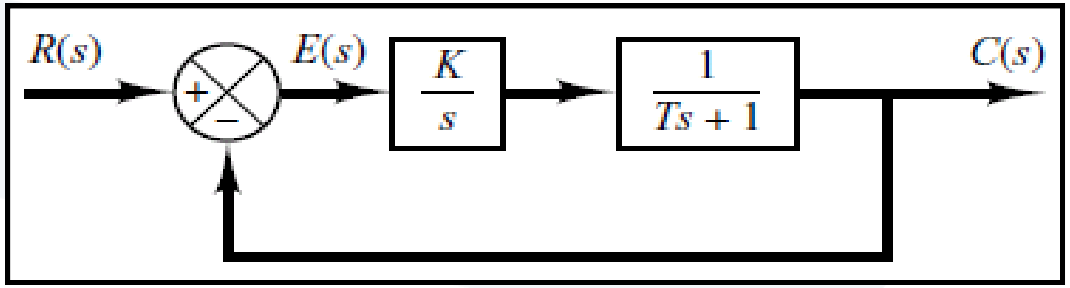
The steady-state error is

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{Ts + 1}{Ts + 1 + K} = \frac{1}{K + 1}$$

Such a system without an integrator in the feedforward path always has a steady-state error in the step response. Such a steady-state error is called an offset. Figure shows the unit-step response and the offset.



Integral Control of Systems



$$\frac{C(s)}{R(s)} = \frac{K}{s(Ts + 1) + K}$$

Hence

$$\frac{E(s)}{R(s)} = \frac{R(s) - C(s)}{R(s)} = \frac{s(Ts + 1)}{s(Ts + 1) + K}$$

the steady-state error for the unit-step response can be obtained by applying the final-value theorem, as follows:

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s^2(Ts + 1)}{Ts^2 + s + K} \frac{1}{s} = 0$$

Integral control of the system thus eliminates the steady-state error in the response to the step input. This is an important improvement over the proportional control alone, which gives offset.

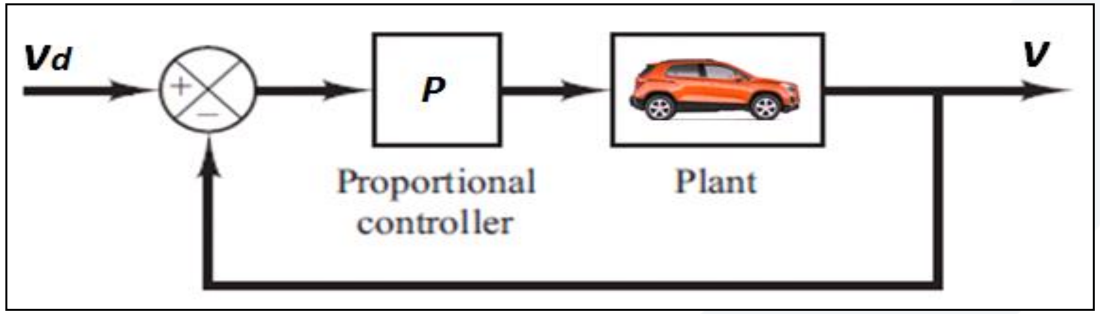
Derivative Control Action

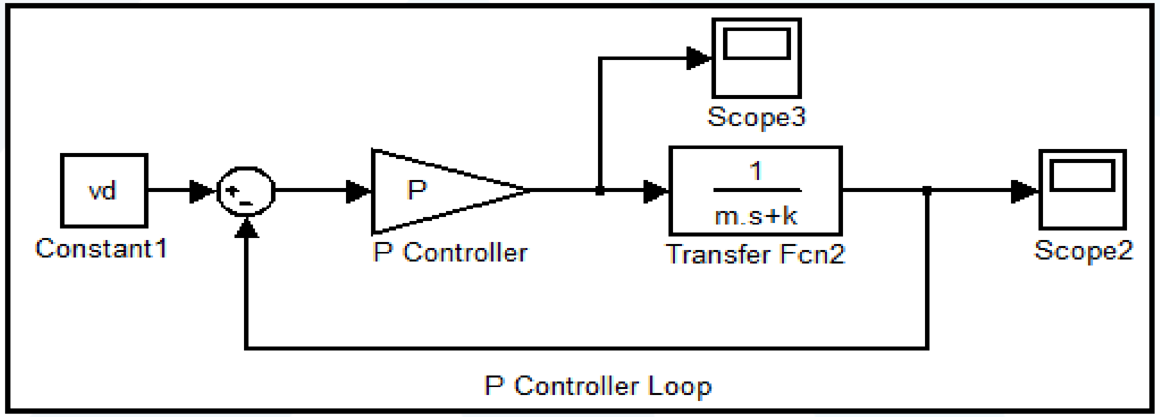
Derivative control actionprovides a means of obtaining a controller with high sensitivity. An advantage of using derivative control action is that it responds to the rate of change of the actuating error and can produce a significant correction before the magnitude of the actuating error becomes too large. Derivative control thus anticipates the actuating error, initiates an early corrective action, and tends to increase the stability of the system.

Example

Mathematical concept of proportional control process

$$mv' = F - kv \qquad v = \frac{F}{k} - \frac{F}{k}e^{-\frac{k}{m}t}$$

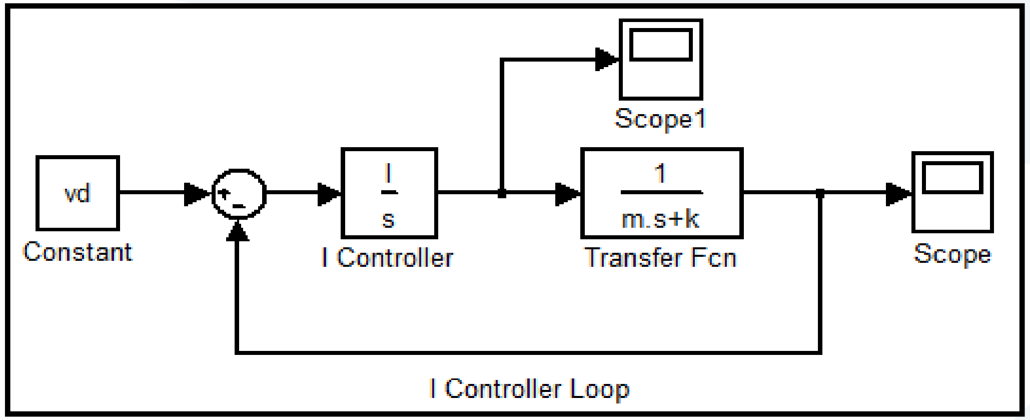




$$mv' = (v_d - v)P - kv$$

$$v = \frac{p}{k+p}v_d - \frac{p}{k+p}v_d e^{-(\frac{k+p}{m})t}$$

Mathematical concept of integral control process



$$\frac{v(s)}{v_d(s)} = \frac{I}{ms^2 + ks + I}$$

$$\frac{e(s)}{v_d(s)} = \frac{v_d(s) - v(s)}{v_d(s)} = \frac{ms^2 + ks}{ms^2 + ks + I}$$

$$e(t) = \lim_{s \to 0} se(s) = \lim_{s \to 0} s \frac{ms^2 + ks}{ms^2 + ks + I} \frac{v_d}{s} = 0$$

$$v = C_1 e^{\frac{-k + \sqrt{k^2 - 4mI}}{2m} t} + C_2 e^{\frac{-k - \sqrt{k^2 - 4mI}}{2m} t} + v_d$$

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