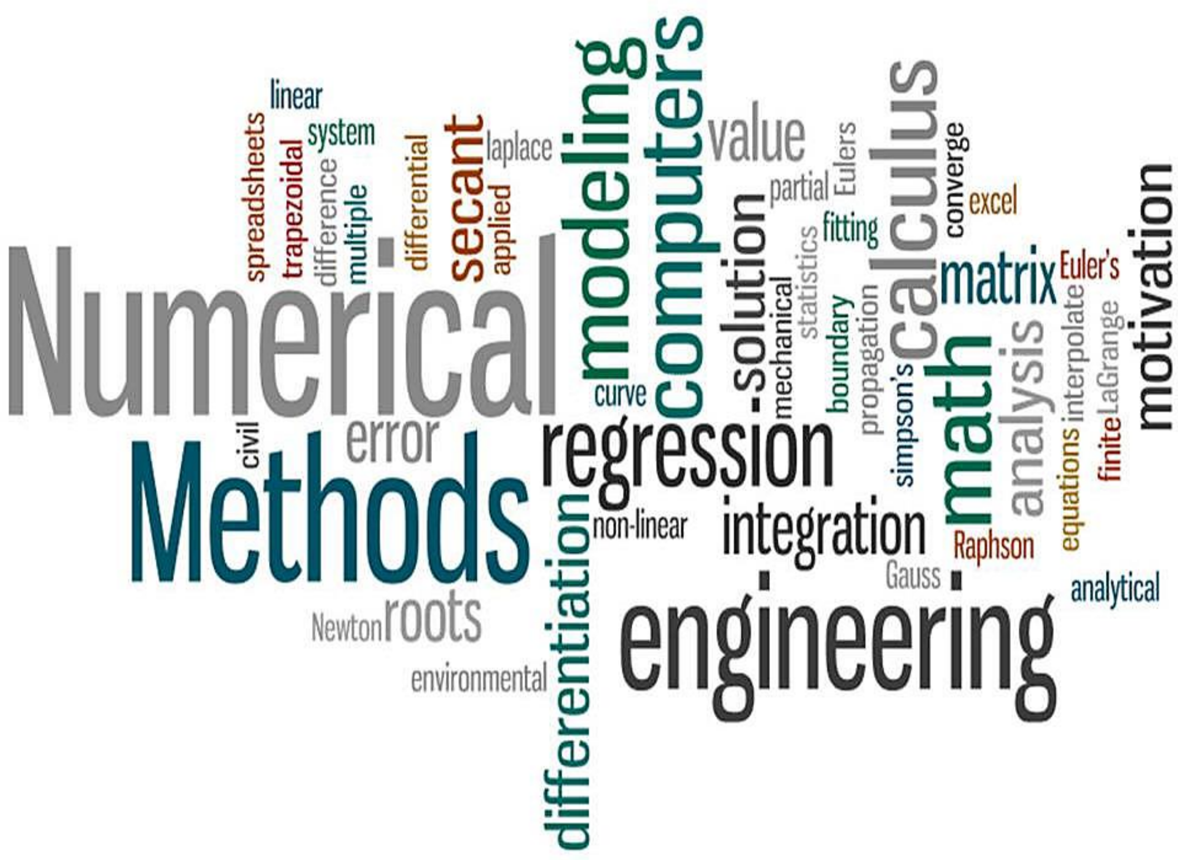
Numerical Analysis and Programming



Numerical Integration

Week-08

Numerical Integration

Newton Cotes Integration Formulas

- Trapezoidal Rules
- Simpson's Rules
- Unequal Segments
- Open Integration Formulas
- Multiple Integrals

Integration of Equations

- Newton Cotes for Equations
- Romberg Integration
- Gauss Quadrature

Newton Cotes Integration Formulas

Integration:

The process of measuring the area under a function plotted on a graph.

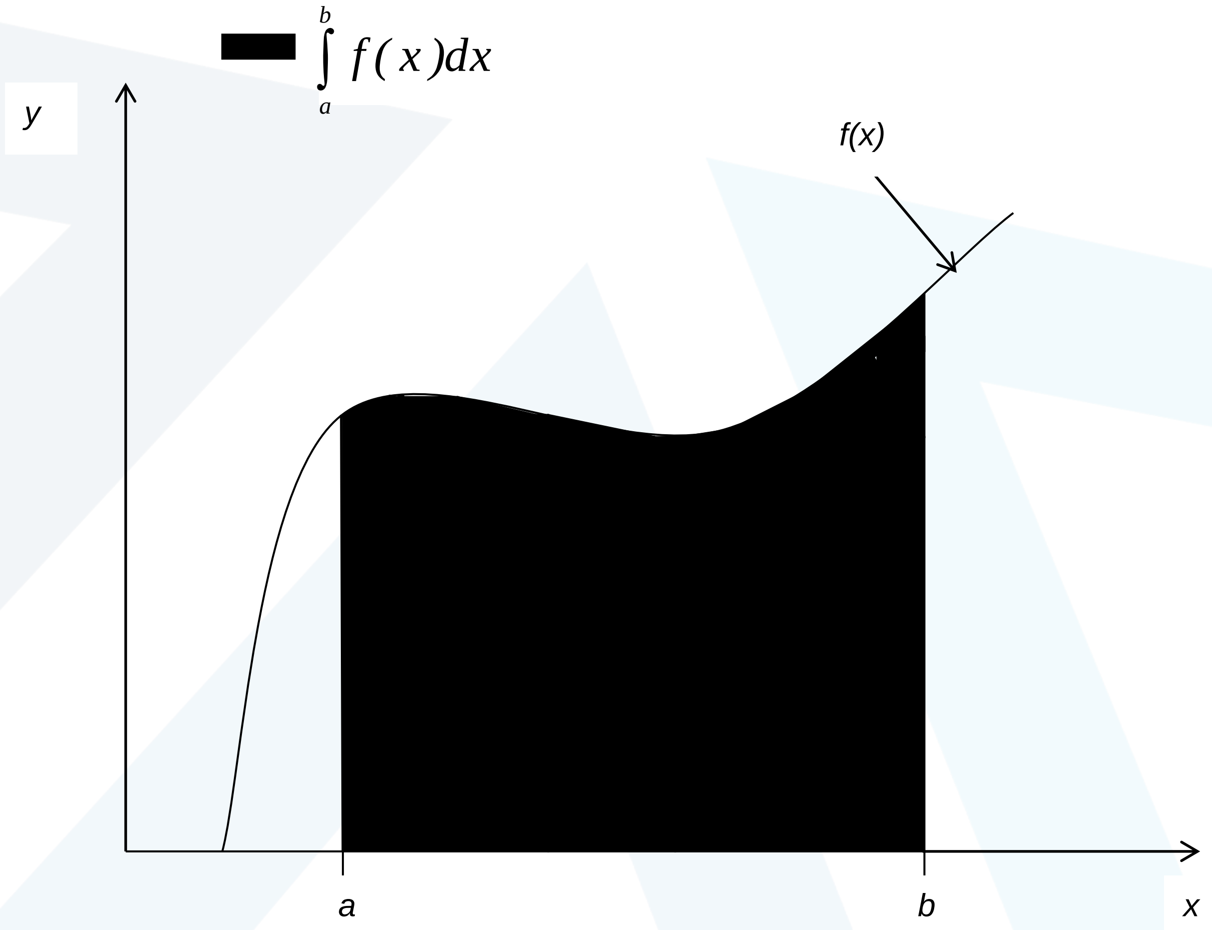
$$I = \int_a^b f(x)dx$$

Where:

f(x) is the integrand

a= lower limit of integration

b= upper limit of integration



Newton Cotes Integration Formulas

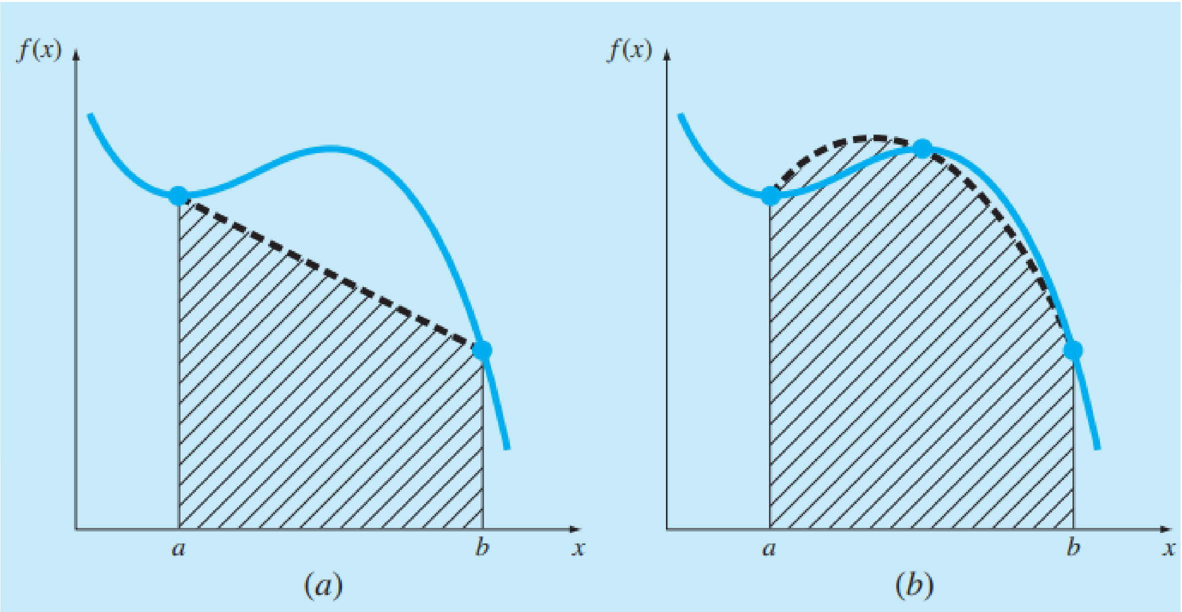
- The Newton-Cotes formulas are the most common numerical integration schemes. They are based on the strategy of replacing a complicated function or tabulated data with an approximating function that is easy to integrate:

$$I = \int_a^b f(x)d(x) \cong \int_a^b f_n(x)d(x) \qquad (1)$$

Where: $f_n(x)$ is a polynomial of the form:

$$f_n(x) = a_0 + a_1 x + \cdots + a_{n-1}x^{n-1} + a_n x^n$$

and n is the order of the polynomial.

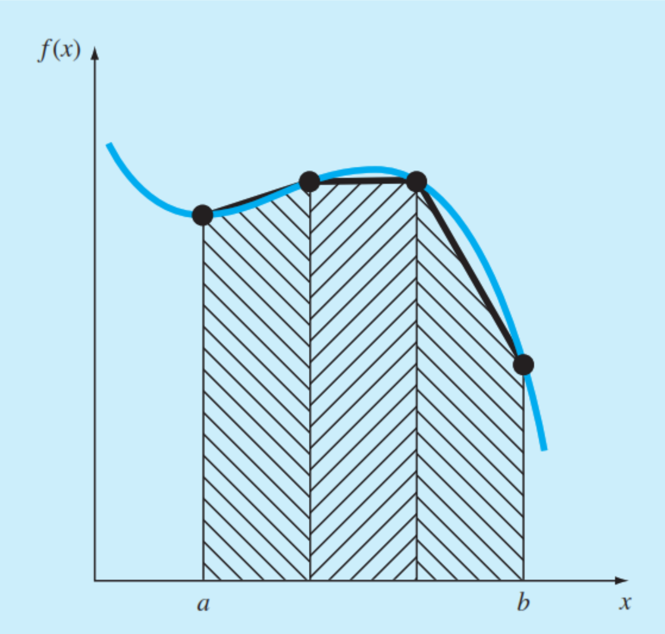


a first-order polynomial (a straight line) is used as an approximation

A parabola is employed for the an approximation

Newton Cotes Integration Formulas

- The integral can also be approximated using a series of polynomials applied piecewise to the function or data over segments of constant length.



three straight-line segments are used for approximation.

The Trapezoidal Rule

- The Trapezoidal rule is the first of the Newton-Cotes closed integration formulas. It corresponds to the case where the polynomial in equation (1) is first-order :

$$I = \int_a^b f(x)d(x) \cong \int_a^b f_1(x)d(x)$$

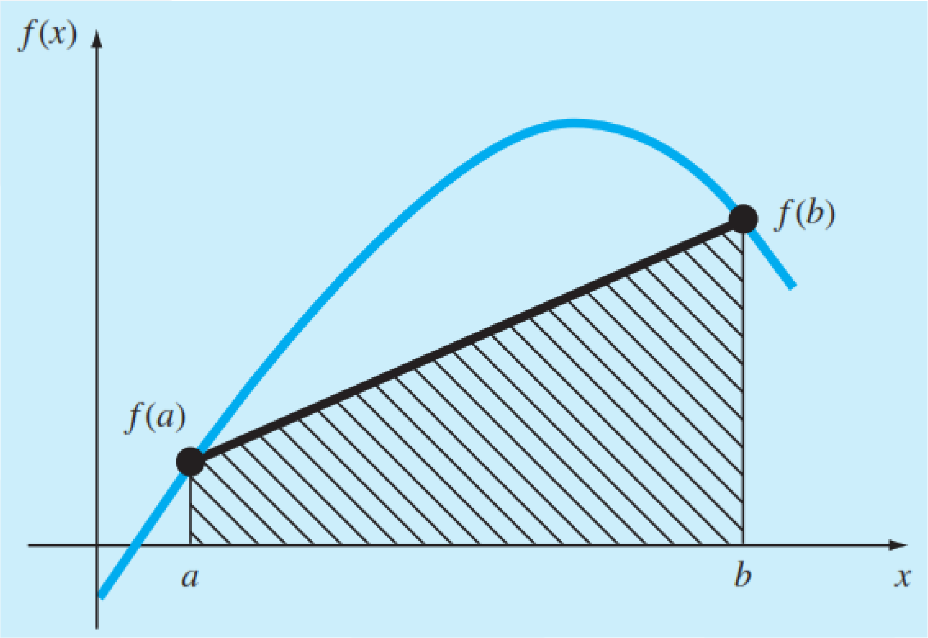
$$f_1(x) = f(a) + \frac{f(b) - f(a)}{b - a}(x - a)$$

The area under this first order polynomial is an estimate of the integral of *f(x)* between the limits of *a* and *b*:

$$I = \int_a^b \left[f(a) + \frac{f(b) - f(a)}{b - a}(x - a) \right] d(x)$$

$$I = (b - a)\frac{f(b) + f(a)}{2}$$

Trapezoidal rule



The Trapezoidal Rule

- **Example**

The vertical distance covered by a rocket from t=8 to t=30 seconds is given by:

$$x = \int_{8}^{30} \left(2000\, ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t \right) dt$$

a) Use single segment Trapezoidal rule to find the distance covered.

b) Find the true error, for part (a).

c) Find the absolute relative true error, for part (a).

The Trapezoidal Rule

- **Example**

a)

$$I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

$$a = 8 \qquad b = 30$$

$$f(t) = 2000\,ln\left[\frac{140000}{140000-2100t}\right] - 9.8t$$

$$f(8) = 2000\,ln\left[\frac{140000}{140000-2100(8)}\right] - 9.8(8) \qquad = 177.27\ m/s$$

$$f(30) = 2000\,ln\left[\frac{140000}{140000-2100(30)}\right] - 9.8(30) \quad = 901.67\ m/s$$

The Trapezoidal Rule

- **Example**

a) $$I = (30-8)\left[\frac{177.27+901.67}{2}\right]$$

$$= 11868\ m$$

b) The exact value of the above integral is

$$x = \int_{8}^{30}\left(2000\,ln\left[\frac{140000}{140000-2100t}\right]-9.8t\right)dt \quad = 11061\ m$$

The Trapezoidal Rule

- **Example**

b)

$$E_t = True\ Value - Approximate\ Value$$

$$= 11061 - 11868$$

$$= -807\ m$$

c) The absolute relative true error, $|\in_t|$, would be

$$|\in_t| = \left|\frac{11061 - 11868}{11061}\right| \times 100 = 7.2959\%$$