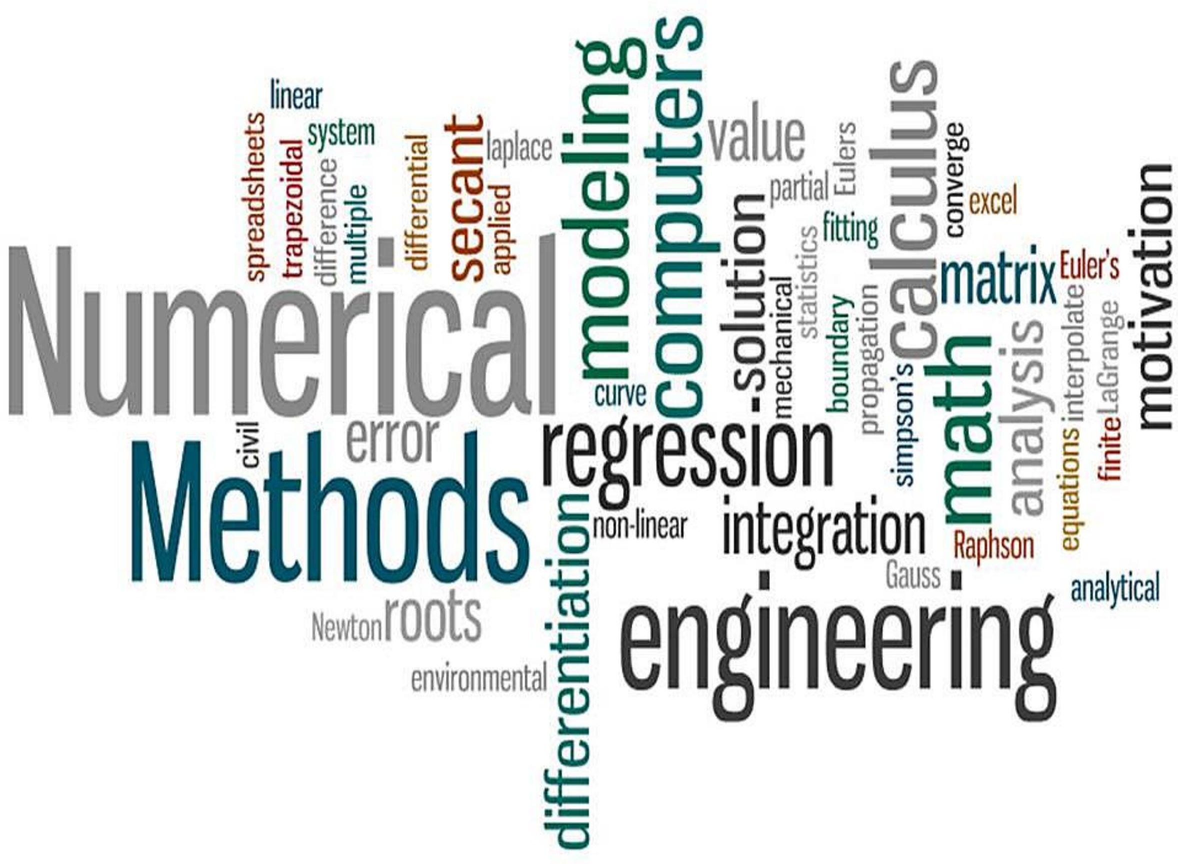
Numerical Analysis and Programming



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Numerical Analysis

Numerical Integration
Week-09

Numerical Integration

Newton Cotes Integration Formulas

- Trapezoidal Rules
- Simpson's Rules
- Unequal Segments
- Open Integration Formulas
- Multiple Integrals

Integration of Equations

- Newton Cotes for Equations
- Romberg Integration
- Gauss Quadrature

Multiple Segment Trapezoidal Rule

In Example 1, the true error using single segment trapezoidal rule was large. We can divide the interval [8,30] into [8,19] and [19,30] intervals and apply Trapezoidal rule over each segment.

$$f(t) = 2000\ln\left(\frac{140000}{140000-2100t}\right) - 9.8t$$

$$\int_{8}^{30} f(t)dt = \int_{8}^{19} f(t)dt + \int_{19}^{30} f(t)dt$$

$$= (19-8)\left[\frac{f(8)+f(19)}{2}\right] + (30-19)\left[\frac{f(19)+f(30)}{2}\right]$$

Multiple Segment Trapezoidal Rule

With

$$f(8)=177.27\ m/s$$

$$f(30)=901.67\ m/s$$

$$f(19)=484.75\ m/s$$

Hence:

$$\int_{8}^{30} f(t)dt=(19-8)\left[\frac{177.27+484.75}{2}\right]+(30-19)\left[\frac{484.75+901.67}{2}\right]$$

$$=11266\ m$$

Multiple Segment Trapezoidal Rule

The true error is:

$$E_t = 11061 - 11266$$
$$= -205\ m$$

The true error now is reduced from -807 m to -205 m.

Extending this procedure to divide the interval into equal segments to apply the Trapezoidal rule; the sum of the results obtained for each segment is the approximate value of the integral.

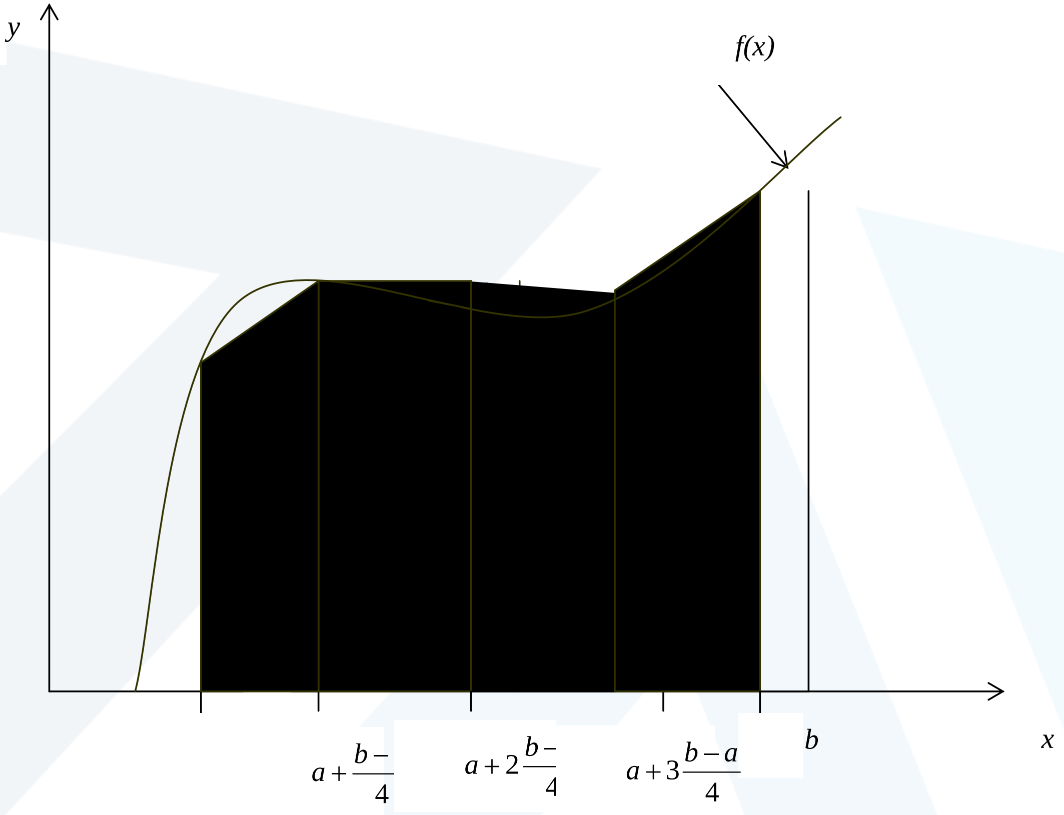
Multiple Segment Trapezoidal Rule

Divide into equal segments as shown in Figure. Then the width of each segment is:

$$h = \frac{b-a}{n}$$

The integral I is:

$$I = \int_a^b f(x)dx$$



Multiple (n=4) Segment Trapezoidal Rule

Multiple Segment Trapezoidal Rule

The integral I can be broken into h integrals as:

$$\int_a^b f(x)dx = \int_a^{a+h} f(x)dx + \int_{a+h}^{a+2h} f(x)dx + \ldots + \int_{a+(n-2)h}^{a+(n-1)h} f(x)dx + \int_{a+(n-1)h}^{b} f(x)dx$$

Applying Trapezoidal rule on each segment gives:

$$\int_a^b f(x)dx = \frac{b-a}{2n}\left[f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

Multiple Segment Trapezoidal Rule

Example-01: The vertical distance covered by a rocket from to seconds is given by:

$$x = \int_{8}^{30} \left(2000\, ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use two-segment Trapezoidal rule to find the distance covered.
b) Find the true error,   for part (a).
c) Find the absolute relative true error,   for part (a).

Multiple Segment Trapezoidal Rule

Example-01:

a) The solution using 2-segment Trapezoidal rule is

$$I = \frac{b-a}{2n}\left[f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

$$n = 2 \qquad a = 8 \qquad b = 30$$

$$h = \frac{b-a}{n} = \frac{30-8}{2} = 11$$

Multiple Segment Trapezoidal Rule

Example-01:

Then:

$$I = \frac{30-8}{2(2)}\left[f(8) + 2\left\{ \sum_{i=1}^{2-1} f(a+ih) \right\} + f(30) \right]$$

$$= \frac{22}{4}\left[f(8) + 2f(19) + f(30) \right]$$

$$= \frac{22}{4}\left[177.27 + 2(484.75) + 901.67 \right]$$

$$= 11266 \ m$$

Multiple Segment Trapezoidal Rule

Example-01:

b) The exact value of the above integral is

$$x = \int_{8}^{30} \left(2000\, ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt \quad = 11061\ m$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$

$$= 11061 - 11266$$

Multiple Segment Trapezoidal Rule

Example-01:

The absolute relative true error, $|\in_t|$, would be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100$$

$$= \left|\frac{11061 - 11266}{11061}\right| \times 100$$

$$= 1.8534\%$$

Multiple Segment Trapezoidal Rule

Example-01:

Table 1 gives the values obtained using multiple segment Trapezoidal rule for

$$x = \int_{8}^{30} \left(2000\, ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

Exact Value=11061 m

n	Value	E_t	$\|\in_t\|\%$	$\|\in_a\|\%$
1	11868	-807	7.296	---
2	11266	-205	1.853	5.343
3	11153	-91.4	0.8265	1.019
4	11113	-51.5	0.4655	0.3594
5	11094	-33.0	0.2981	0.1669
6	11084	-22.9	0.2070	0.09082
7	11078	-16.8	0.1521	0.05482
8	11074	-12.9	0.1165	0.03560

Table 1: Multiple Segment Trapezoidal Rule Values

Multiple Segment Trapezoidal Rule

Example-02: Use Multiple Segment Trapezoidal Rule to find the area under the curve

$$f(x)=\frac{300x}{1+e^{x}} \quad \text{from} \quad x=0 \quad \text{to} \quad x=10$$

Using two segments, we get $h=\frac{10-0}{2}=5$ and

$$f(0)=\frac{300(0)}{1+e^{0}}=0 \qquad f(5)=\frac{300(5)}{1+e^{5}}=10.039 \qquad f(10)=\frac{300(10)}{1+e^{10}}=0.136$$

Multiple Segment Trapezoidal Rule

Example-02: Use Multiple Segment Trapezoidal Rule to find the area under the curve

Then:

$$I = \frac{b-a}{2n}\left[f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

$$= \frac{10-0}{2(2)}\left[f(0) + 2\left\{ \sum_{i=1}^{2-1} f(0+5) \right\} + f(10) \right]$$

$$= \frac{10}{4}\left[f(0) + 2f(5) + f(10) \right]$$

$$= 50.535$$

Multiple Segment Trapezoidal Rule

Example-02: Use Multiple Segment Trapezoidal Rule to find the area under the curve

So what is the true value of this integral?

Making the absolute relative true error:

$$|\in_t| = \left|\frac{246.59 - 50.535}{246.59}\right| \times 100\%$$

$$= 79.506\%$$

Multiple Segment Trapezoidal Rule

Example-02: **Table 2:** Values obtained using Multiple Segment Trapezoidal Rule for:

$$\int_0^{10} \frac{300x}{1+e^x}\,dx$$

n	Approximate Value	E_t	$\lvert\in_t\rvert$
1	0.681	245.91	99.724%
2	50.535	196.05	79.505%
4	170.61	75.978	30.812%
8	227.04	19.546	7.927%
16	241.70	4.887	1.982%
32	245.37	1.222	0.495%
64	246.28	0.305	0.124%