

Steel Structures 2 Summer Sem. 2023-2024

أ.د. نايل محمد حسن

Lecture 1-2

- ✓ **Flexural Members**
- ✓ **-I- Restrained Beams**

Flexural Members

-I- Restrained Beams

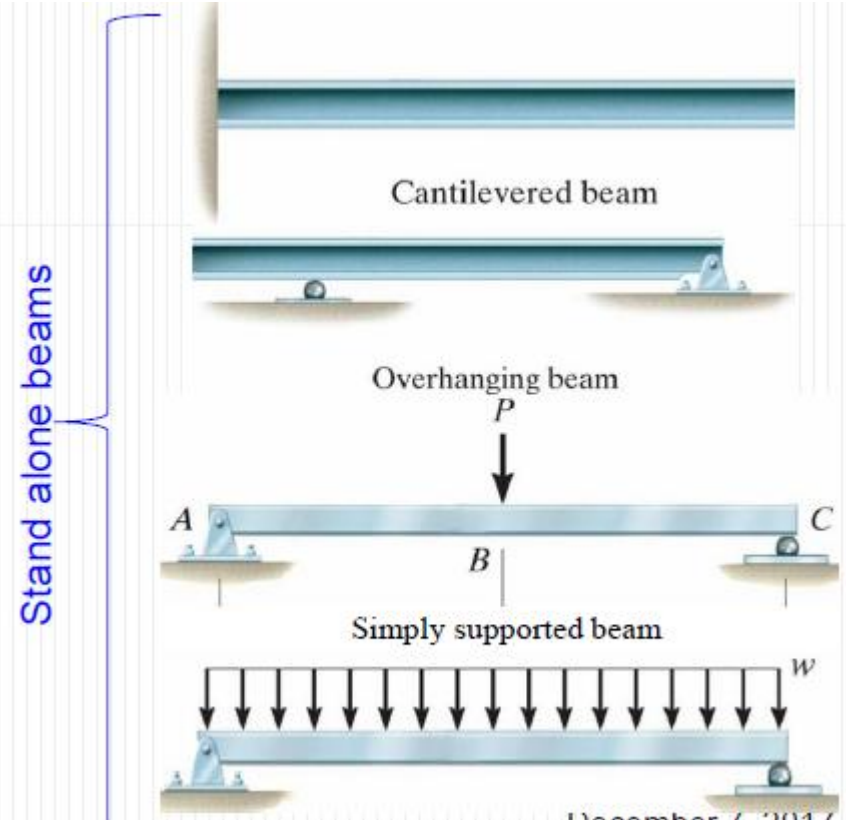
Flexural Members -I- Restrained Beams

Beams in structures

- ❑ Beam is predominately subjected to bending.
- ❑ A beam is a structural member which is subjected to transverse loads, and accordingly must be designed to withstand shear and moment.
- ❑ Generally, it will be bent about its major axis.

Flexural Members : Introduction

Beams in structures



Flexural Members : Introduction

Beams in Buildings



Flexural members are the second most common structural members in frame structures.

Flexural Members : Introduction

Beams in Buildings-Construction and installation



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Flexural Members : Introduction

Beams in Buildings-Construction and installation



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Flexural members are the second most common structural members in frame structures.

Flexural Members : Introduction

Beams in Bridges

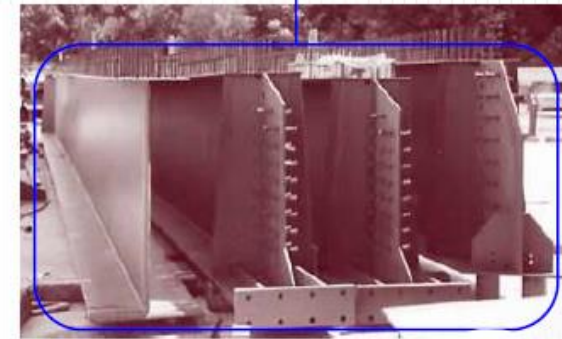
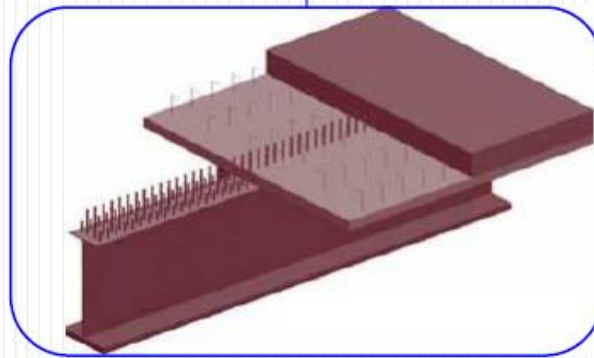
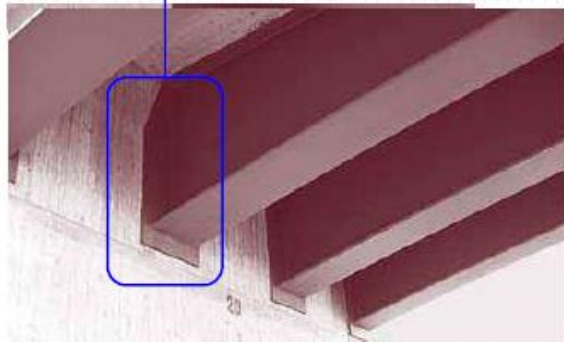
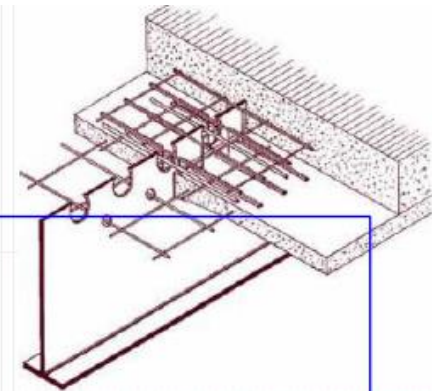
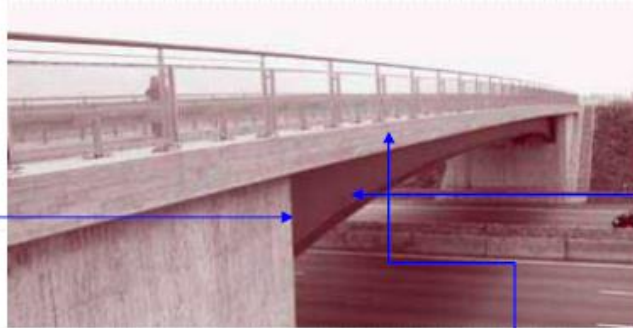


Flexural members are the second most common structural members in frame structures.

Flexural Members : Introduction

Beams in Bridges

Beam and deck connections in Bridges



Flexural members are the second most common structural members in frame structures.

Flexural Members : Introduction

Beams in Bridges-Construction and installation

Beams in Bridges-Construction and installation

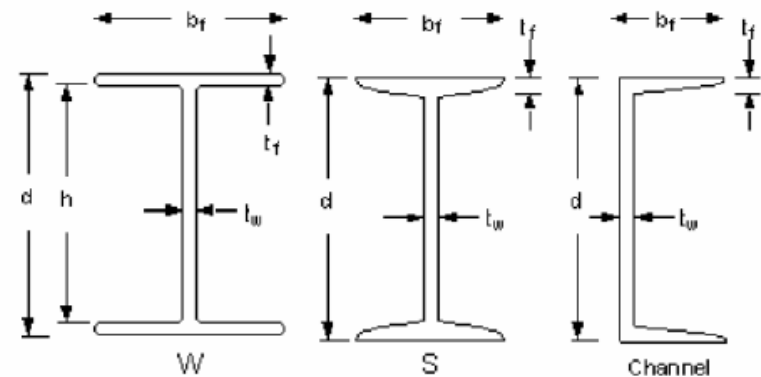


Flexural members are the second most common structural members in frame structures.

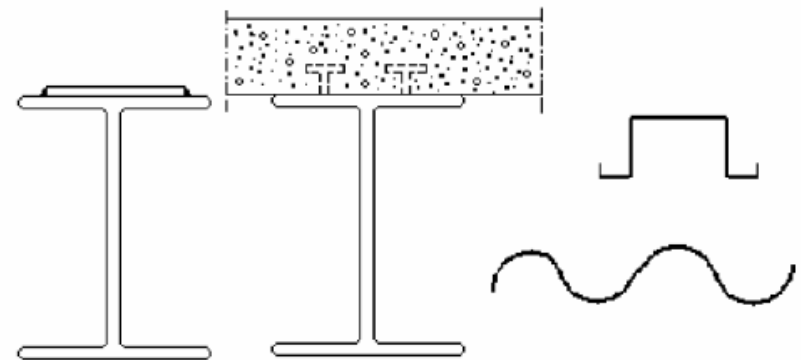
Introduction: Section Profiles for Flexural Members

Beam cross-sections may take many different forms, as shown below, and these represent various methods of obtaining an efficient and economical member.

- ▶ Thus, most steel beams are **not of solid cross-section**, but have their material distributed more efficiently in **thin walls**.
- ▶ Thin-walled **sections** may be **open**, and while these tend to be **weak in torsion**, they are often **cheaper to manufacture** than the **stiffer closed sections**.
- ▶ The **most economic method** of manufacturing steel beams is by **hot-rolling**, but only a **limited number** of open cross-sections is available.
- ▶ A **substitute** may be **fabricated** by **connecting** together a series of rolled plates.



(a)



(b)

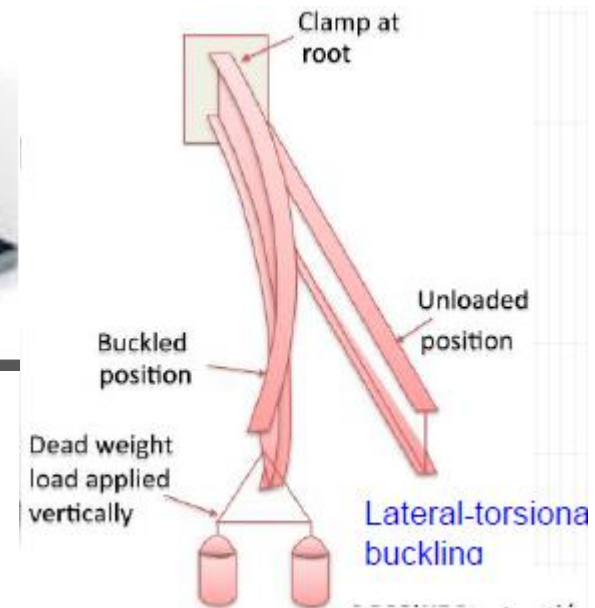
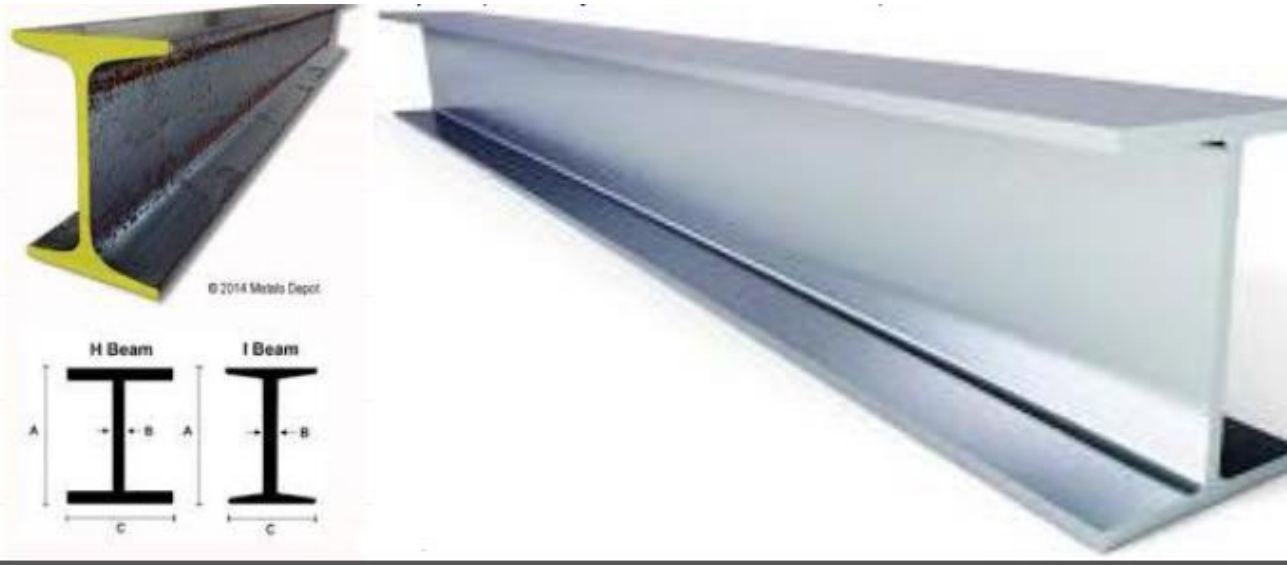
(c)

(d)

Introduction: Classification of Flexural Members

The resistance of a steel beam in bending depends on;

- ▶ the cross section resistance or
- ▶ the occurrence of lateral instability.



Introduction: Classification of Flexural Members

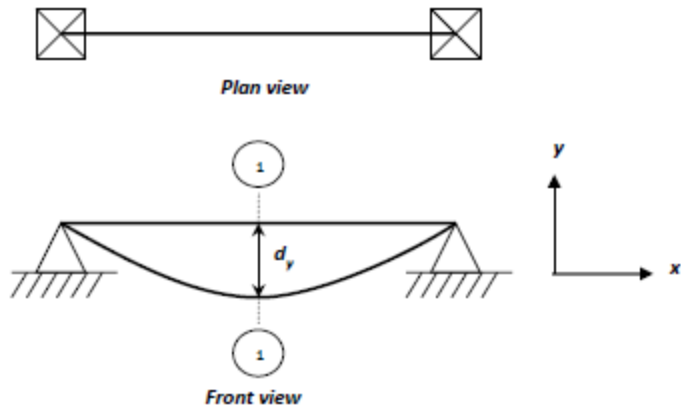
Whenever one of the following situations occurs in a beam, lateral-torsional buckling cannot develop and assessment of the beam can be based just on the cross section resistance:

- **The cross section of the beam is bent about its minor z axis;**
- **The beam is laterally restrained by means of secondary steel members, by a concrete slab or any other method that prevents lateral displacement of the compressed parts of the cross section;**
- **The cross section of the beam has high torsional stiffness and similar flexural stiffness about both principal axes of bending as, for example, closed hollowcross sections.**

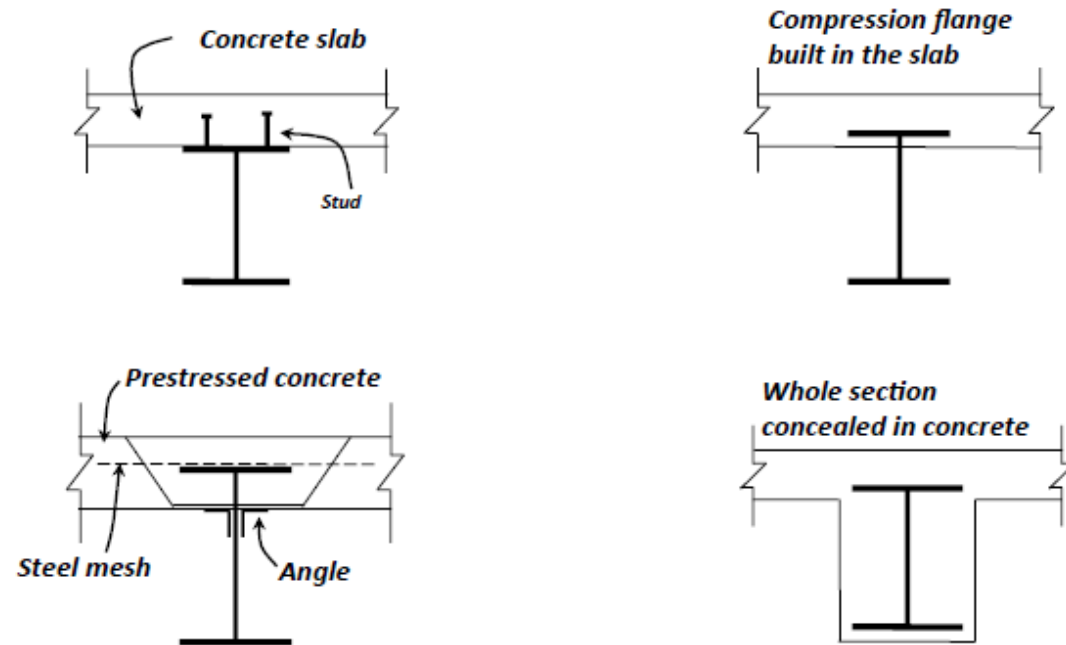
Introduction: Laterally Restrained Beams

Types of restraining condition of beam

1- **Restrained Beam** A beam where the compression flange is restrained against lateral deflection and rotation. Only vertical deflection exists.

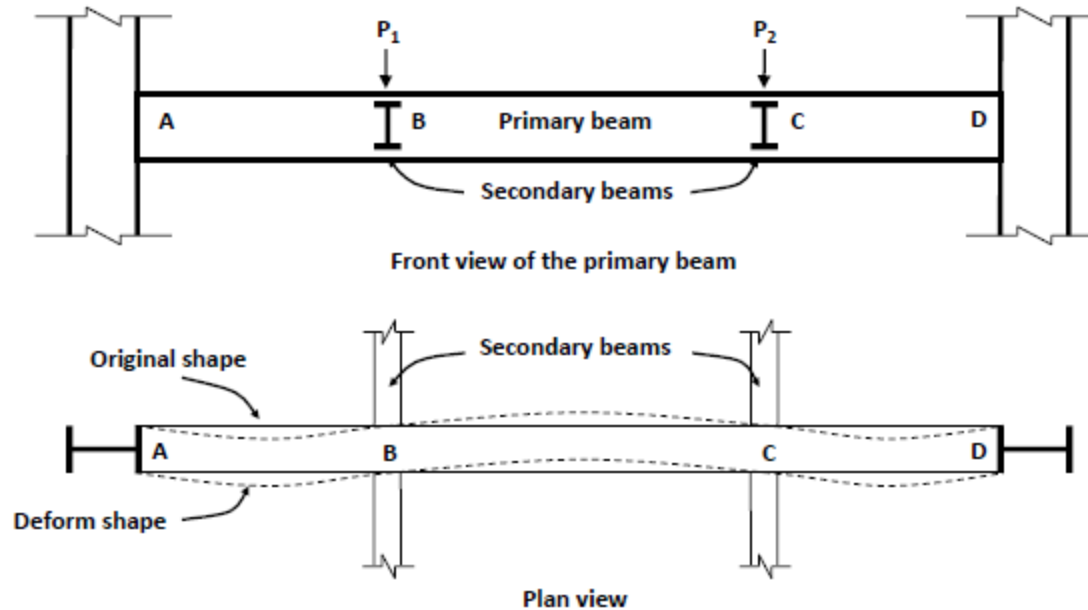


2- A full lateral restraint may be provided by concrete floor which sufficiently connected to the beam, or by sufficient bracing members added.



Introduction: Laterally Restrained Beams

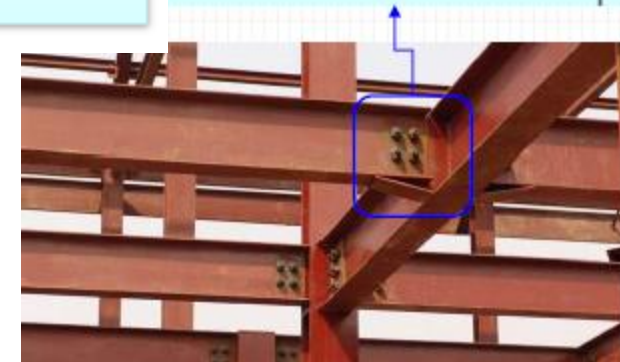
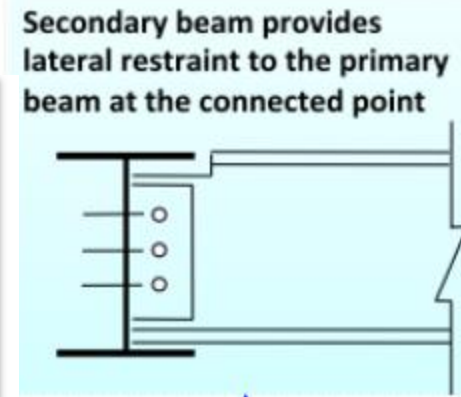
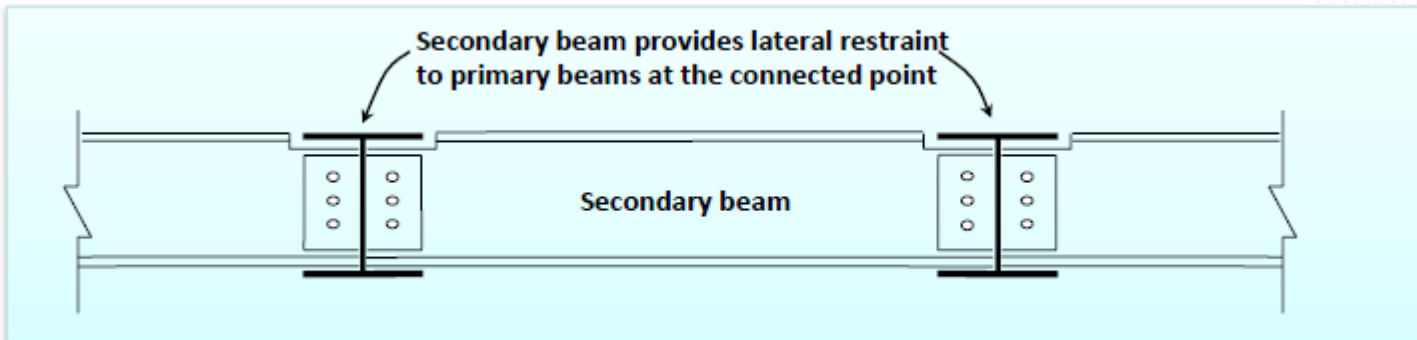
Laterally restraint may be of along the span or at some points along the span.



Points A, B, C and D are restrained from deform laterally by the secondary beams and the connection at column

Introduction: Laterally Restrained Beams

By means of secondary steel members:



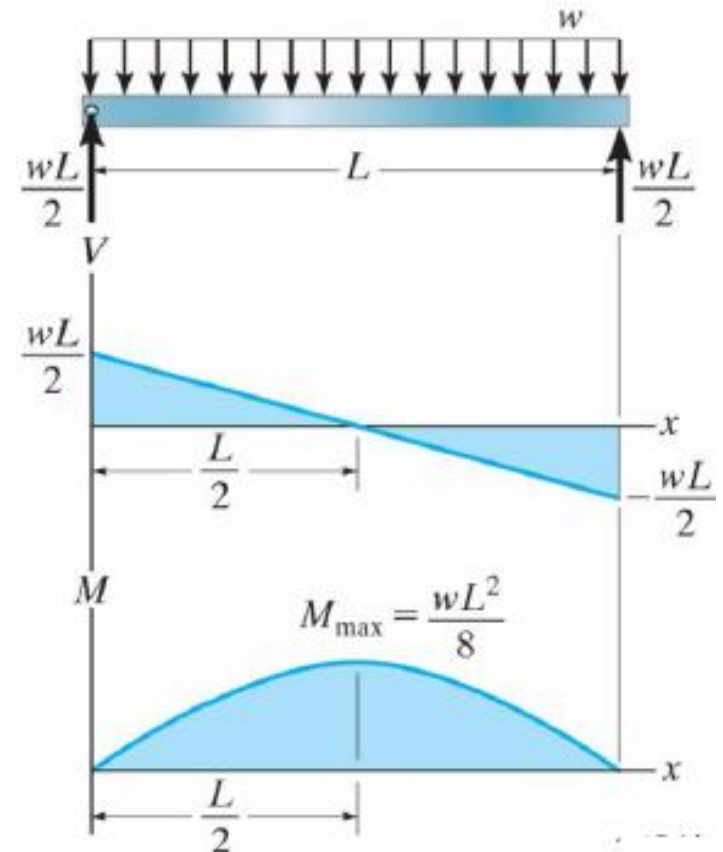
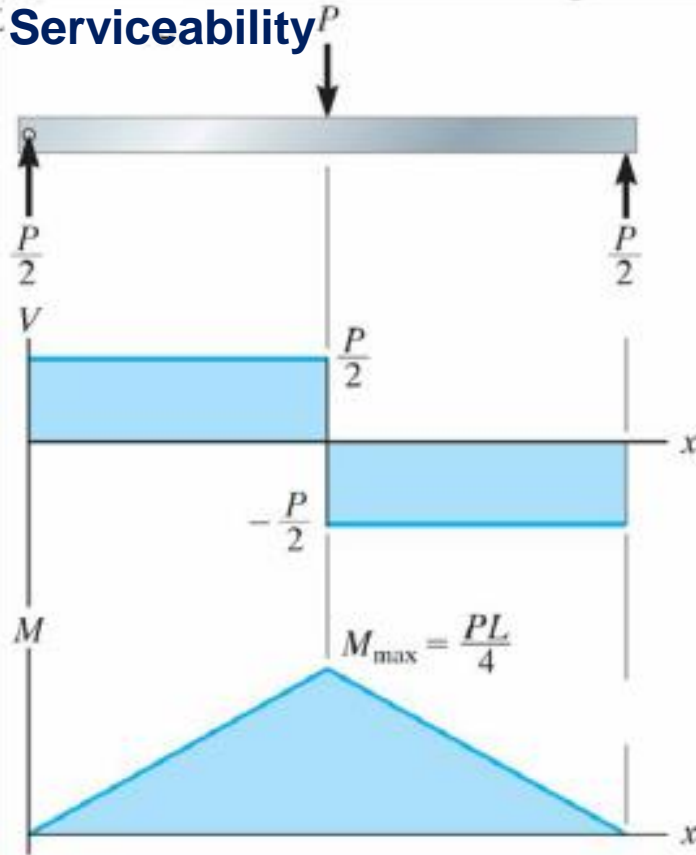
Introduction: Laterally Restrained Beams



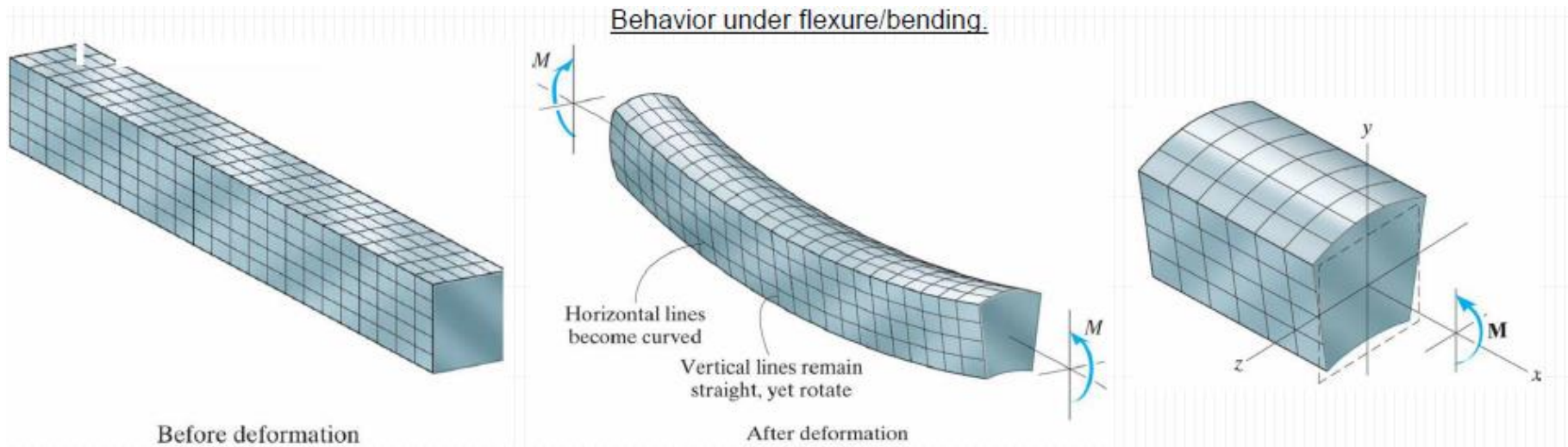
Introduction: Laterally Restrained Beams

Beam under a transverse load is analyzed and designed for the following criteria .

- Bending (Uniaxial or Biaxial)
- Shear Combined effect of Shear and Bending
- And Serviceability



Introduction: Laterally Restrained Beams



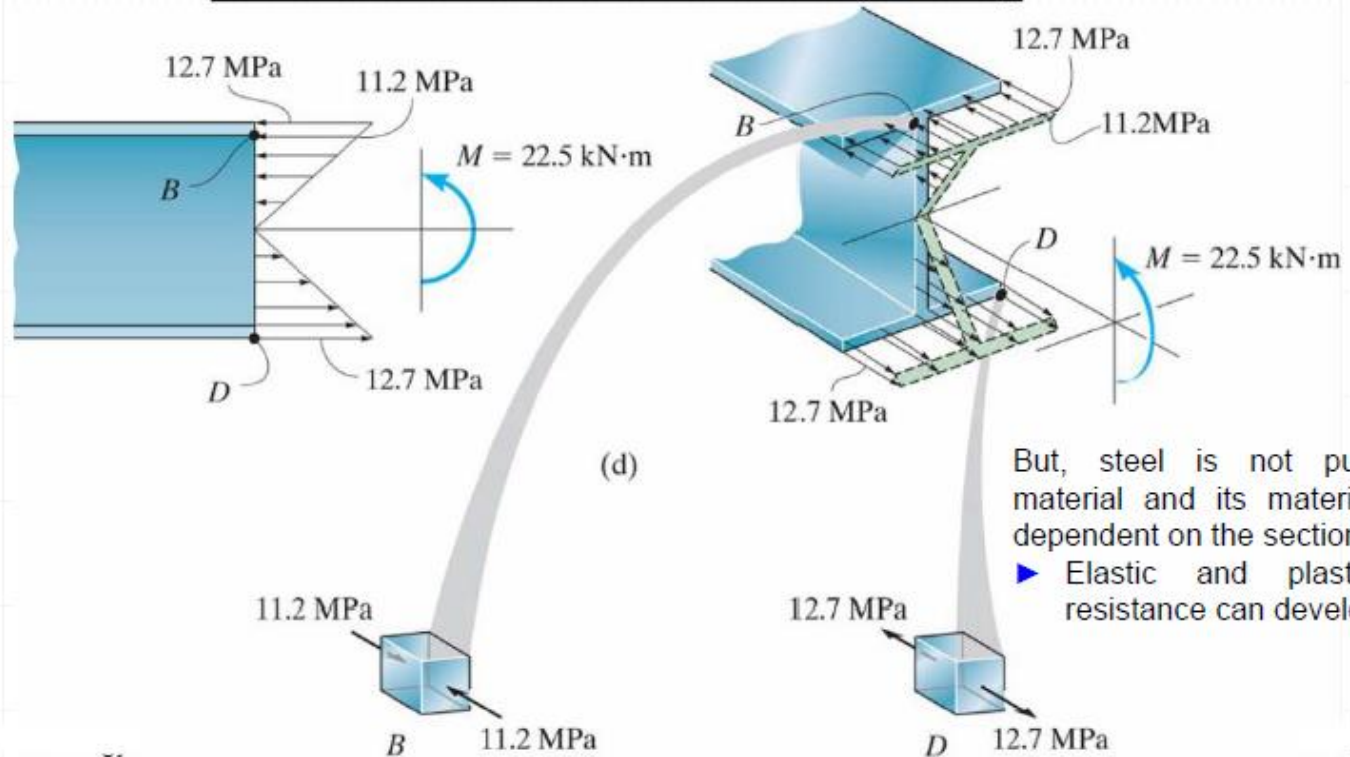
The assumption “plane section remains plane” also applies for steel section:

- Strain varies linearly.

Introduction: Laterally Restrained Beams

Stress distribution for elastic section under flexure/bending.

Example:

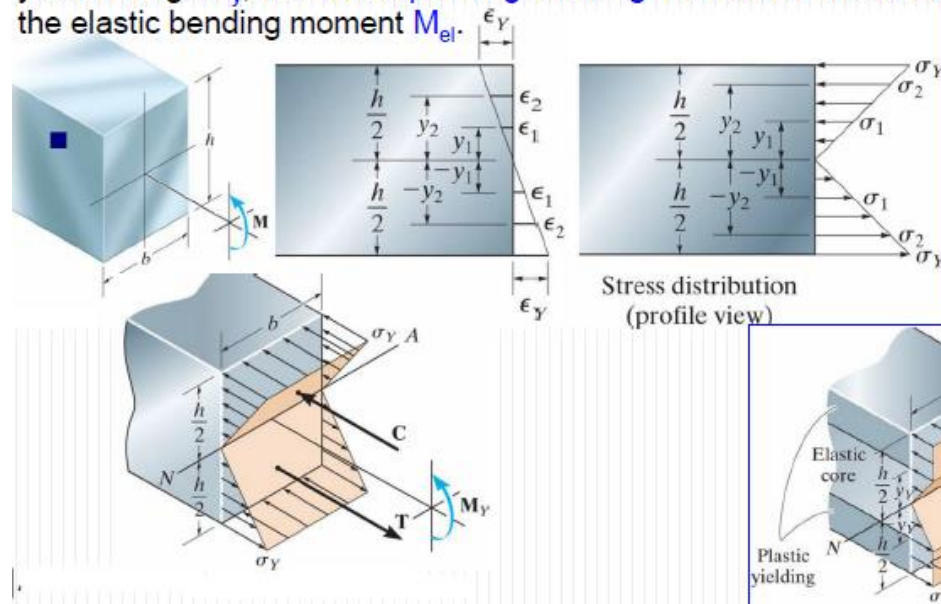


Introduction: Laterally Restrained Beams

Elastic and plastic bending moment resistance of steel section.

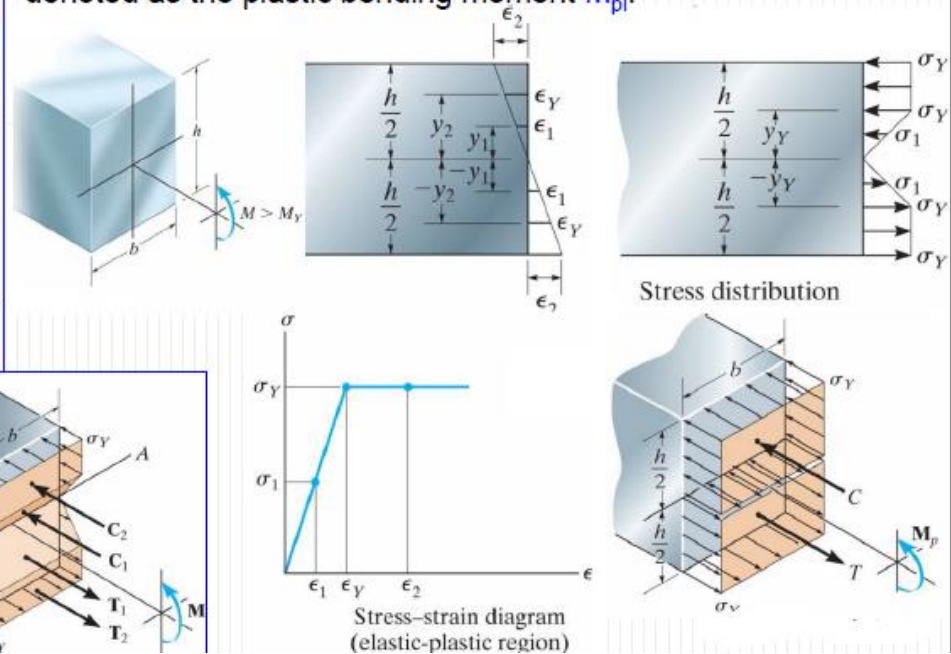
Elastic bending moment resistance.

of a cross section is attained when the normal stress in the point furthest away from the elastic neutral axis (e. n. a.) reaches the yield strength f_y ; the corresponding bending moment is denoted the elastic bending moment M_{el} .



Plastic bending moment resistance.

The bending moment that is able to totally plastify a section is denoted as the plastic bending moment M_{pl} .



Introduction: Laterally Restrained Beams Bending

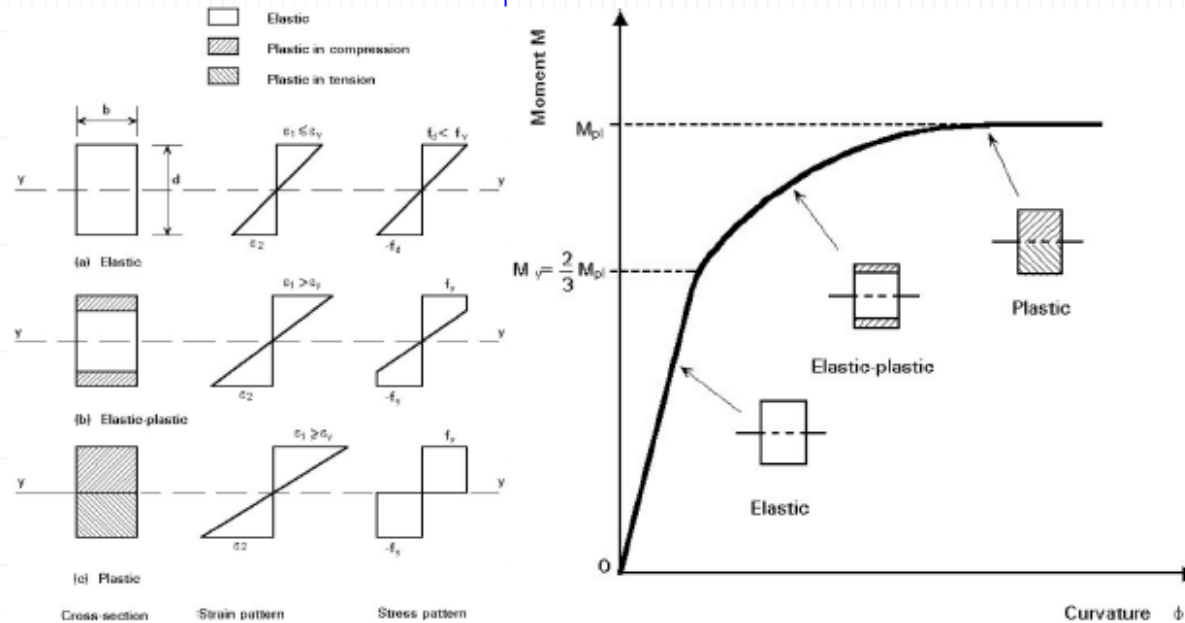
Elastic and plastic bending moment resistance of steel section.

Elastic bending moment resistance.

Plastic bending moment resistance.

An alternative way to capture the sense of elastic and plastic moment resistance is through the **moment curvature relationship**

Moment-curvature behavior of a **rectangular** cross-section in bending

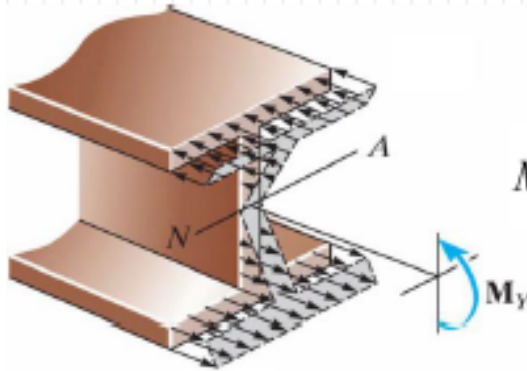


Introduction: Laterally Restrained Beams Bending

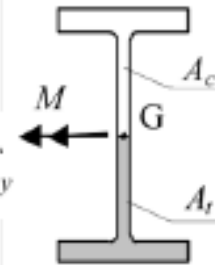
Elastic and plastic bending moment resistance of steel section.

Elastic bending moment resistance.

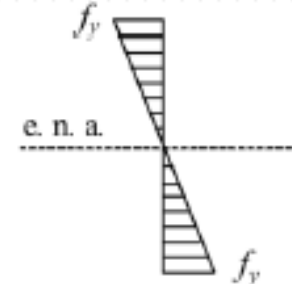
A steel cross section (assuming equal yield strengths in tension and compression), the elastic neutral axis (e.n.a.) is located at the centroid only if the section is symmetrical.



$$M_{el} = \frac{I}{v} f_y = W_{el} f_y$$



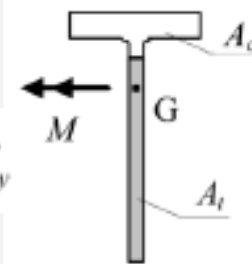
I or H section



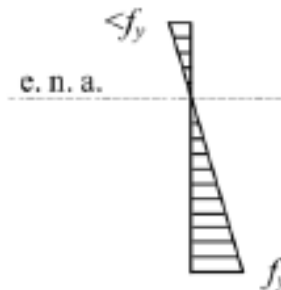
$M = M_{el}$

In case of non-symmetric cross sections, such as a T-section, the neutral axis moves in order to divide the section in two equal areas.

$$M_{el} = \frac{I}{v} f_y = W_{el} f_y$$



T section

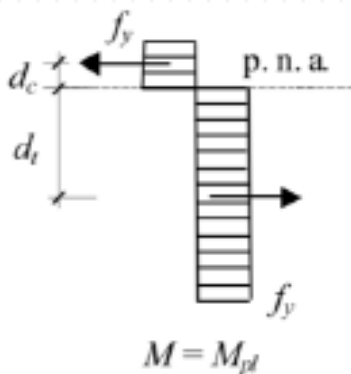
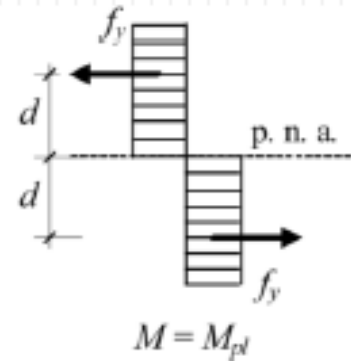


$M = M_{el}$

Introduction: Laterally Restrained Beams Bending

Plastic bending moment resistance

Similarly, the plastic neutral axis (p.n.a.) is located at the centroid for these sections



$$M_{pl} = A_c f_y d_c + A_t f_y d_t = (S_c + S_t) f_y = W_{pl} f_y$$

where,

I is the second moment of area about the elastic neutral axis (coincident with the centroid of the cross section);

v is the maximum distance from an extreme fiber to the same axis;

$W_{el} = I / v$ is the elastic bending modulus;

A_c and A_t are the areas of the section in compression and in tension, respectively (of equal value);

f_y is the yield strength of the material;

d_c and d_t are the distances from the centroid of the areas of the section in compression and in tension, respectively, to the plastic neutral axis;

W_{pl} is the plastic bending modulus, given by the sum of first moment of areas A_c and A_t , in relation to the plastic neutral axis ($W_{el} = S_c + S_t$).

Elastic and plastic bending moment resistance of steel section.

Introduction: Laterally Restrained Beams Bending in EC1993-1-1

Uniaxial bending.

In the absence of shear forces, the design value of the bending moment M_{Ed} at each cross section should satisfy:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1.0$$

where $M_{c,Rd}$ is the design resistance for bending.

The design resistance for bending about one principal axis of a cross section is determined as follows: Class 1 or 2 cross sections

$$M_{c,Rd} = W_{pl} f_y / \gamma_{M0}$$

- Class 3 cross sections

$$M_{c,Rd} = W_{el,min} f_y / \gamma_{M0}$$

- Class 4 cross sections

$$M_{c,Rd} = W_{eff,min} f_y / \gamma_{M0}$$

Where,

W_{pl}

is the plastic bending modulus

$W_{el,min}$

is the minimum elastic section bending modulus

$W_{eff,min}$

is the minimum elastic bending modulus of the reduced effective section

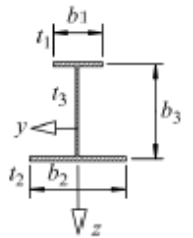
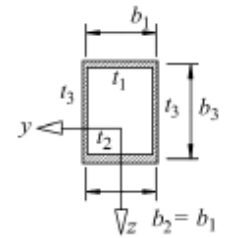

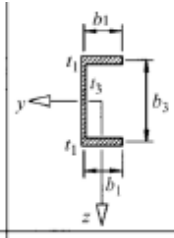
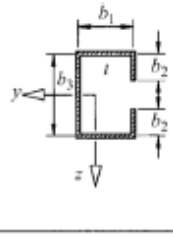
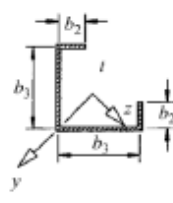
f_y

is the yield strength of the material

γ_{M0}

is the partial safety factor

Introduction: Laterally Restrained Beams Bending in EC1993-1-1

Section Geometry				Section Geometry			
A	$A_1 + A_2 + A_3$	$A_1 + A_2 + 2A_3$	πdt	A	$2A_1 + A_3$	$2A_1 + 2A_2 + 2A_3$	$\frac{2\sum A_n}{2}$
I_y	$\sum_3 A_n z_n^2 + I_3$	$A_1 z_1^2 + A_2 z_2^2 + 2A_3 z_3^2 + 2I_3$	$\pi d^3 t / 8$	I_y	$A_1 b_3^2 / 2 + I_3$	$\frac{A_1 b_3^2 + I_3 + I_2 + A_2 (b_3 - b_2)^2}{2}$	$\frac{A_2 (b_3 - b_2)^2}{2} + A_3 b_3^2 / 4 + \sum_2 A_n$
I_z	$I_1 + I_2$	$I_1 + I_2 + A_3 b_1^2 / 2$	$\pi d^3 t / 8$	I_z	$2A_1 y_1^2 + A_3 y_3^2 + 2I_1$	$2A_1 y_1^2 + 2A_2 y_2^2 + A_3 y_3^2 + 2I_1$	$A_2 b_3^2 / 4 + A_3 b_2^2 / 4 + \sum_2 I_n$
$W_{el,y,1,2}$	$I_y / z_{1,2}$	I_y / z_n	$\pi d^2 t / 4$	$W_{el,y}$	$2I_y / b_3$	$2I_y / b_3$	$\sqrt{2} I_y / b_3$
$W_{el,z,1,2}$	$2I_z / b_{1,2}$	$2I_z / b_1$	$\pi d^2 t / 4$	$W_{el,z}$	$I_z / (b_1 - y_3)$ and I_z / y_3	I_z / y_2 and I_z / y_3	$2\sqrt{2} I_z / (b_2 + b_3)$
$W_{pl,y,1,2}$	$\sum_2 (A_n z_{pn} + z_{pn}^2 t_3 / 2)$	$\sum_2 (A_n z_{pn} + z_{pn}^2 t_3)$	$d^2 t$	$W_{pl,y}$	$A_1 b_3 + A_3 b_3 / 4$	$A_1 b_3 + A_2 (b_3 - b_2) + A_3 b_3 / 4$	$(b_3^2 + 2b_2 b_3 - b_2^2) t / 2\sqrt{2}$
$W_{pl,z,1,2}$	$\sum_2 A_n b_n / 4$	$\sum_2 A_n b_n / 4 + A_3 b_1$	$d^2 t$	$W_{pl,z}$	$(b_1 - y_p)^2 t_1 + y_p^2 t_1 + A_3 y_p$	$\{(b_1 - y_p)^2 + y_p^2 + 2b_2 (b_1 - y_p) + (b_3 y_p)\} t$	$(b_2 + b_3)^2 t / 2\sqrt{2}$
y_0	0	0	0	y_0	$y_3 + \frac{A_1 b_3^2 b_1}{4I_y}$	$y_3 + \{b_3^2 (b_1 + 2b_2) - 8b_2^3 / 3\} \frac{A_1}{4I_y}$	$\frac{(b_2 + b_3) / 2\sqrt{2} + (3b_3 - 2b_2) b_2^2 b_3 t}{3\sqrt{2} I_y}$
z_0	$b_3 \left\{ \frac{(I_2 - I_1)}{2I_z} - \frac{(A_2 - A_1)}{2A} \right\}$	$b_3 \left\{ \frac{(I_2 - I_1)}{2I_z} - \frac{(A_2 - A_1)}{2A} \right\}$	0	z_0	0	0	0
$A_n = b_n t_n, \quad I_n = b_n^3 t_n / 12$ $z_{1,2} = b_3 (A_{2,1} + A_3 / 2) / A$ $z_3 = (z_2 - z_1) / 2$				$A_n = b_n t_n, \quad I_n = b_n^3 t_n / 12$			

Introduction: Laterally Restrained Beams Bending in EC1993-1-1

Net area in bending

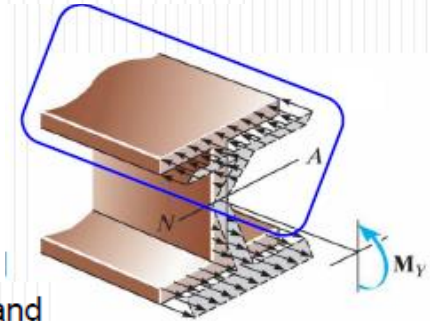
For plate members in Tension Zone

- ▶ Holes in the tension flange for bolts or other connection members may be ignored if the following condition is satisfied,

$$A_{f,net} 0.9 f_u / \gamma_{M2} \geq A_f f_y / \gamma_{M0}$$

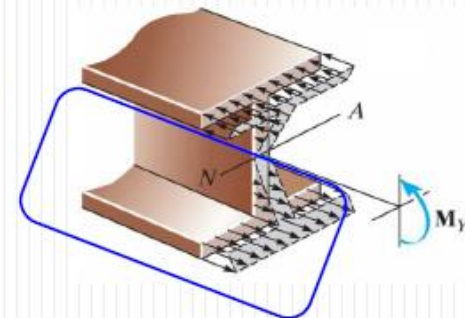
where $A_{f,net}$ and A_f are the net section and the gross area of the tension flange, respectively, and γ_{M2} is a partial safety factor (defined according to (EC3-1-8)).

- ▶ A similar procedure must be considered for holes in the tensioned part of a web, as described in clause 6.2.5(5) of EC3-1-1.



For plate members in Compression Zone

- ▶ The holes in the compressed parts of a section may be ignored, except if they are slotted or oversize, provided that they are filled by fasteners (bolts, rivets, etc...).





Steel Structures 2 Summer Sem. 2023-2024

أ.د. نايل محمد حسن

Lecture 3-4

Flexural Members

- ✓ -II- Laterally Restrained Beams
- ✓ II- Unrestrained Beams

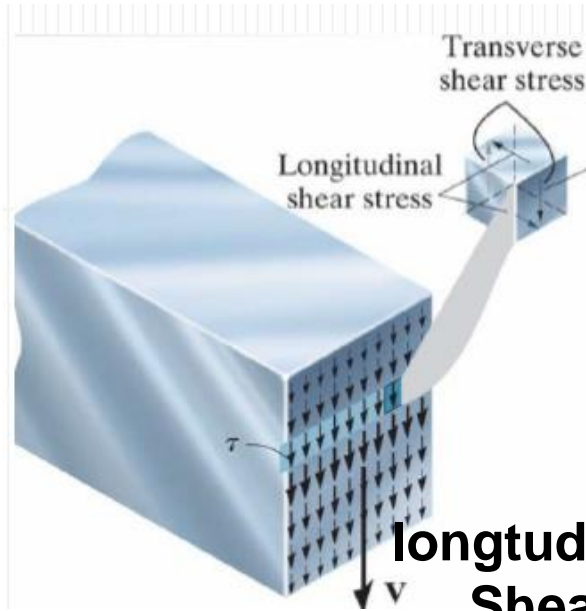


Introduction: Laterally Restrained Beams

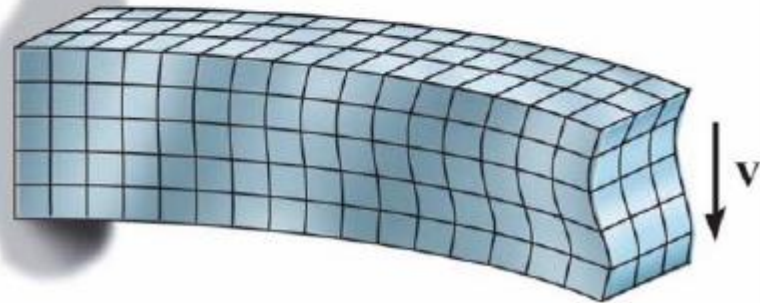
Behavior under Shear.



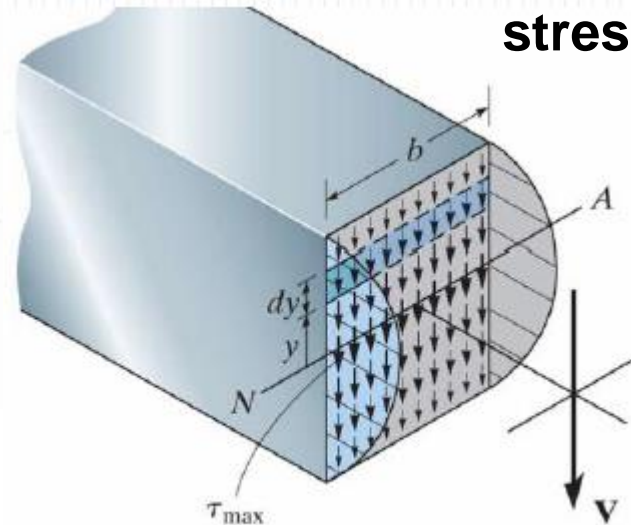
(a) Before deformation



**Transverse
Shear
stress**



(b) After deformation



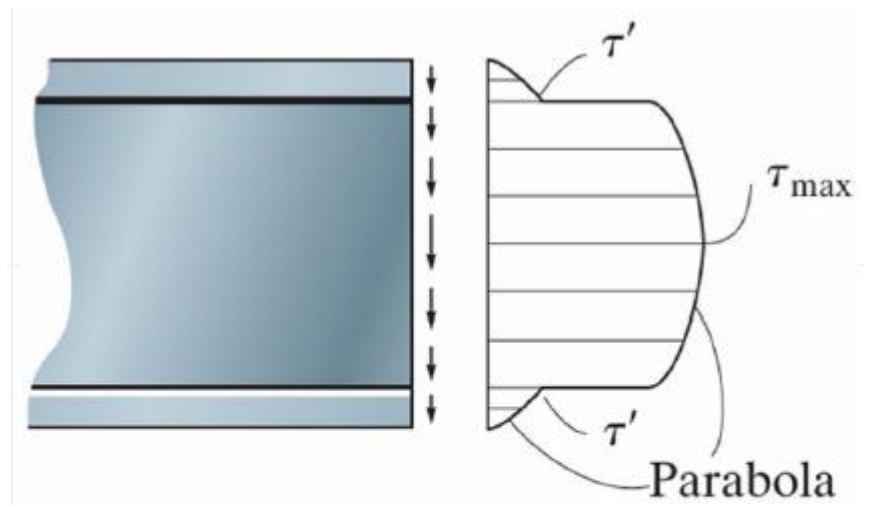
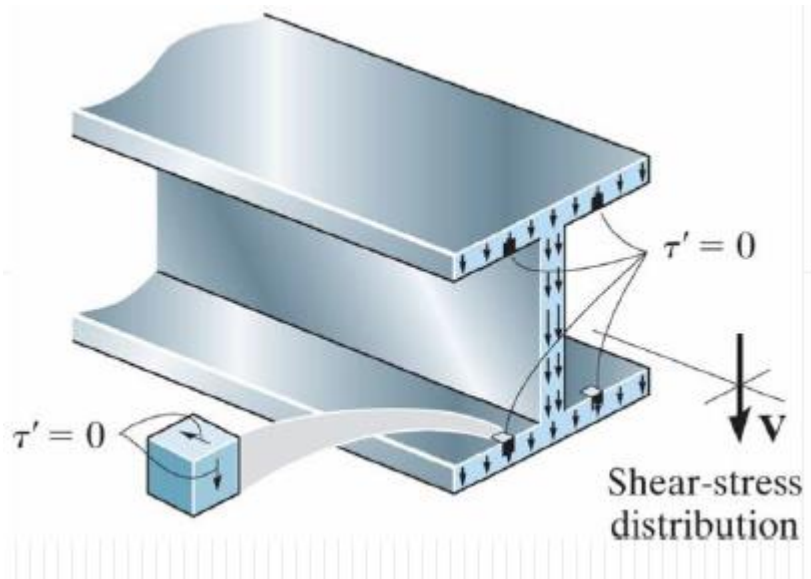
**Longitudinal
Shear
stress**



Shear-stress distribution

Introduction: Laterally Restrained Beams

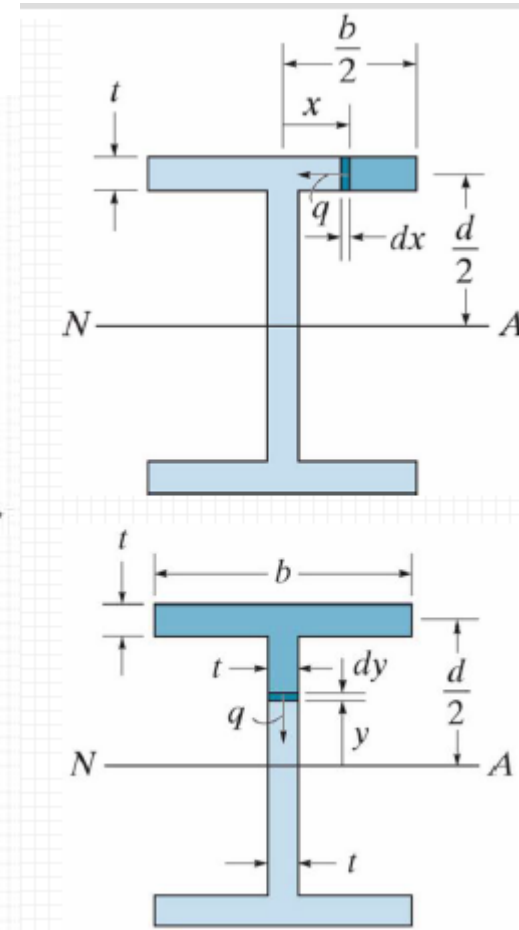
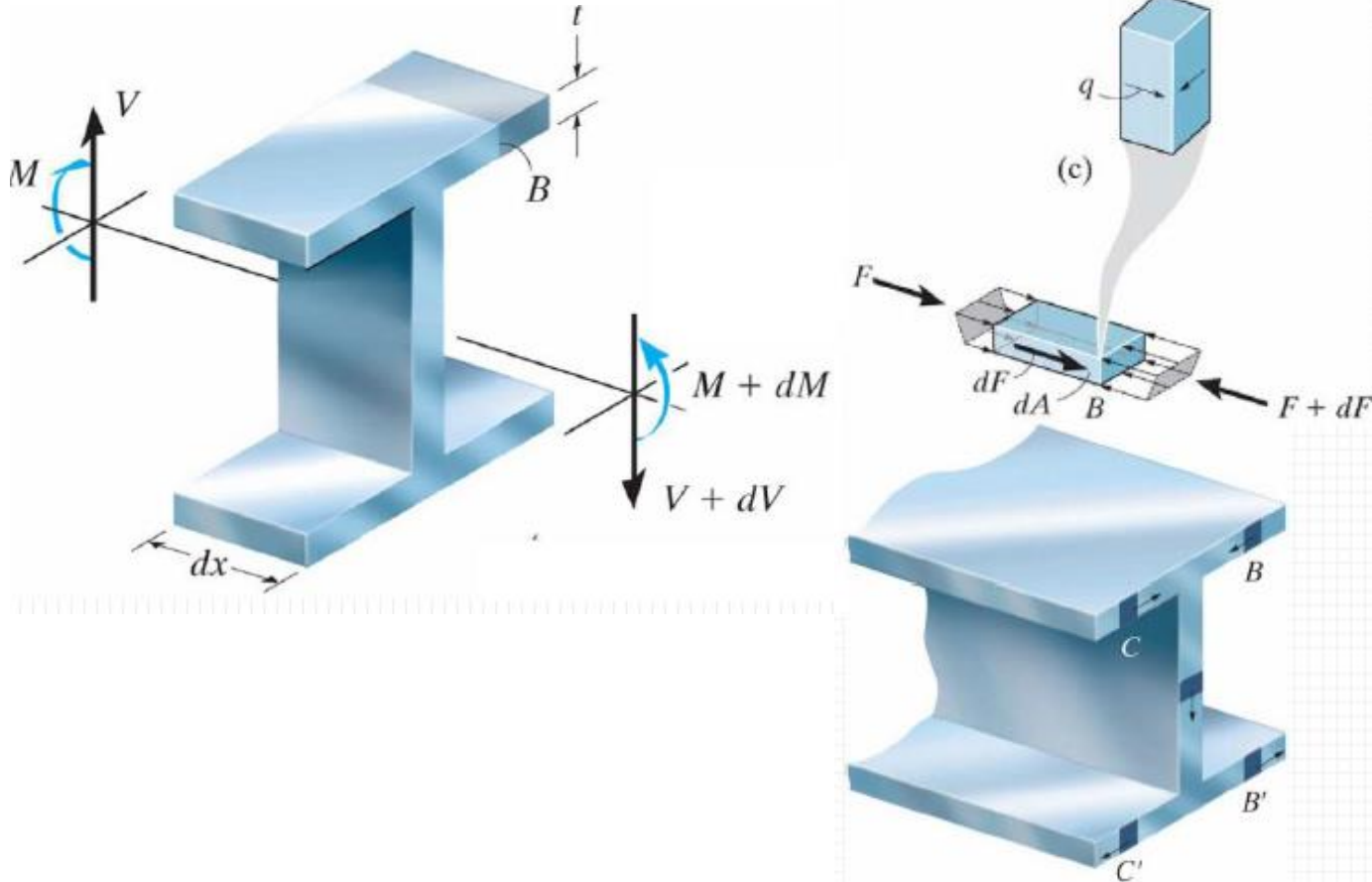
Shear Stress Distribution.



Intensity of shear-stress distribution
(profile view)

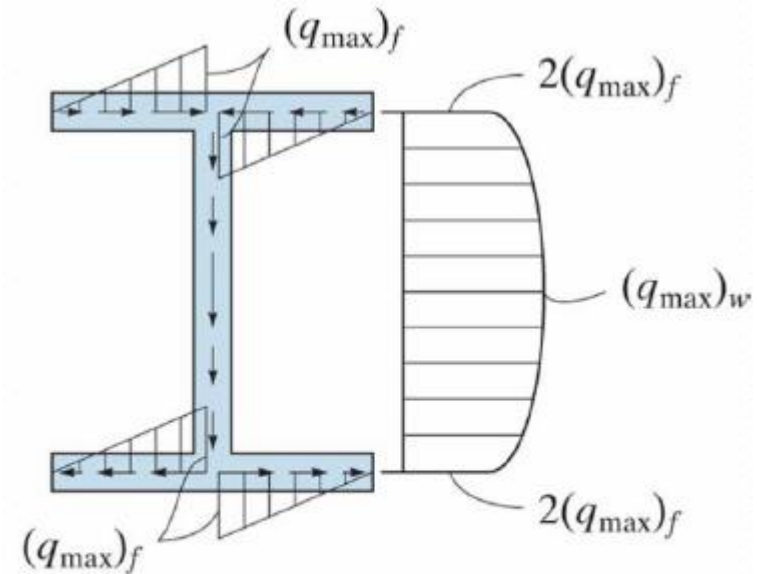
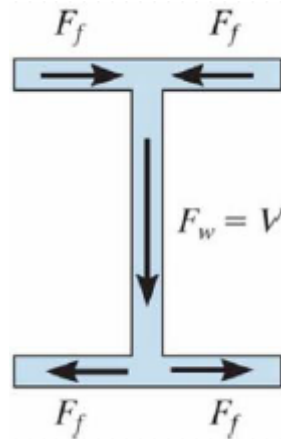
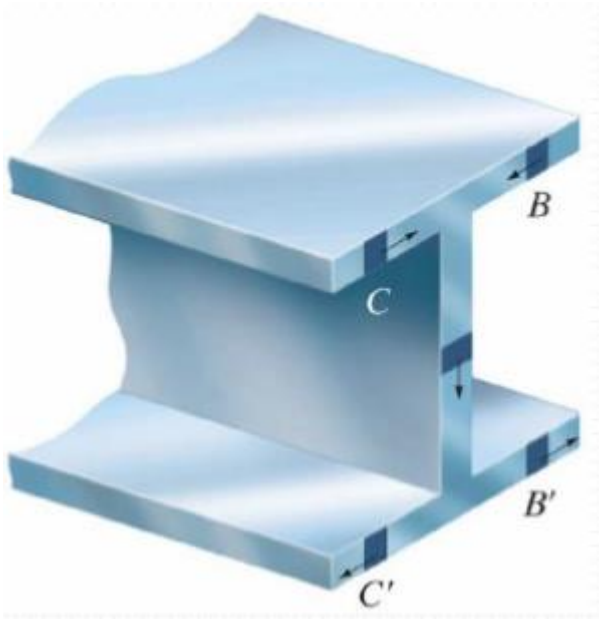
Introduction: Laterally Restrained Beams

Shear Flow



Introduction: Laterally Restrained Beams

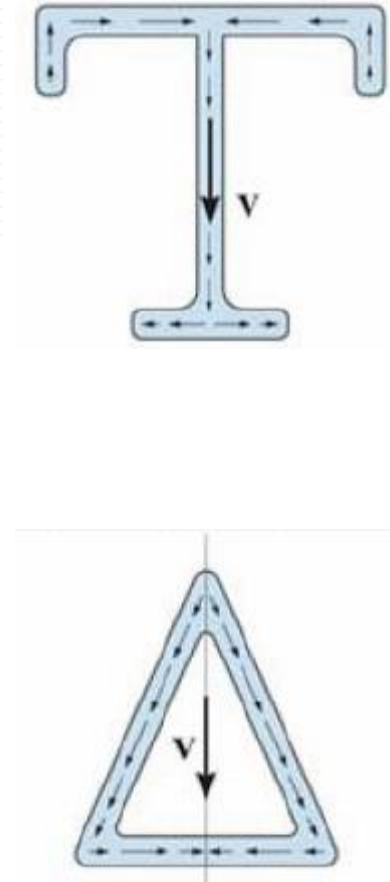
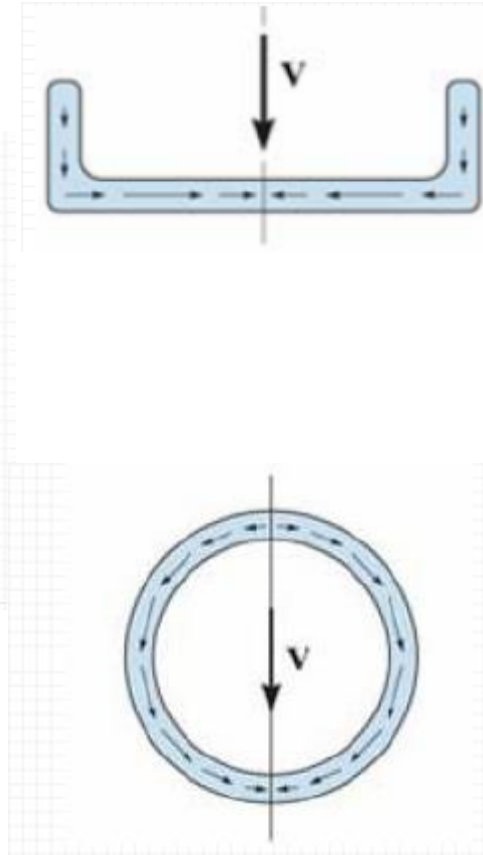
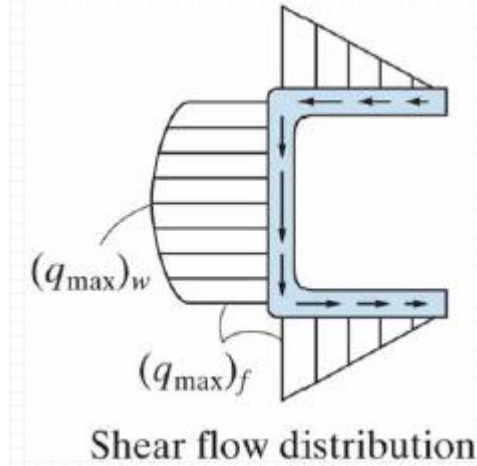
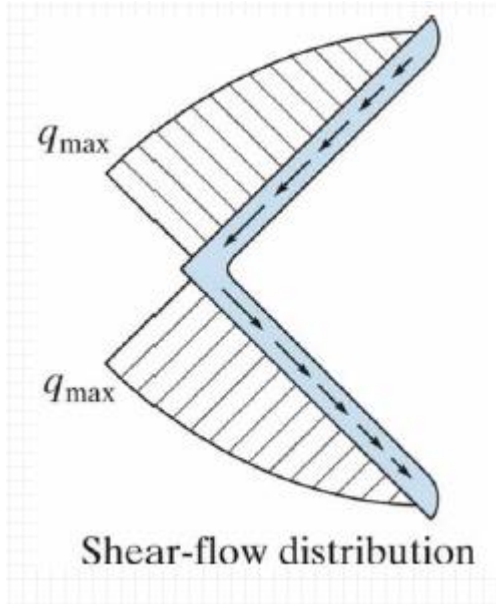
Shear Flow



Shear-flow distribution

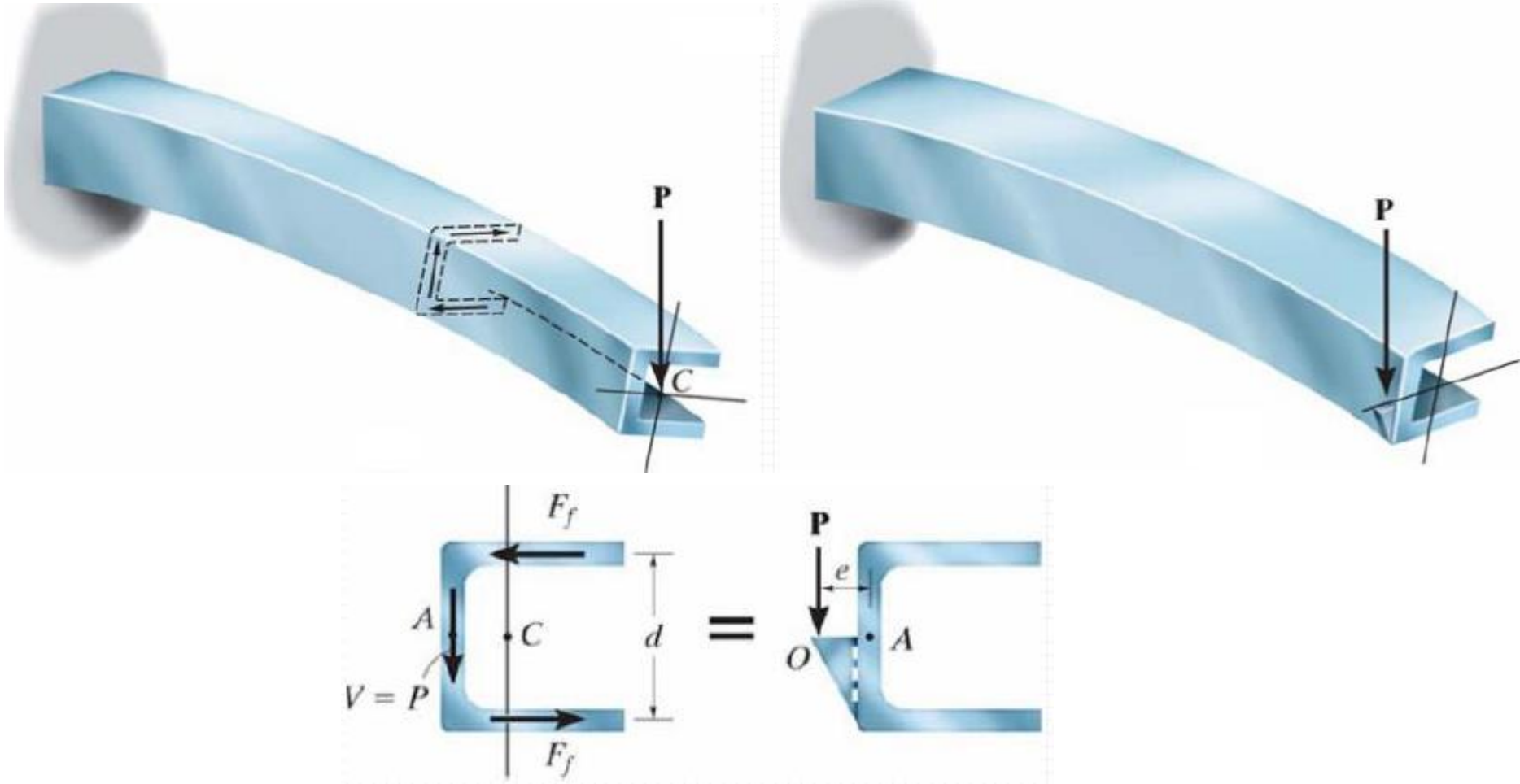
Introduction: Laterally Restrained Beams

Shear Flow



Introduction: Laterally Restrained Beams-Bending-In EC1993-1-1

Shear Center



Introduction: Laterally Restrained Beams

Shear Flow Effect



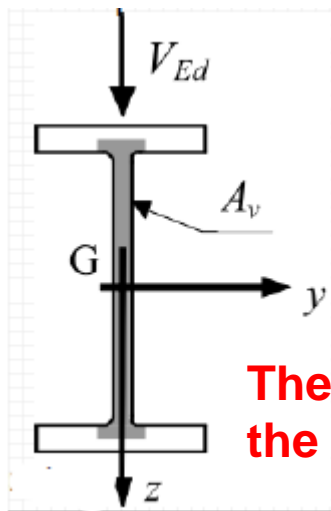
Introduction: Laterally Restrained Beams-Shear-In EC1993-1-1

According to clause 6.2.6 (EC1993-1-1), the design value of the shear force, V_{Ed} , must satisfy the following condition:

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1.0 \quad \text{Where: } V_{c,Rd} \text{ is the design shear resistance.}$$

Considering **plastic design**, in the **absence of torsion** the design shear resistance, $V_{c,Rd}$, is given by the design plastic shear resistance, $V_{pl,Rd}$, given by the following expression:

$$V_{pl,Rd} = A_v \left(f_y / \sqrt{3} \right) / \gamma_{M0} \quad \text{where } A_v \text{ is the shear area,}$$



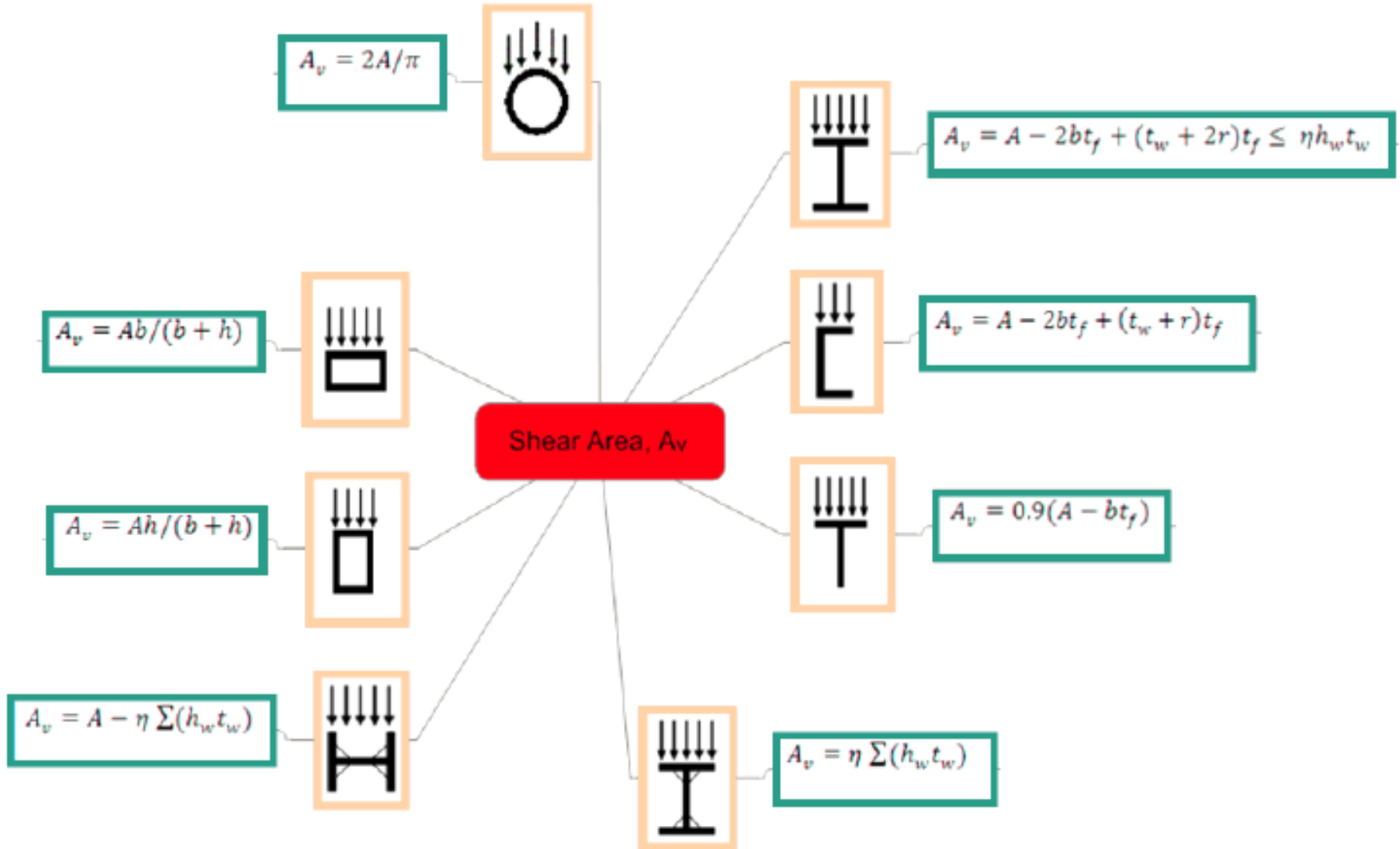
A_v is defined in a qualitative manner for an I section subjected to shear as

$$A - 2bt_f + (t_w + 2r) t_f \text{ but not less than } \eta h_w t_w$$

η may be conservatively taken equal 1.0.

The shear area corresponds approximately to the area of the parts of the cross section that are parallel to the direction of the shear force.

Introduction: Laterally Restrained Beams-Shear-In EC1993-1-1
 Similarly EC1993-1-1 clause 6.2.6(3) provides expressions for the calculation of the shear area for standard steel sections:



Introduction: Laterally Restrained Beams-Shear-In EC1993-1-1

When verification of , $V_{c,Rd}$, can not be performed using the design plastic shear resistance, $V_{pl,Rd}$, a conservative verification , a conservative verification excluding partial plastic shear distribution can be done, which is permitted in elastic design

$$\frac{\tau_{Ed}}{f_y / (\sqrt{3} \gamma_{M0})} \leq 1.0$$

where, τ_{Ed} is the design value of the local shear stress at a given point, obtained from:

$$\tau_{Ed} = \frac{V_{Ed} S}{I t}$$

V_{Ed} is the design value of the shear force;

S is the first moment of area about the centroidal axis of that portion of the cross section between the point at which the shear is required and the boundary of the cross section;

I is the second moment of area about the neutral axis;

t is the thickness of the section at the given point.

For some I or H sections, the shear stress can be calculated more simply from

$$\tau_{Ed} = \frac{V_{Ed}}{A_w} \quad \text{if } A_f / A_w \geq 0,6$$

Where: A_f is the area of one flange;

A_w is the area of the web: $A_w = h_w \cdot t_w$.

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Where the **shear force is present** allowance should be made for its **effect on the moment resistance**.

For Elastic Analysis.

The following condition (from von Mises criterion for a state of plane stress) has then to be verified:

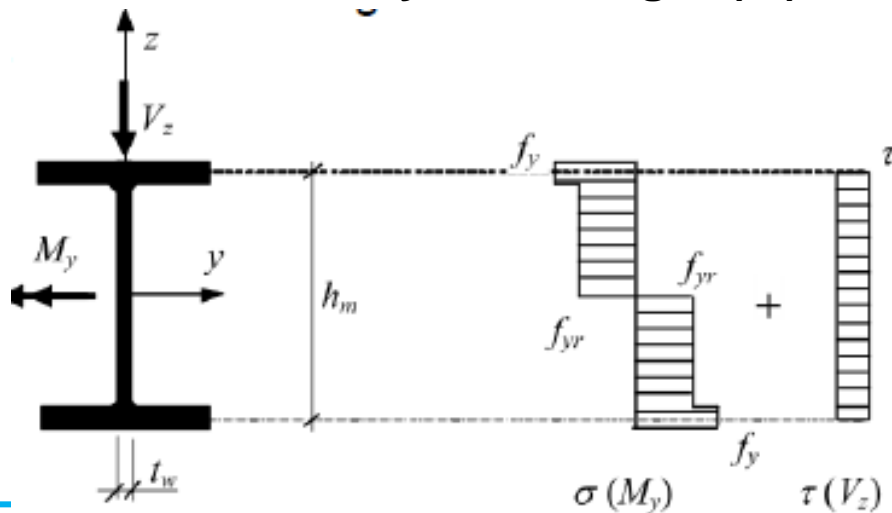
$$\sigma_{von-Mises} = \sqrt{\sigma^2 + 3\tau^2} \leq \frac{f_y}{\gamma_{M0}}$$

Where, σ is elastic normal stresses

τ is elastic shear stresses

For Plastic Analysis

The model used by EC3-1-1 evaluates a reduced bending moment obtained from a reduced yield strength (f_{yr}) along the shear area.



Where, f_{yr} is obtained as;

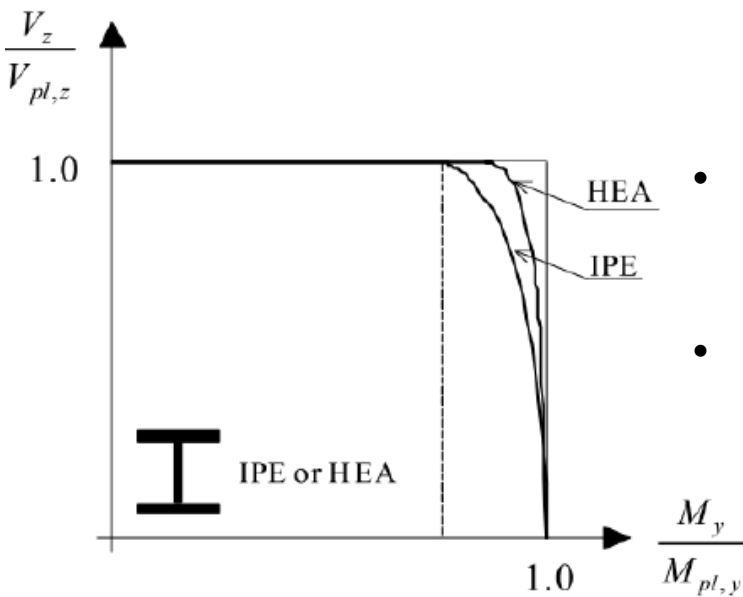
$$f_{yr} = (1 - \rho) f_y$$

$$\text{Where, } \rho = \left(\frac{2 V_{Ed}}{V_{pl,Rd}} - 1 \right)^2$$

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Where the **shear force is present** allowance should be made for its **effect on the moment resistance**.

For Elastic Analysis.
Bending moment–shear force interaction diagrams for I or H sections



- In general, it may be assumed that for **low** values of **shear** it is **not** necessary to reduce the design plastic bending resistance.
- When $V_{Ed} < 50\%$ of the plastic shear resistance $V_{pl,Rd}$, it is not necessary to reduce the design moment resistance $M_{c,Rd}$, except where shear buckling reduces the cross section resistance.
- If $V_{Ed} \geq 50\%$ of the plastic shear resistance $V_{pl,Rd}$, the value of the design moment resistance should be evaluated using the reduced yielding strength (f_{yr}).
- In I or H sections with equal flanges, under major axis bending, the reduced design plastic moment resistance $M_{y,V,Rd}$ may be obtained from:

$$M_{y,V,Rd} = \left(W_{pl,y} - \frac{\rho A_w^2}{4t_w} \right) \frac{f_y}{\gamma_{M0}}, \quad \text{but } M_{y,V,Rd} \leq M_{y,c,Rd}$$

Where, $A_w = h_w \times t_w$ is the area of the web,

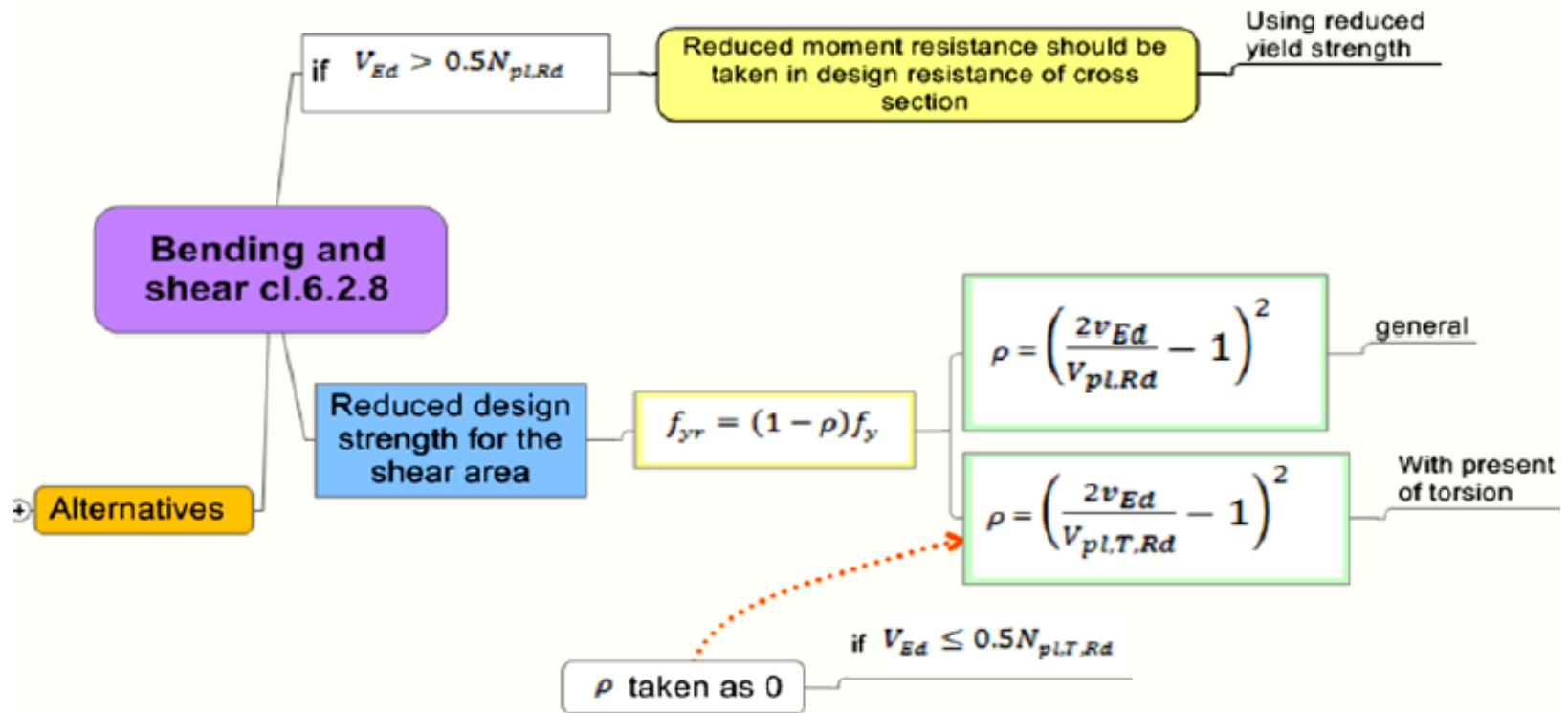
$M_{y,c,Rd}$ is the design resistance for bending moment about the y-axis.

Introduction: Laterally Restrained Beams-Shear-In EC1993-1-1

Combined bending and shear resistance checking

If $V_{Ed} < 0.5V_{c,Rd}$ the combined bending and shear resistance is not required

If $V_{Ed} > 0.5V_{c,Rd}$, then the reduced moment resistance = design resistance of the cross section calculated using reduced yield strength



Design According to EC3: Restrained Beams

To summarize a beam is considered restrained if:

- The section is bent about its minor axis
- Full lateral restraint is provided
- Closely spaced bracing is provided making the slenderness of the weak axis low
- The compressive flange is restrained against torsion
- The section has a high torsional and lateral bending stiffness

There are a number of factors to consider when designing a beam, and they all must be satisfied for the beam design to be adopted:

- Bending Moment Resistance
- Shear Resistance
- Combined Bending and Shear
- Serviceability

Design According to EC3: Restrained Beams

Bending Moment Resistance Summary:

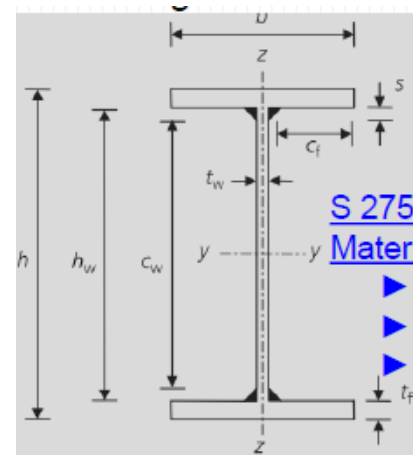
1. Determine the design moment, M_{Ed}
2. Choose a section and determine the section classification
3. Determine $M_{c,Rd}$, using the equation for the respective cross section. Ensure that the correct value of W , (the section modulus) is used.
4. Carry out the cross-sectional moment resistance check by ensuring $M_{c,Rd} > M_{Ed}$ is satisfied.

Shear Resistance Summary:

1. Calculate the shear area, A_v
2. Substitute the value of A_v into equation to get the design plastic shear resistance
3. Carry out the cross-sectional plastic shear resistance check by ensuring $V_{pl,Rd} > V_{Ed}$ is satisfied.

Worked Example: Example on cross-section resistance in bending

Example 4.1. A welded I section is to be designed in bending. The proportions of the section have been selected such that it may be classified as an effective **Class 2 cross-section**. The chosen section is of **grade S275 steel**, and has **two 200 x 16mm flanges**, an overall section height of **600mm** and a 6mm web. The weld size (leg length) **s** is **6.0mm**. Assuming full lateral restraint, calculate the bending moment resistance.



S 275 for $t \leq 16\text{mm}$

Material Properties:

▶ $f_y = 275 \text{ MPa}$

▶ $f_u = 430 \text{ MPa}$

▶ $E = 210 \text{ GPa}$

Solution [1]. Section classification

Step 1.1: Identify the element type.

Flange is **outstand** and the web is **Internal element**

Step 1.2: Evaluate the slenderness ratio (c/t)

$$c_f = (b - t_w - 2s)/2 = 91.0 \text{ mm}$$

$$c_f/t_f = 91.0/16.0 = 5.69$$

$$c_w = h - 2t_f - 2s = 556.0 \text{ mm}$$

$$c_w/t_w = 556.0/6.0 = 92.7$$

Outstand

Internal

Step 1.3: Evaluate the parameter ϵ .

$$\epsilon = \sqrt{235/f_y} = \sqrt{235/275} = 0.92$$

Step 1.4: Determine class of the outstand element in bending

$$\text{Limit for Class 1 flange} = 9\epsilon = 8.32$$

$$8.32 > 5.69 \quad \therefore \text{flange is Class 1}$$

Step 1.5: Determine class of the internal element in bending

$$\text{Limit for Class 3 web} = 124\epsilon = 114.6$$

$$114.6 > 92.7 \quad \therefore \text{web is Class 3}$$

Step 1.6: Determine class of the cross section

Overall section classification is :- **CLASS 3**

However, as stated in clause 6.2.2.4 (EC3-1-1), a section with a **Class 3 web** and **Class 1 or 2 flanges** may be classified as an **effective Class 2 cross-section**.

$$h = 600.0 \text{ mm} \quad W_{el,y} = 2536249 \text{ mm}^3$$

$$b = 200.0 \text{ mm}$$

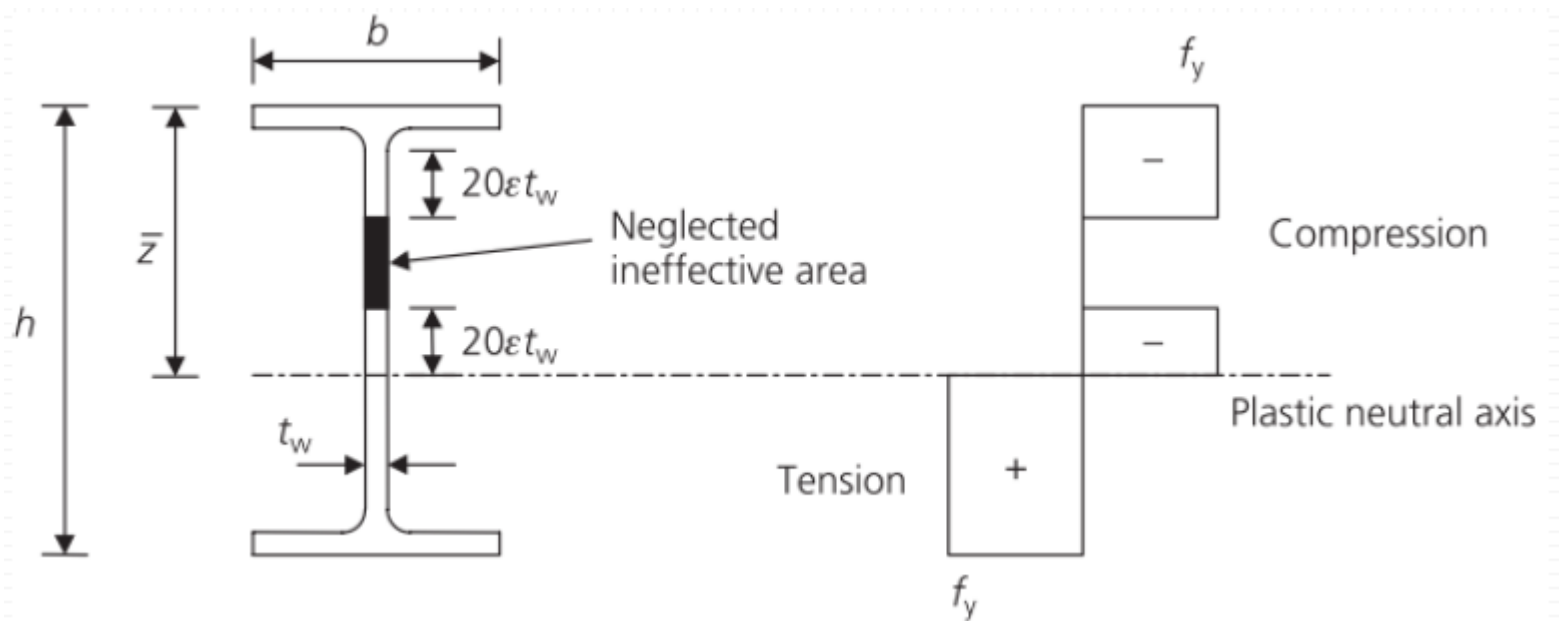
$$t_w = 6.0 \text{ mm}$$

$$t_f = 16.0 \text{ mm}$$

$$s = 6.0 \text{ mm}$$

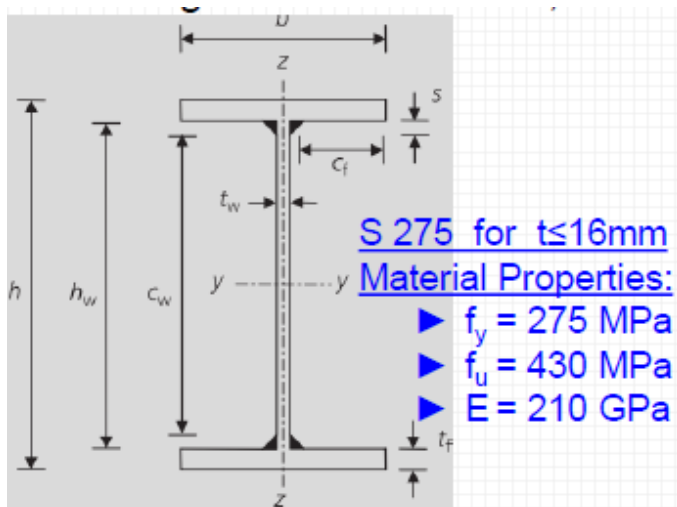
Worked Example: Example on cross-section resistance in bending

Eurocode 3 (clauses 5.5.2(11) and 6.2.2.4) makes special allowances for cross-sections with Class 3 webs and Class 1 or 2 flanges by permitting the cross-sections to be classified as effective Class 2 cross-sections. Accordingly, part of the compressed portion of the web is neglected, and plastic section properties for the remainder of the cross-section are determined. The effective section is prescribed without the use of a slenderness-dependent reduction factor ρ , and is therefore relatively straightforward.



Worked Example: Example on cross-section resistance in bending

Example 4.1. A welded **I section** is to be designed in bending. The proportions of the section have been selected such that it may be classified as an effective **Class 2 cross-section**. The chosen section is of **grade S275 steel**, and has **two 200 x 16mm flanges**, an overall section height of **600mm** and a 6mm web. The weld size (leg length) **s** is **6.0mm**. Assuming full lateral restraint, calculate the bending moment resistance.

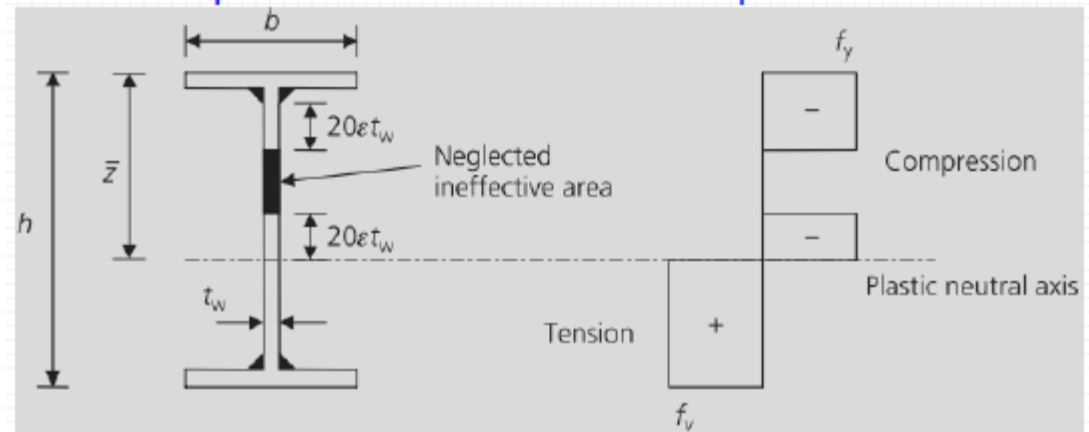


- ▶ $h = 254.1\text{mm}$ ▶ $W_{el,y} = 2536249\text{mm}^3$
- ▶ $b = 200.0\text{mm}$
- ▶ $t_w = 6.0\text{mm}$
- ▶ $t_f = 16.0\text{mm}$
- ▶ $s = 6.0\text{mm}$

Solution [2]. Effective Class 2 cross-section properties

Step 2.1: Plastic neutral axis of effective section.

Based on equal areas above and below the plastic neutral axis



$$\begin{aligned} \bar{z} &= h - t_f - s - (2 \times 20 \epsilon t_w) \\ &= 600.0 - 16.0 - 6.0 - (2 \times 20 \times 0.92 \times 6.0) \\ &= 356.1 \text{ mm} \end{aligned}$$

Bending resistance

$$M_{c,y,Rd} = \frac{W_{pl,y,eff} f_y}{\gamma_{M0}}$$

Worked Example: Example on cross-section resistance in bending

Solution [2]. Effective Class 2 cross-section properties

Step2.2: Plastic modulus of effective section.

$$\begin{aligned}
 W_{pl,y,eff} &= bt_f(h - t_f) + t_w \{ (20\epsilon t_w + s) [\bar{z} - t_f - (20\epsilon t_w + s)/2] \} \\
 &\quad + t_w (20\epsilon t_w \times 20\epsilon t_w / 2) + t_w [(h - t_f - \bar{z})(h - t_f - \bar{z}) / 2] \\
 &= 2\,259\,100 \text{ mm}^3
 \end{aligned}$$

Solution [3]. Bending resistance of cross-section

$$\begin{aligned}
 M_{c,y,Rd} &= \frac{W_{pl,y,eff} f_y}{\gamma_{M0}} \quad \text{for effective class 2 sections} \\
 &= \frac{2\,259\,100 \times 275}{1.0} = 621.3 \times 10^6 \text{ N mm} = 621.3 \text{ kNm}
 \end{aligned}$$

Based the
provision
of EC3-1-1

CLASS 3 → Effective CLASS 2

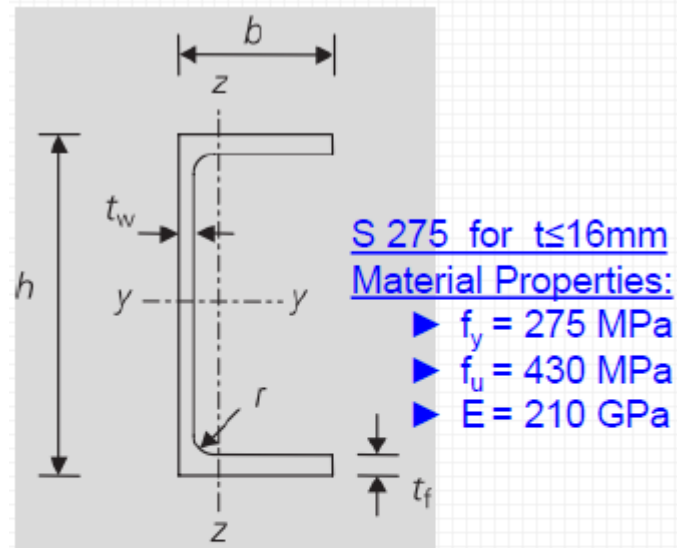
Hence used

$$\begin{aligned}
 W_{pl,y,eff} &= 2,259,100 \text{ mm}^3 \\
 &\text{instead of,} \\
 W_{el,y} &= 2,124,800 \text{ mm}^3
 \end{aligned}$$

Bending
resistance
↑ by 6%

Worked Example: Example on shear resistance

Example 4.2. Determine the shear resistance of a **229x89** rolled channel section in grade **S275 steel** loaded parallel to the web.



- ▶ $h = 228.6\text{mm}$
- ▶ $b = 88.9\text{mm}$ ▶ $A = 4160\text{mm}^2$
- ▶ $t_w = 8.6\text{mm}$
- ▶ $t_f = 13.30\text{mm}$
- ▶ $r = 13.7\text{mm}$

Solution

Step 1: Compute the Shear area A_v .

Shear resistance is determined according to

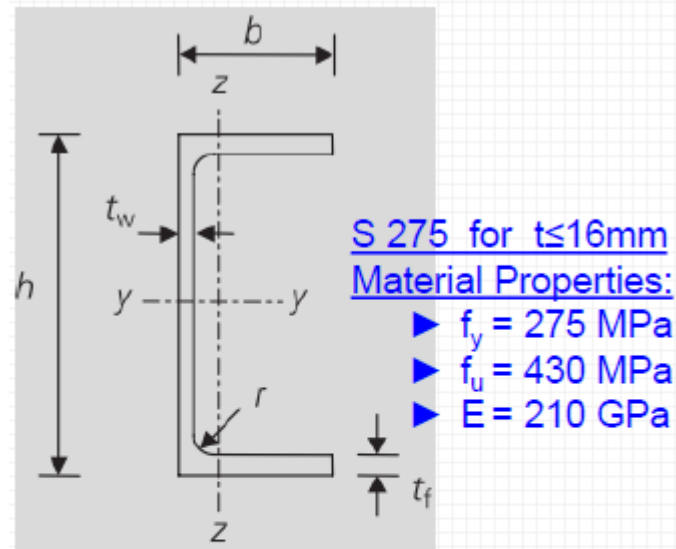
$$V_{pl,Rd} = \frac{A_v (f_y / \sqrt{3})}{\gamma_{M0}}$$

And for a rolled channel section, loaded parallel to the web, the shear area is given by

$$\begin{aligned} A_v &= A - 2bt_f + (t_w + r)t_f \\ &= 4160 - (2 \times 88.9 \times 13.3) + (8.6 + 13.7) \times 13.3 \\ &= 2092\text{ mm}^2 \end{aligned}$$

Worked Example: Example on shear resistance

Example 4.2. Determine the shear resistance of a **229x89** rolled channel section in **grade S275 steel** loaded parallel to the web.



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- ▶ $t_w = 8.6\text{mm}$
- ▶ $t_f = 13.30\text{mm}$
- ▶ $r = 13.7\text{mm}$

Step2: Determine the Shear resistance $V_{pl,Rd}$

$$V_{pl,Rd} = \frac{2092 \times (275/\sqrt{3})}{1.00} = 332\,000 \text{ N} = 332 \text{ kN}$$

Step3: Check for shear buckling

Shear buckling need not be considered, provided:

$$\frac{h_w}{t_w} \leq 72 \frac{\epsilon}{\eta} \quad \text{for unstiffened webs}$$

$$\epsilon = \sqrt{235/f_y} = \sqrt{235/275} = 0.92$$

$$\eta = 1.0$$

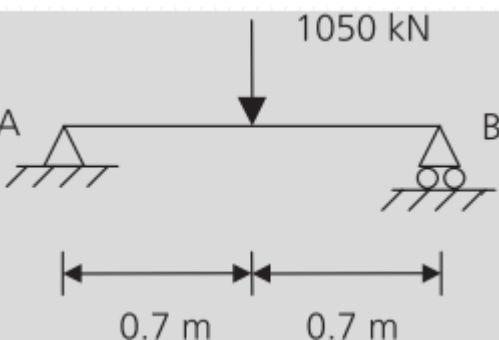
$$72 \frac{\epsilon}{\eta} = 72 \times \frac{0.92}{1.0} = 66.6$$

$$\text{Actual } h_w/t_w = 23.5 \leq 66.6$$

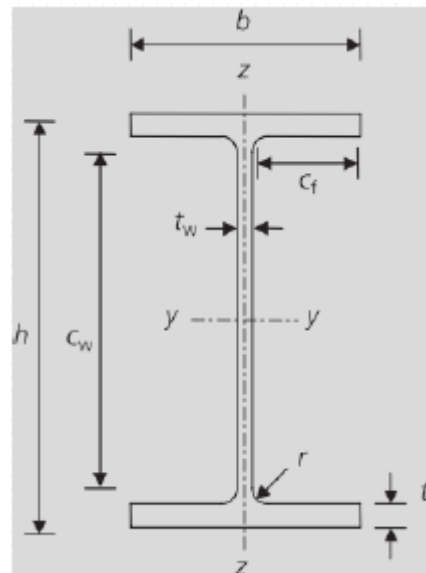
∴ No shear buckling check required

Worked Example: cross-section resistance under combined bending and shear

Example 4.3. A short-span (1.4m), simply supported, laterally restrained beam is to be designed to carry a central point load of 1050 kN, as shown. The arrangement results in a maximum design shear force V_{ed} of 525 kN and a maximum design bending moment M_{ed} of 367.5 kNm. Check the suitability of 406x178x74 UKB in grade S275 steel.



- ▶ $h = 412.8\text{mm}$
- ▶ $b = 179.5\text{mm}$
- ▶ $t_w = 9.5\text{mm}$
- ▶ $t_f = 16.0\text{mm}$
- ▶ $r = 10.2\text{mm}$
- ▶ $A = 9450\text{mm}^2$
- ▶ $W_{pl,y} = 1501000\text{mm}^3$



S 275 for $t \leq 16\text{mm}$
Material Properties:

- ▶ $f_y = 275\text{ MPa}$
- ▶ $f_u = 430\text{ MPa}$
- ▶ $E = 210\text{ GPa}$

Solution [1]. Section classification

Step1.1: Identify the element type.

Flange is **outstand** and the web is **Internal element**

Step1.2: Evaluate the slenderness ratio (c/t)

$$c_f = (b - t_w - 2r)/2 = 74.8\text{ mm}$$

Outstand

$$c_f/t_f = 74.8/16.0 = 4.68$$

$$c_w = h - 2t_f - 2r = 360.4\text{ mm}$$

Internal

$$c_w/t_w = 360.4/9.5 = 37.94$$

Step1.3: Evaluate the parameter ϵ .

$$\epsilon = \sqrt{235/f_y} = \sqrt{235/275} = 0.92$$

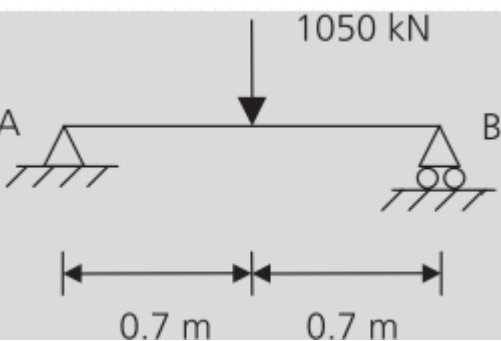
Step1.4: Determine class of the outstand element in bending

$$\text{Limit for Class 1 flange} = 9\epsilon = 8.32$$

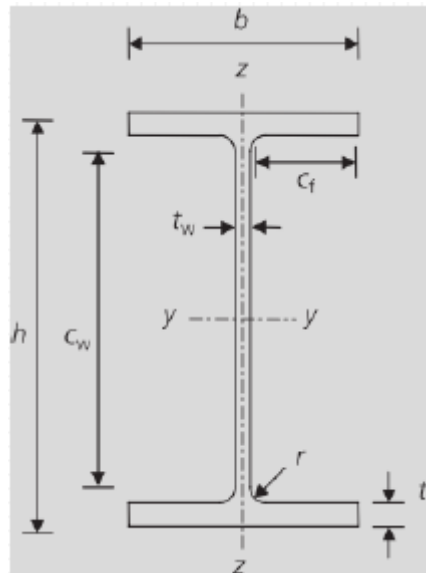
$$8.32 > 4.68 \quad \therefore \text{flange is Class 1}$$

Worked Example: cross-section resistance under combined bending and shear

Example 4.3. A short-span (1.4m), simply supported, laterally restrained beam is to be designed to carry a central point load of 1050 kN, as shown. The arrangement results in a maximum design shear force V_{ed} of 525 kN and a maximum design bending moment M_{ed} of 367.5 kNm. Check the suitability of 406x178x74 UKB in grade S275 steel.



- ▶ $h = 412.8\text{mm}$
- ▶ $b = 179.5\text{mm}$
- ▶ $t_w = 9.5\text{mm}$
- ▶ $t_f = 16.0\text{mm}$
- ▶ $r = 10.2\text{mm}$
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- ▶ $W_{pl,y} = 1501000\text{mm}^3$



S 275 for $t \leq 16\text{mm}$
Material Properties:

- ▶ $f_y = 275\text{ MPa}$
- ▶ $f_u = 430\text{ MPa}$
- ▶ $E = 210\text{ GPa}$

Solution [1]. Section classification

Step 1.5: Determine class of the internal element in bending

Limit for Class 1 web = $72\varepsilon = 66.56$

$66.56 > 37.94$ ∴ web is Class 1

Step 1.6: Determine class of the cross section

Overall section classification is :- CLASS 1

Solution [2]. Bending resistance of cross-section

The design bending resistance of the cross-section

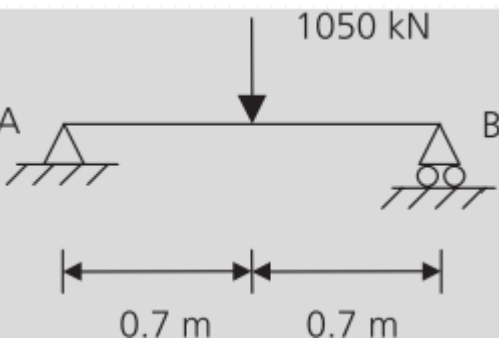
$$M_{c,y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} \quad \text{for Class 1 or 2 cross-sections}$$

$$M_{c,y,Rd} = \frac{1501 \times 10^3 \times 275}{1.00} = 412 \times 10^6 \text{ N mm} = 412 \text{ kNm}$$

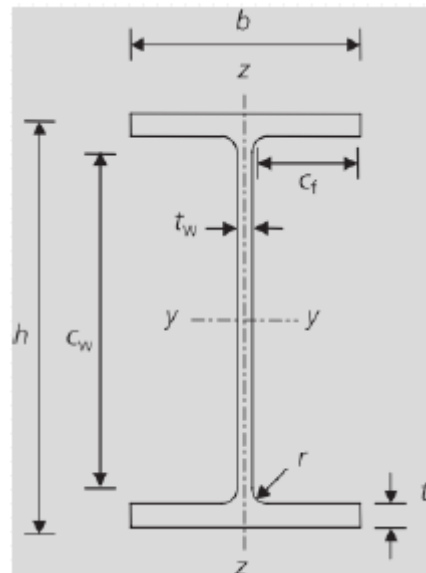
$412 \text{ kNm} > 367.5 \text{ kNm}$ Cross-section resistance in bending is acceptable

Worked Example: cross-section resistance under combined bending and shear

Example 4.3. A short-span (1.4m), simply supported, laterally restrained beam is to be designed to carry a central point load of 1050 kN, as shown. The arrangement results in a maximum design shear force V_{ed} of 525 kN and a maximum design bending moment M_{ed} of 367.5 kNm. Check the suitability of 406x178x74 UKB in grade S275 steel.



- ▶ $h = 412.8\text{mm}$
- ▶ $b = 179.5\text{mm}$
- ▶ $t_w = 9.5\text{mm}$
- ▶ $t_f = 16.0\text{mm}$
- ▶ $r = 10.2\text{mm}$
- ▶ $A = 9450\text{mm}^2$
- ▶ $W_{pl,y} = 1501000\text{mm}^3$



S 275 for $t \leq 16\text{mm}$
Material Properties:

- ▶ $f_y = 275\text{ MPa}$
- ▶ $f_u = 430\text{ MPa}$
- ▶ $E = 210\text{ GPa}$

Solution [3]. Shear resistance of cross-section

Step 3.1: Compute the Shear area A_v .

Shear resistance is determined according to

$$V_{pl,Rd} = \frac{A_v (f_y / \sqrt{3})}{\gamma_{M0}}$$

For a rolled I section, loaded parallel to the web, the shear area A_v is given by

$$A_v = A - 2bt_f + (t_w + 2r)t_f \quad (\text{but not less than } \eta h_w t_w)$$

$$\eta = 1.0$$

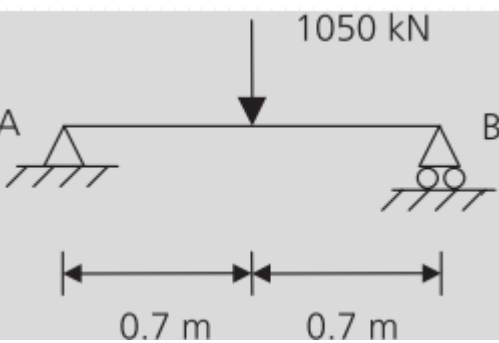
$$h_w = (h - 2t_f) = 412.8 - (2 \times 16.0) = 380.8\text{ mm}$$

$$A_v = 9450 - (2 \times 179.5 \times 16.0) + [9.5 + (2 \times 10.2)] \times 16.0$$

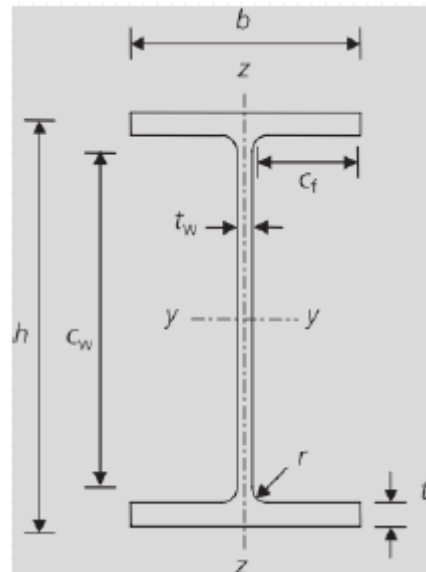
$$= 4184\text{ mm}^2 \quad (\text{but not less than } 1.0 \times 380.8 \times 9.5 = 3618\text{ mm}^2)$$

Worked Example: cross-section resistance under combined bending and shear

Example 4.3. A short-span (1.4m), simply supported, laterally restrained beam is to be designed to carry a central point load of 1050 kN, as shown. The arrangement results in a maximum design shear force V_{ed} of 525 kN and a maximum design bending moment M_{ed} of 367.5 kNm. Check the suitability of 406x178x74 UKB in grade S275 steel.



- ▶ $h = 412.8\text{mm}$
- ▶ $b = 179.5\text{mm}$
- ▶ $t_w = 9.5\text{mm}$
- ▶ $t_f = 16.0\text{mm}$
- ▶ $r = 10.2\text{mm}$
- ▶ $A = 9450\text{mm}^2$
- ▶ $W_{pl,y} = 1501000\text{mm}^3$



S 275 for $t \leq 16\text{mm}$
Material Properties:

- ▶ $f_y = 275\text{ MPa}$
- ▶ $f_u = 430\text{ MPa}$
- ▶ $E = 210\text{ GPa}$

Solution [3]. Shear resistance of cross-section

Step 3.2: Determine the Shear resistance $V_{pl,Rd}$

$$V_{pl,Rd} = \frac{4184 \times (275/\sqrt{3})}{1.00} = 664\,300\text{ N} = 664.3\text{ kN}$$

Step 3.3: Check for shear buckling

Shear buckling need not be considered, provided:

$$\frac{h_w}{t_w} \leq 72 \frac{\epsilon}{\eta} \quad \text{for unstiffened webs}$$

$$72 \frac{\epsilon}{\eta} = 72 \times \frac{0.92}{1.0} = 66.6$$

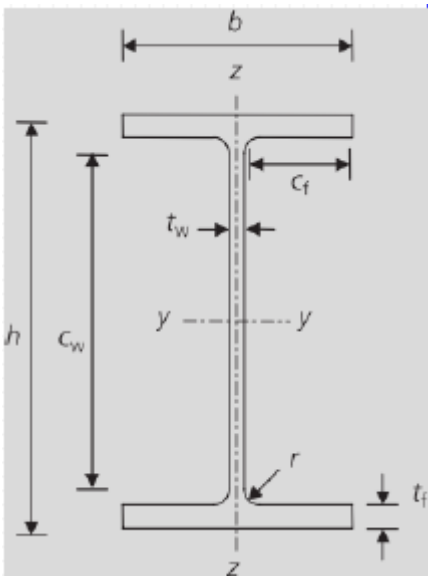
$$\text{Actual } h_w/t_w = 380.8/9.5 = 40.1$$

$$40.1 \leq 66.6 \quad \therefore \text{no shear buckling check required}$$

$$664.3 > 525\text{ kN} \quad \therefore \text{shear resistance is acceptable}$$

Worked Example: cross-section resistance under combined bending and shear

Example 4.3. A short-span (1.4m), simply supported, laterally restrained beam is to be designed to carry a central point load of 1050 kN, as shown. The arrangement results in a maximum design shear force V_{ed} of 525 kN and a maximum design bending moment M_{ed} of 367.5 kNm. Check the suitability of 406x178x74 UKB in grade S275 steel.



S 275 for $t \leq 16\text{mm}$

Material Properties:

- ▶ $f_y = 275 \text{ MPa}$
- ▶ $f_u = 430 \text{ MPa}$
- ▶ $E = 210 \text{ GPa}$

Solution [4]. Resistance of cross-section to combined bending and shear

Step 4.1: Determine the influence of the design shear force

The applied shear force is greater than half the plastic shear resistance of the cross-section,

Step 4.2: Determine the reduced moment resistance

$$M_{y,V,Rd} = \frac{(W_{pl,y} - \rho A_w^2 / 4 t_w) f_y}{\gamma_{M0}} \quad \text{but } M_{y,V,Rd} \leq M_{y,c,Rd}$$

$$\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1 \right)^2 = \left(\frac{2 \times 525}{689.2} - 1 \right)^2 = 0.27$$

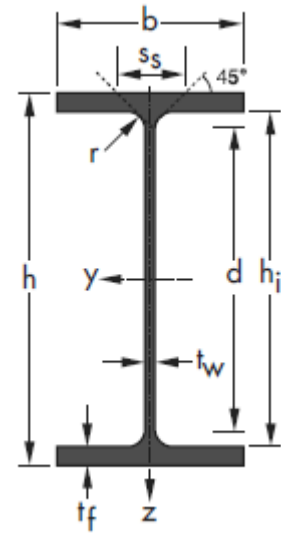
$$A_w = h_w t_w = 380.8 \times 9.5 = 3617.6 \text{ mm}^2$$

$$\Rightarrow M_{y,V,Rd} = \frac{(1\,501\,000 - 0.27 \times 3617.6^2 / 4 \times 9.5) \times 275}{1.0} = 386.8 \text{ kNm} > 367.5 \text{ kNm}$$

Cross-section resistance to combined bending and shear is acceptable

Worked Example: cross-section resistance under combined bending and shear

Désignation Designation Bezeichnung	Dimensions Abmessungen					Dimension Dimensio Konstr			
	G kg/m	h mm	b mm	t _w mm	t _f mm	r mm	A mm ²	h _i mm	d mm
UB 406 x 140 x 39 ⁺	39,0	398	141,8	6,4	8,6	10,2	49,65	380,8	360,4
UB 406 x 140 x 46 ⁺	46,0	403,2	142,2	6,8	11,2	10,2	58,64	380,8	360,4
UB 406 x 178 x 54 ⁺	54,1	402,6	177,7	7,7	10,9	10,2	68,95	380,8	360,4
UB 406 x 178 x 60 ⁺	60,1	406,4	177,9	7,9	12,8	10,2	76,52	380,8	360,4
UB 406 x 178 x 67 ⁺	67,1	409,4	178,8	8,8	14,3	10,2	85,54	380,8	360,4
UB 406 x 178 x 74 ⁺	74,2	412,8	179,5	9,5	16	10,2	94,51	380,8	360,4



G kg/m	l _y	W _{el.y}	W _{pl.y} ♦	i _y	A _{vz}	I _z	W _{el.z}	W _{pl.z} ♦	i _z	s _s	I _t	I _w	
	mm ⁴	mm ³	mm ³	mm	mm ²	mm ⁴	mm ³	mm ³	mm	mm	mm ⁴	mm ⁶	
	x 10 ⁴	x 10 ³	x 10 ³	x 10	x 10 ²	x 10 ⁴	x 10 ³	x 10 ³	x 10		x 10 ⁴	x 10 ⁹	
UB 406 x 140 x 39	39,0	12508	628,6	723,7	15,87	27,57	409,8	57,80	90,85	2,87	35,55	10,99	154,9
UB 406 x 140 x 46	46,0	15685	778,0	887,6	16,35	29,83	538,1	75,68	118,1	3,03	41,15	19,07	206,2
UB 406 x 178 x 54	54,1	18720	930,0	1055	16,48	33,28	1021	114,9	178,3	3,85	41,45	23,50	391,0
UB 406 x 178 x 60	60,1	21600	1063	1199	16,80	34,60	1203	135,3	209,0	3,97	45,45	33,49	465,2
UB 406 x 178 x 67	67,1	24330	1189	1346	16,87	38,58	1365	152,7	236,6	3,99	49,35	46,40	531,7
UB 406 x 178 x 74	74,2	27310	1323	1501	17,00	41,85	1545	172,2	267,0	4,04	53,45	63,10	607,1