Structural Mechanics (2)

Week No-01

Displacements Methods

Indeterminate Structures
(Slope Deflection Method)

Indeterminate Structures
(Moment Distributed Method)

Slope-Deflection for Beams and Frames

Slope-Deflection Method for Beams and Frames

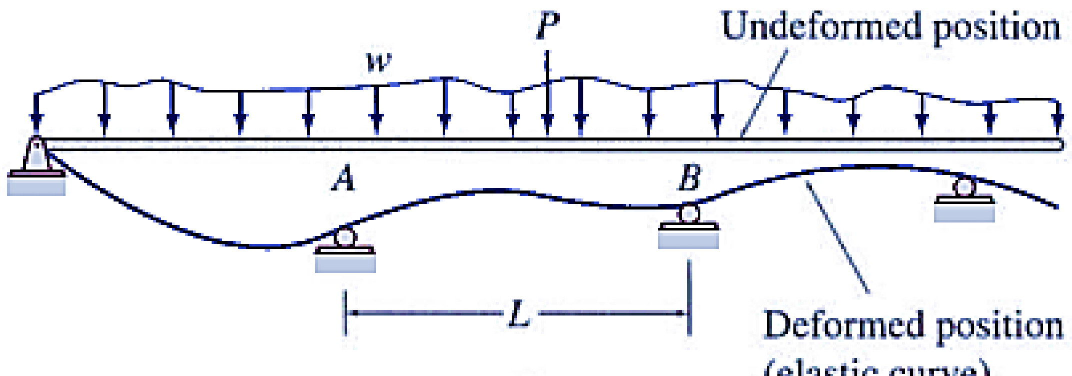
- Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.
- Analysis of Continuous Beams.
- Analysis of Frames without Sidesway.
- Analysis of Frames with Sidesway.

Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.

When a continuous beam or a frame is subjected to external loads, internal moments generally developed at the ends of its individual members.

The slope-deflection equations relate the moments at the ends of a member to the rotations and displacements of its ends and the external loads applied to the member.

To derive the slope-deflection equations, let us focus our attention on an arbitrary member *AB* of the continuous beam shown in following figure>



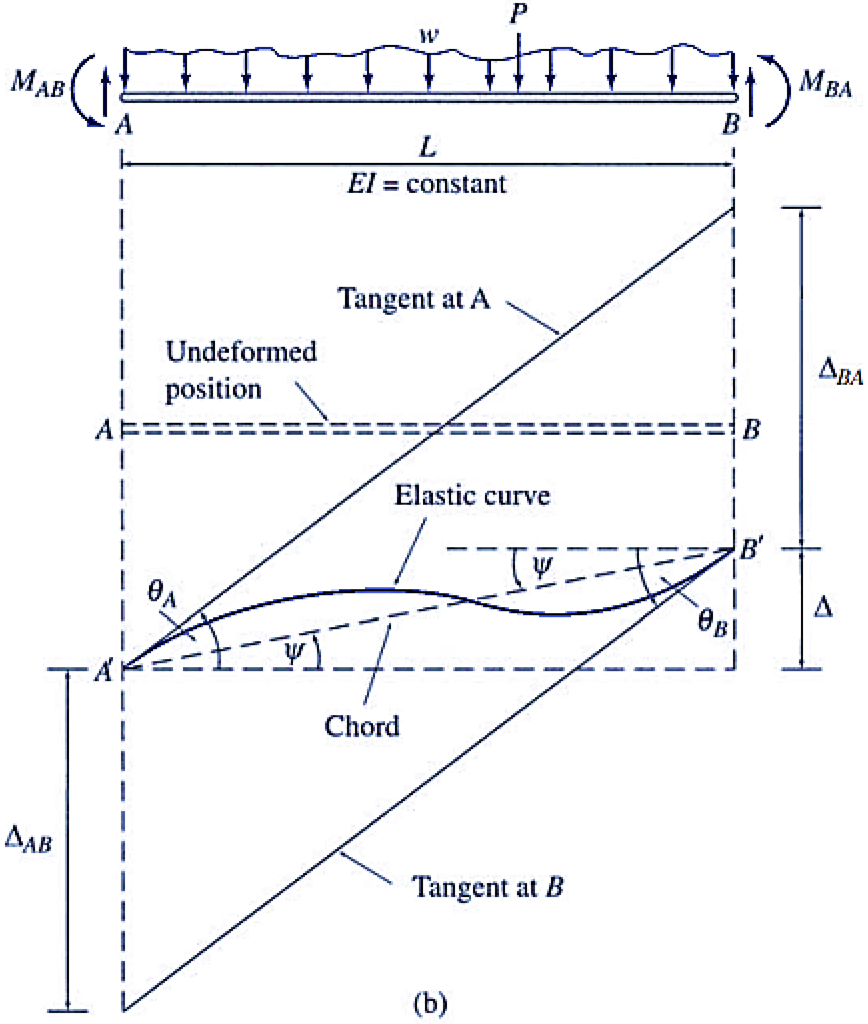
(a)

Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.

When the beam is subjected to external loads and support settlements, member AB deforms, as shown in the figure, and internal moments are induced at its ends.

The free-body diagram and the elastic curve for member AB are shown using an exaggerated scale.

As indicated in this figure, double-subscript notation is used for member end moments, with the first subscript identifying the member end at which the moment acts and the second subscript indicating the other end of the member. Thus, M_{AB} denotes the moment at end A of member AB, whereas M_{BA} represents the moment at end B of member AB. Also, as shown in figure, θ_A and θ_B denote, respectively, the rotations of ends A and B of the member with respect to the undeformed (horizontal) position of the member.



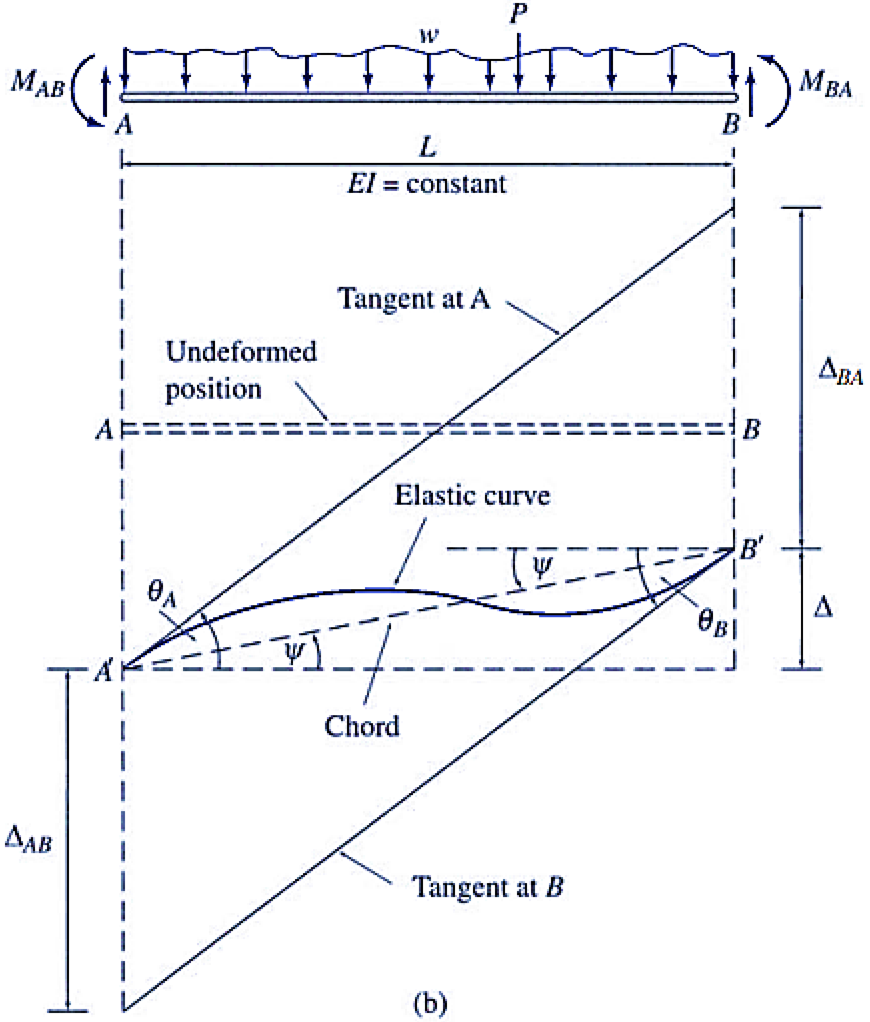
Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.

Δ denotes the relative translation between the two ends of the member in the direction perpendicular to the undeformed axis of the member; and the angle ψ denotes the rotation of the member's chord (i.e., the straight line connecting the deformed positions of the member ends) due to the relative translation Δ. Since the deformations are assumed to be small, the chord rotation can be expressed as

$$\psi = \frac{\Delta}{L}$$

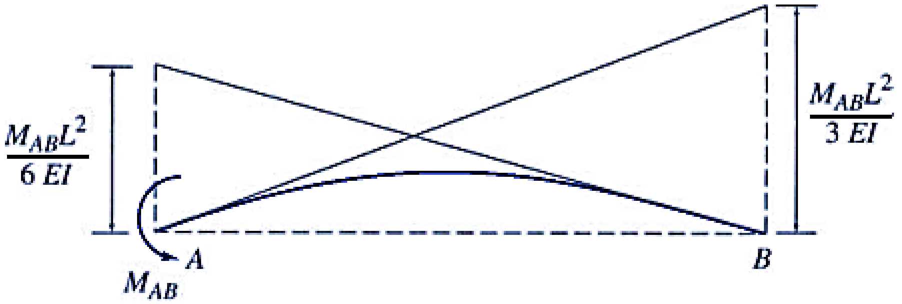
The sign convention used in this chapter is as follows:

The member end moments, end rotations, and chord rotation are positive when counterclockwise.

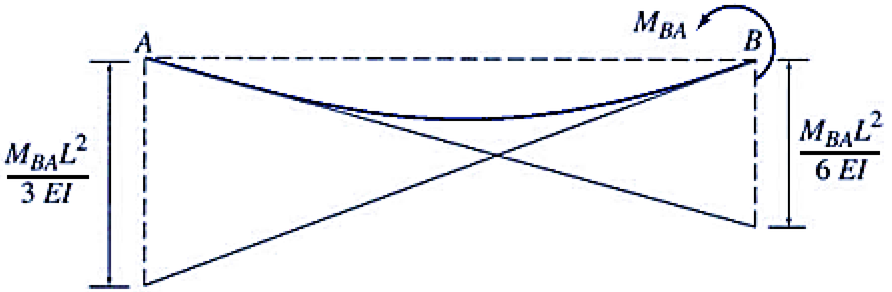


(b)

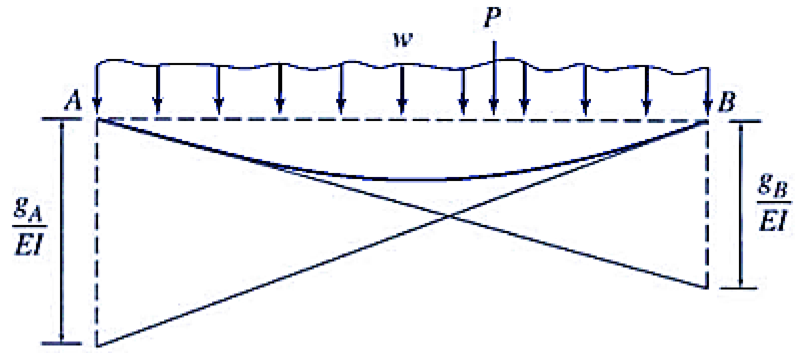
Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.



Tangential Deviations Due to M_{AB}

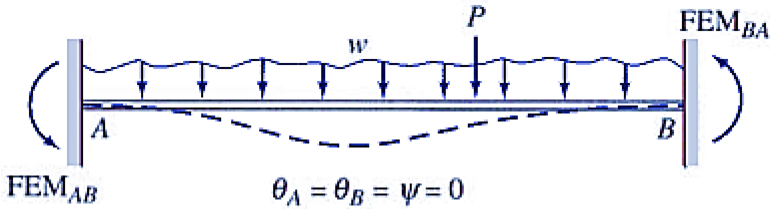


Tangential Deviations Due to M_{BA}

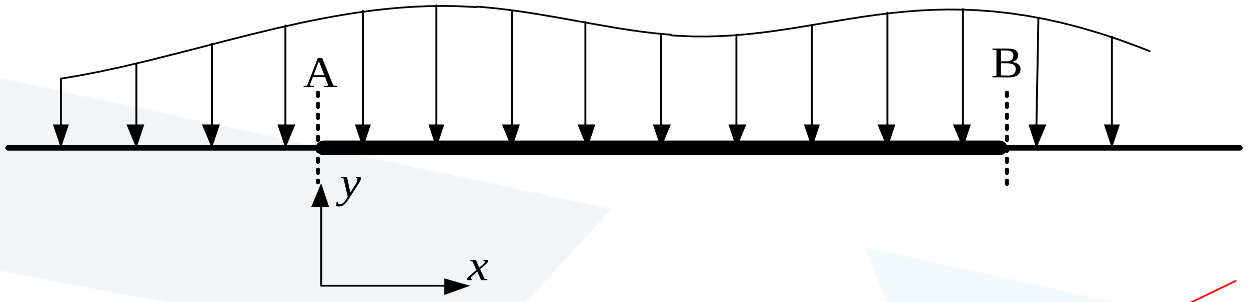


Tangential Deviations Due to External Loading

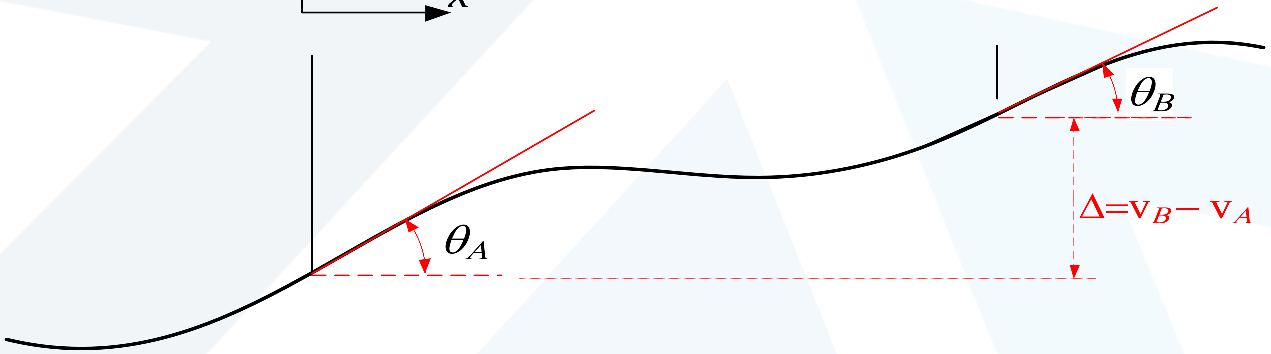
(d)



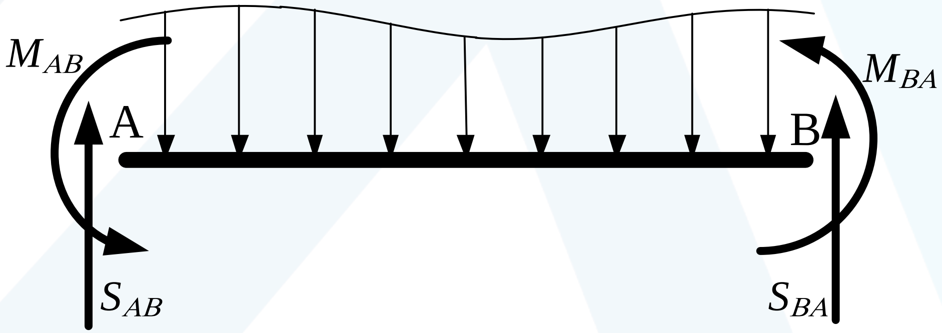
The slope-deflection method is a displacement method: Primary unknowns are rotations (slopes) & deflections that determine the deviation of the deformed position from the undeformed one.



Positive end rotations & relative deflection



Positive end moments & shear forces

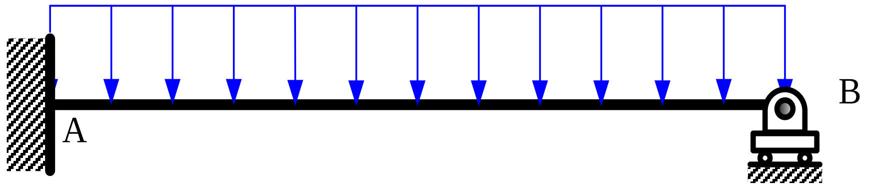


End Moments & forces are external not internal!!

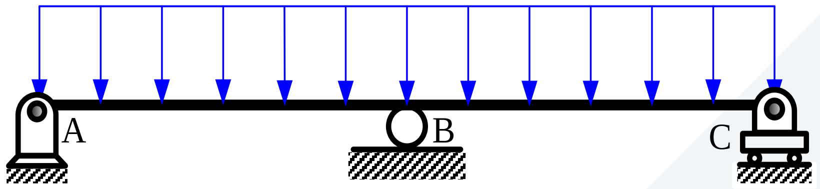
Degrees of Kinematic Indeterminacy in Beams & Frames

The degree of kinematic indeterminacy (DKI) is the number of unknown kinematic (displacement) boundary values for every considered segment in the structure, respecting the nature of supports & connections, and neglecting axial deformation.

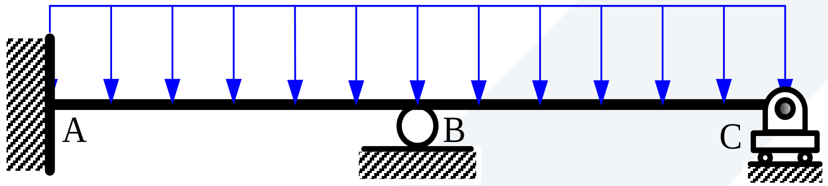
While the degree of static indeterminacy (DSI) is the number of unknown static (force) boundary values exceeding the available number of equilibrium equations and conditions in the entire structure.



A propped cantilever is statically indeterminate beam to degree one. Also it has one kinematic unknown θ_B. Axial deformation is neglected so there is no translation at B

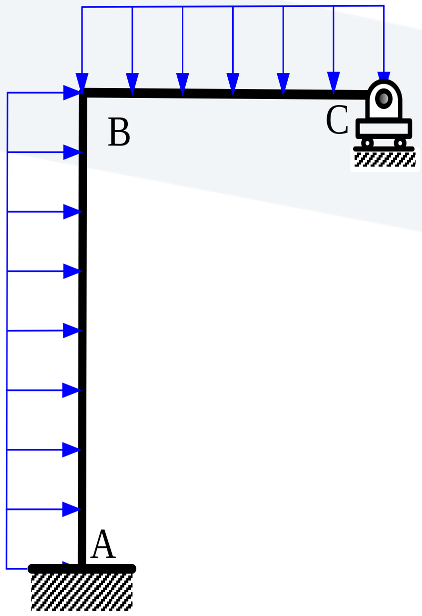


This continuous beam is statically indeterminate to degree one. however it has three kinematic unknowns θ_A, θ_B, and θ_C

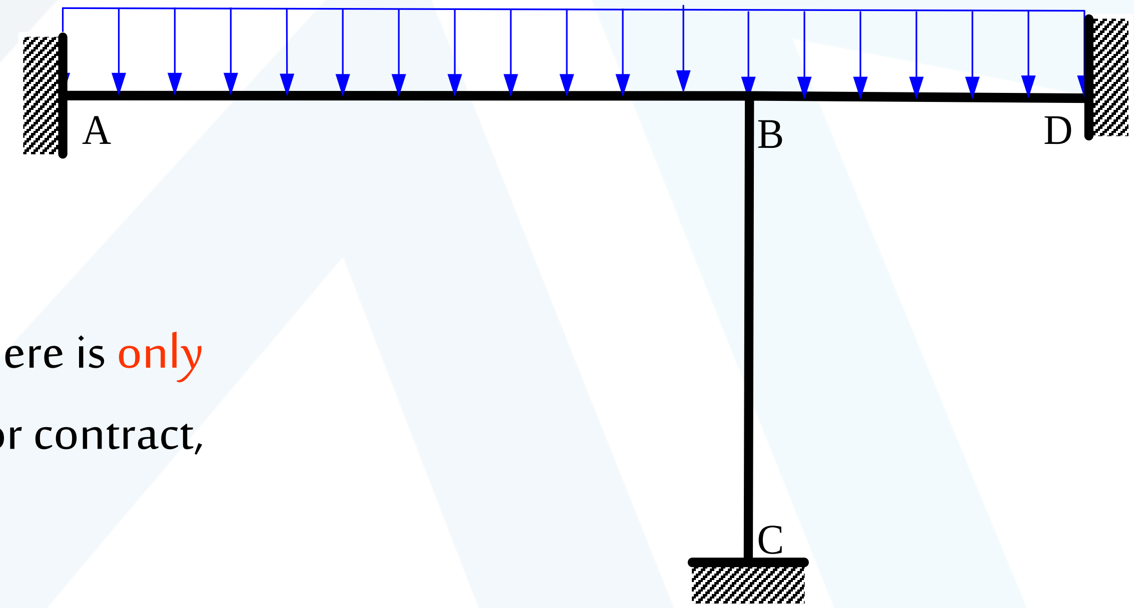


This continuous beam is statically indeterminate to degree two. Also there is two kinematic unknowns θ_B, and θ_C

Degrees of Kinematic Indeterminacy in Beams & Frames

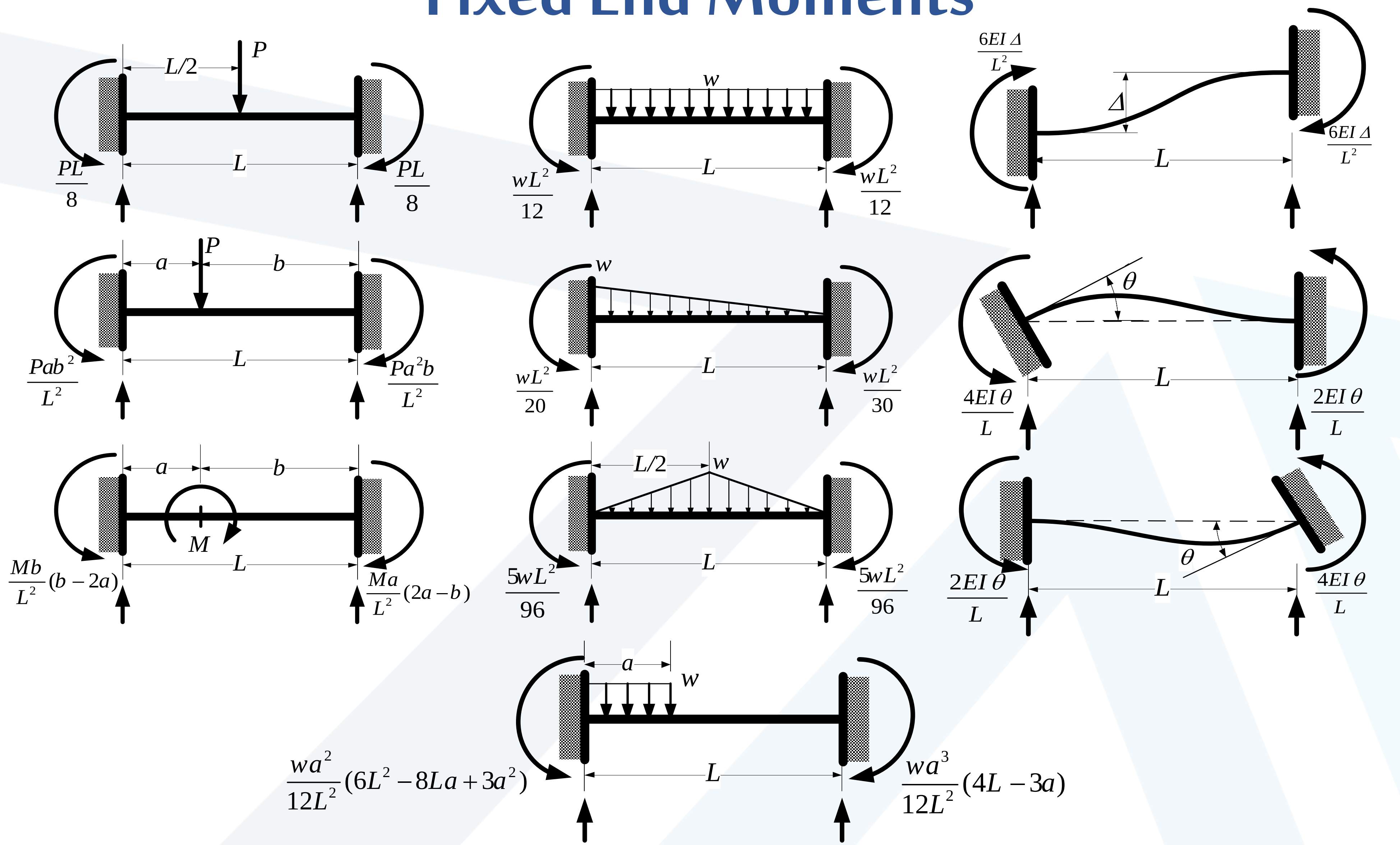


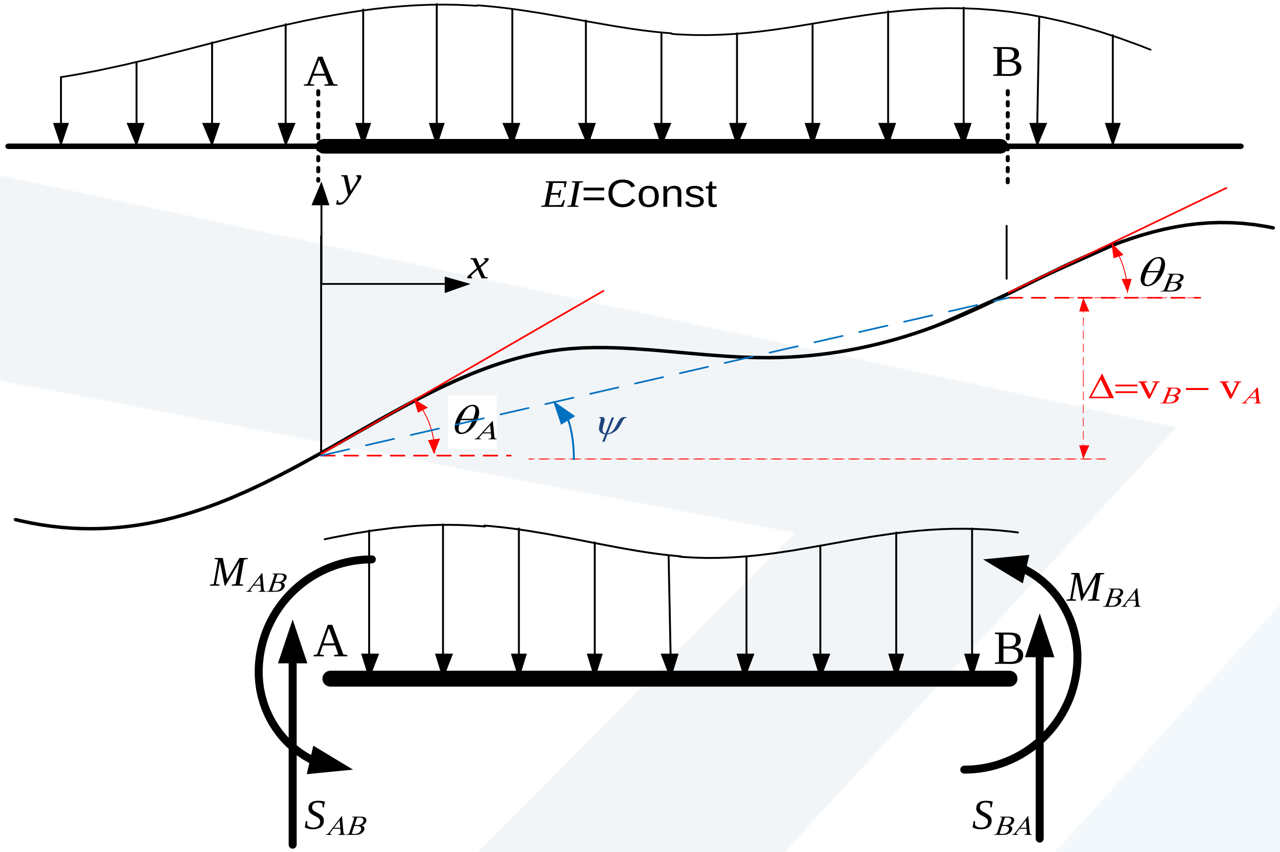
The frame is statically indeterminate to degree one. However, there are 3 kinematic unknowns, θ_B, θ_C and Δ, the horizontal displacement of B and C. Since we do not assume axial deformations to take place in the beams or columns: B and C must have the same horizontal displacement, and the vertical displacement at B is zero.



The frame is statically indeterminate to degree six! However, there is only one kinematic unknown, θB. Since members cannot elongate or contract, point B cannot move up or down, right or left.

Fixed End Moments





$$M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi) + \mathrm{FEM}_{AB}$$

$$M_{BA} = \frac{2EI}{L}(\theta_A + 2\theta_B - 3\psi) + \mathrm{FEM}_{BA}$$

$$\psi = \frac{\Delta}{L}$$

Slope-deflection equations
or 1, 2, 3 equations

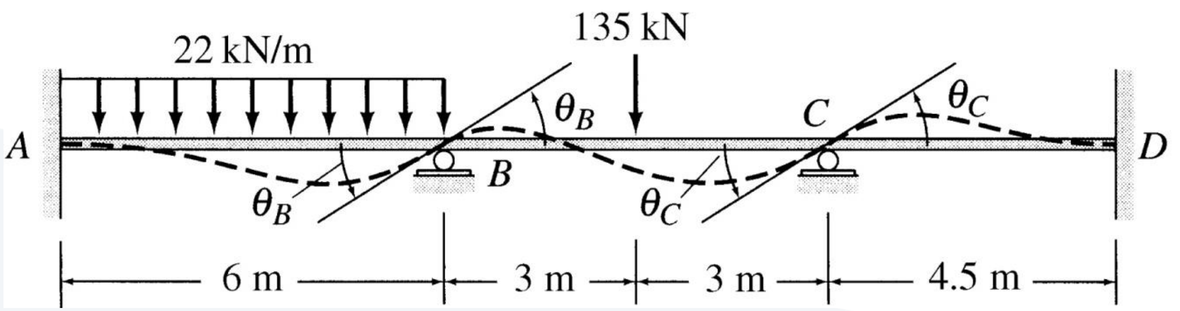
FEM: fixed end moment due to external loads while the other end values are zero

Ex.1: Compute the reactions then draw the SF & BM diagrams

$$\theta_A = \theta_D = 0$$

$$\& \quad \Delta = 0$$

$$\theta_B \neq 0 \ \& \ \ \theta_C \neq 0$$



EI = constant
E = 200 GPa
$I = 210 \times 10^6$ mm^4

$$M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi) + \text{FEM}_{AB} = \frac{2EI}{L}(0 + \theta_B - 0) + \frac{wL^2}{12} = \frac{EI}{3}\theta_B + 66$$

$$M_{BA} = \frac{2EI}{L}(\theta_A + 2\theta_B - 3\psi) + \text{FEM}_{BA} = \frac{2EI}{L}(0 + 2\theta_B - 0) - \frac{wL^2}{12} = \frac{2EI}{3}\theta_B - 66$$

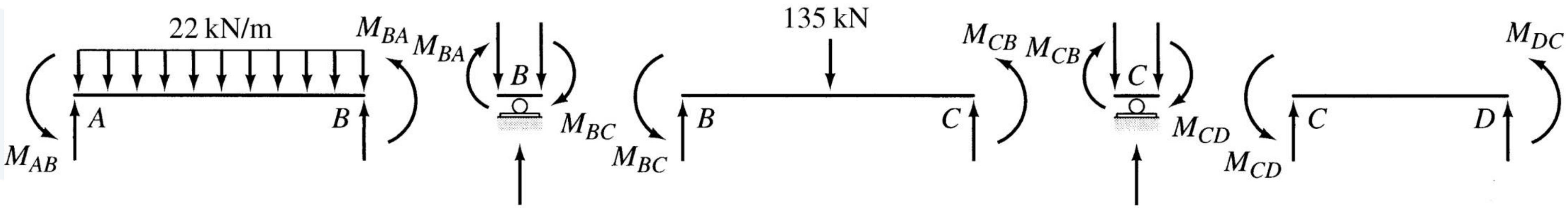
$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 3\psi) + \text{FEM}_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 0) + \frac{PL}{8} = \frac{2EI}{6}(2\theta_B + \theta_C) + 101.25$$

$$M_{CB} = \frac{2EI}{L}(\theta_B + 2\theta_C - 3\psi) + \text{FEM}_{CB} = \frac{2EI}{L}(\theta_B + 2\theta_C - 0) - \frac{PL}{8} = \frac{2EI}{6}(\theta_B + 2\theta_C) - 101.25$$

$$M_{CD} = \frac{2EI}{L}(2\theta_C + \theta_D - 3\psi) + \text{FEM}_{CD} = \frac{2EI}{L}(2\theta_C + 0 - 0) + 0 = \frac{8EI}{9}\theta_C$$

$$M_{DC} = \frac{2EI}{L}(\theta_C + 2\theta_D - 3\psi) + \text{FEM}_{CD} = \frac{2EI}{L}(\theta_C + 0 - 0) - 0 = \frac{4EI}{9}\theta_C$$

Ex.1: Compute the reactions then draw the SF & BM diagrams



$$M_{AB}=\frac{EI}{3}\theta_B+66$$

$$M_{BA}=\frac{2EI}{3}\theta_B-66$$

$$M_{BC}=\frac{2EI}{6}(2\theta_B+\theta_C)+101.25$$

$$M_{CB}=\frac{2EI}{6}(\theta_B+2\theta_C)-101.25$$

$$M_{CD}=\frac{8EI}{9}\theta_C$$

$$M_{DC}=\frac{4EI}{9}\theta_C$$

$$M_{BA}+M_{BC}=0 \;\;\&\;\; M_{CB}+M_{CD}=0$$

$$1.33\theta_B+0.33\theta_C=-35.25/EI$$

$$0.33\theta_B+1.56\theta_C=101.25/EI$$

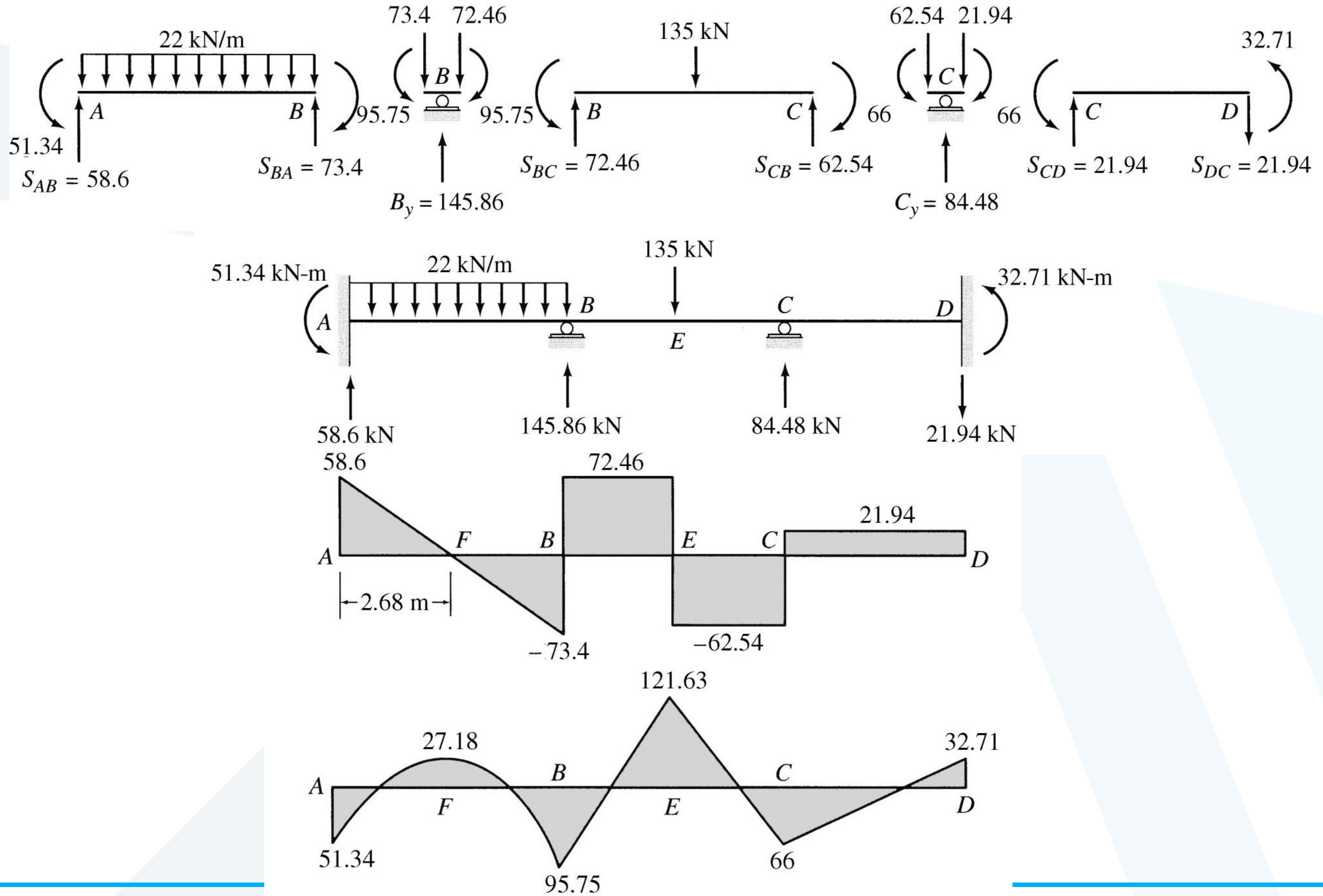
$$\theta_B=-44.41/EI \;\;\&\;\; \theta_C=74.34/EI$$

$$M_{AB}=51.34\,\text{kN.m} \qquad M_{BA}=--95.75\,\text{kN.m}$$

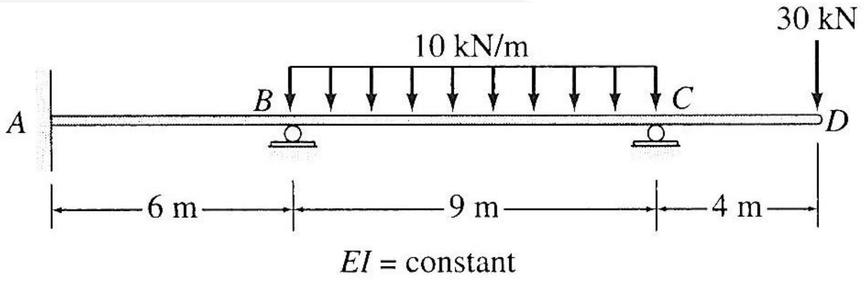
$$M_{BC}=95.75\,\text{kN.m} \qquad M_{CB}=--66\,\text{kN.m}$$

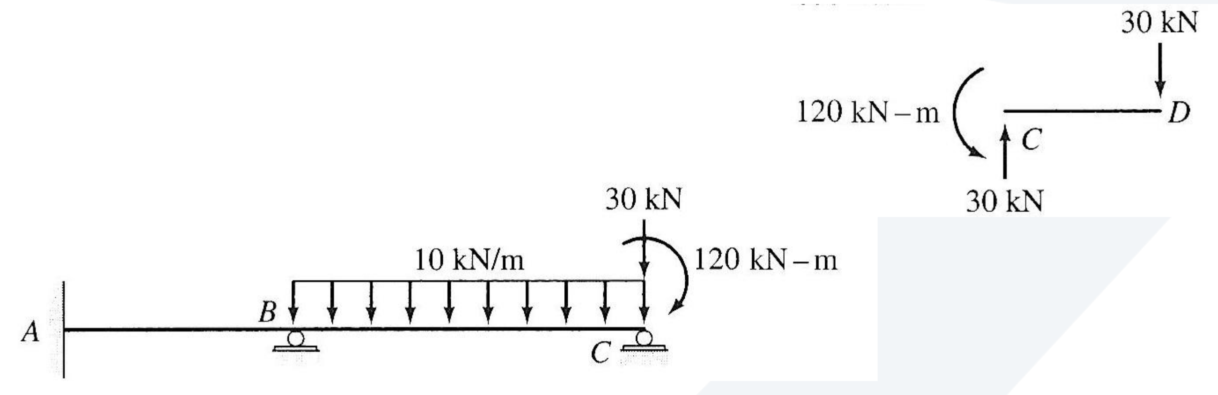
$$M_{CD}=66\,\text{kN.m} \qquad M_{DC}=32.71\,\text{kN.m}$$

Ex.1: Compute the reactions then draw the SF & BM diagrams



Ex.2: Determine the member end moments and reactions for the following continuous beam.





$\theta_A = 0 \ \ \& \ \ \Delta = 0$

$\theta_B \neq 0 \ \ \& \ \ \theta_C \neq 0$

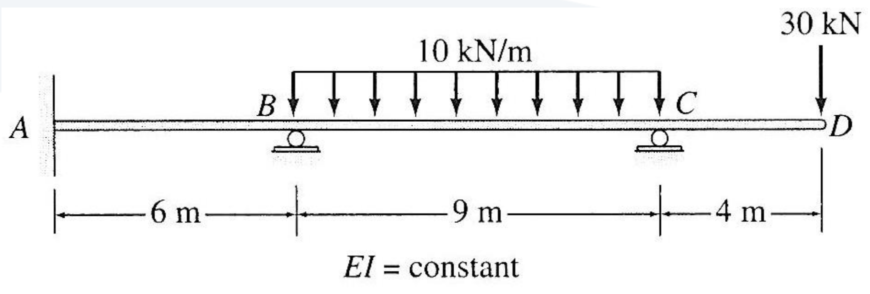
Fixed-End Moments:

$$\text{FEM}_{AB} = \text{FEM}_{AB} = 0$$

$$\text{FEM}_{BC} = \frac{10(9)^2}{12} = 67.5\ kN.m \curvearrowleft or + 67.5\ kN.m$$

$$\text{FEM}_{CB} = 67.5\ kN.m \curvearrowright or - 67.5\ kN.m$$

Ex.2: Determine the member end moments and reactions for the following continuous beam.



Slope Deflection Equations:

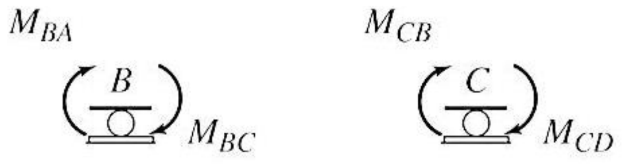
$$M_{AB} = \frac{2EI}{6}(\theta_B) = 0.333EI\ \theta_B$$

$$M_{BA} = \frac{2EI}{6}(2\theta_B) = 0.667EI\ \theta_B$$

$$M_{BC} = \frac{2EI}{9}(2\theta_B + \theta_C) + 67.5 = 0.444EI\ \theta_B + 0.222EI\ \theta_C + 67.5$$

$$M_{CB} = \frac{2EI}{9}(2\theta_C + \theta_B) - 67.5 = 0.222EI\ \theta_B + 0.444EI\ \theta_C - 67.5$$

Equilibrium Equations:



$$M_{BA} + M_{AB} = 0$$

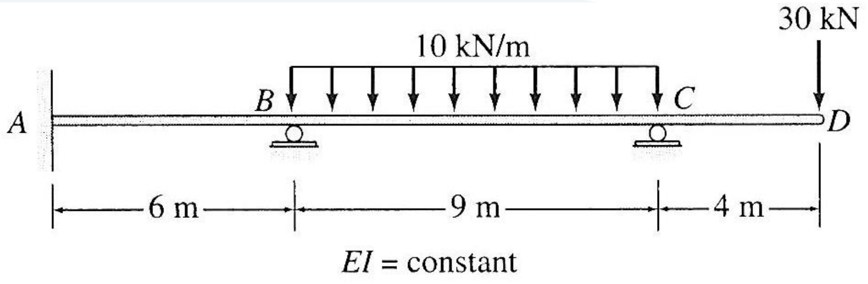
$$M_{CB} + 120 = 0$$

$$0.111EI\theta_B + 0.222EI\theta_C = -67.5$$

$$0.222EI\theta_B + 0.444EI\theta_C = -52.5$$

$$\boxed{\begin{aligned} EI\theta_B &= -41.25\ kN.m^2 \\ EI\theta_C &= -97.62\ kN.m^2 \end{aligned}}$$

Ex.2: Determine the member end moments and reactions for the following continuous beam.



$$EI\theta_B = -41.25\ kN.m^2$$
$$EI\theta_C = -97.62\ kN.m^2$$

Member End Moments:

$$M_{AB} = 0.333EI\ \theta_B = 0.333(-41.25) = -13.7\ \mathrm{kN.m}\quad (\mathbf{13.7\ kN.m}\ \curvearrowright)$$

$$M_{BA} = 0.667EI\ \theta_B = 0.667(-41.25) = -27.5\ \mathrm{kN.m}\quad (\mathbf{27.5\ kN.m}\ \curvearrowright)$$

$$M_{BC} = 0.444EI\ \theta_B + 0.222EI\ \theta_C + 67.5 = 0.444(-41.25) + 0.222(-97.62) + 67.5 = 27.5\ \mathrm{kN.m}\ (\mathbf{27.5\ kN.m}\ \curvearrowleft)$$

$$M_{CB} = 0.222EI\ \theta_B + 0.444EI\ \theta_C - 67.5 = 0.222(-41.25) + 0.444(-97.62) - 67.5 = -120\ \mathrm{kN.m}\ (\mathbf{120\ kN.m}\ \curvearrowright)$$

Ex.2: Determine the member end moments and reactions for the following continuous beam.