جامعة المنارة

كلية: الهندسة

اسم المقرر: الرياضيات المتقطعة

رقم الجلسة (االثانية)

عنوان الجلسة

التكافؤ المنطقي

Logical Equivalence

Logical Equivalence Laws:

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الاسم	التكافؤ
قوانين الذاتية Identity laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
قوانين الهيمنة Domination laws	$p \vee T \equiv T$ $p \wedge F \equiv F$
قوانين التماثل Idempotent laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
قانون النفي المزدوج Double negation laws	$\sim(\sim p) \equiv p$
قوانين التبديل Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
قوانين التجميع Associative laws	$p \vee (q \vee r) \equiv (p \vee q) \vee r$ $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
قوانين التوزيع Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
قوانين دومورغان De Morgan's laws	$\sim(p \wedge q) \equiv \sim p \vee \sim q$ $\sim(p \vee q) \equiv \sim p \wedge \sim q$
قوانين الامتصاص Absorption laws	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
قوانين النفي Negation laws	$p \vee \sim p \equiv T$ $p \wedge \sim p \equiv F$

Other logical equivalences:

$p \rightarrow q \equiv \neg p \vee q$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Verify this logical equivalence

$$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$$

$\neg(p \lor (\neg p \land q)) \equiv (\neg p \land \neg(\neg p \land q))$ **De Morgan law**

$\equiv (\neg p \land (p \lor \neg q))$ **De Morgan law**

$\equiv (\neg p \land p) \lor (\neg p \land \neg q)$ **distributive law**

$\equiv F \lor (\neg p \land \neg q)$ **negation law**

$\equiv \neg p \land \neg q$ **identity law**

Verify this logical equivalence:

$(p \longrightarrow r) \vee (q \longrightarrow r) \equiv (p \wedge q) \longrightarrow r$

$(p \longrightarrow r) \vee (q \longrightarrow r) \equiv (\neg p \vee r) \vee (\neg q \vee r)$ conditional law

$\equiv \neg p \vee r \vee \neg q \vee r$ associative law

$\equiv \neg p \vee \neg q \vee r \vee r$ commutative law

$\equiv (\neg p \vee \neg q) \vee (r \vee r)$ associative law

$\equiv (\neg p \vee \neg q) \vee r$ idempotent law

$\equiv \neg (p \wedge q) \vee r$ De Morgan law

$\equiv (p \wedge q) \longrightarrow r$ conditional law

Verify this logical equivalence

$$(r \vee p) \longrightarrow (r \vee q) \equiv r \vee (p \longrightarrow q)$$

$(r \vee p) \longrightarrow (r \vee q) \equiv \neg(r \vee p) \vee (r \vee q)$ conditional law

$\equiv (\neg r \wedge \neg p) \vee (r \vee q)$ De Morgan law

$\equiv (\neg r \wedge \neg p) \vee r) \vee q$ associative law

$\equiv ((\neg r \vee r) \wedge (\neg p \vee r)) \vee q$ distributive law

$\equiv (T \wedge (\neg p \vee r)) \vee q$ negation law

$\equiv (\neg p \vee r) \vee q$ identity law

$\equiv \neg p \vee r \vee q$ associative law

$\equiv r \vee \neg p \vee q$ commutative law

$\equiv r \vee (\neg p \vee q)$ associative law

$\equiv r \vee (p \longrightarrow q)$ conditional law

Verify this logical equivalence:

$\neg q \longrightarrow (\neg p \vee r) \equiv p \longrightarrow (q \vee r)$

$\neg q \longrightarrow (\neg p \vee r) \equiv q \vee (\neg p \vee r)$ conditional law

$\equiv q \vee \neg p \vee r$ associative law

$\equiv \neg p \vee q \vee r$ commutative law

$\equiv \neg p \vee (q \vee r)$ associative law

$\equiv p \longrightarrow (q \vee r)$ conditional law

Verify this logical equivalence:

$$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$$

$$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv \neg(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$$ distributive law

$$\equiv \neg(\neg p \wedge T) \vee (p \wedge q)$$ negation law

$$\equiv \neg(\neg p) \vee (p \wedge q)$$ identity law

$$\equiv p \vee (p \wedge q)$$ double negation law

$$\equiv p$$ absorption law

Show that this statement is Tautology

$(p \wedge (p \longrightarrow q)) \longrightarrow q$ (مصدوقة)

$(p \wedge (p \longrightarrow q)) \longrightarrow q \equiv (p \wedge (\neg p \vee q)) \longrightarrow q$ conditional law

$\equiv ((p \wedge \neg p) \vee (p \wedge q)) \longrightarrow q$ distributive law

$\equiv (F \vee (p \wedge q)) \longrightarrow q$ negation law

$\equiv (p \wedge q) \longrightarrow q$ identity law

$\equiv \neg(p \wedge q) \vee q$ conditional law

$\equiv (\neg p \vee \neg q) \vee q$ De Morgan law

$\equiv \neg p \vee (\neg q \vee q)$ associative law

$\equiv \neg p \vee T$ negation law

$\equiv T$ (tautology) domination law

Show that this statement is Tautology

$(\neg q \wedge (p \vee q)) \longrightarrow p$ (مصدوقة)

$(\neg q \wedge (p \vee q)) \longrightarrow p \equiv ((\neg q \wedge p) \vee (\neg q \wedge q)) \longrightarrow p$ distributive law

$\equiv ((\neg q \wedge p) \vee F) \longrightarrow p$ negation law

$\equiv (\neg q \wedge p) \longrightarrow p$ identity law

$\equiv \neg(\neg q \wedge p) \vee p$ conditional law

$\equiv (q \vee \neg p) \vee p$ De Morgan law

$\equiv q \vee (\neg p \vee p)$ associative law

$\equiv q \vee T$ negation law

$\equiv T$ (tautology) domination law

Show that this statement is Contradiction

$(p \longrightarrow q) \wedge (\neg q \wedge p)$ (تناقض)

$(p \longrightarrow q) \wedge (\neg q \wedge p)$	$\equiv (\neg p \vee q) \wedge (\neg q \wedge p)$	conditional law
	$\equiv ((\neg p \vee q) \wedge \neg q) \wedge p$	associative law
	$\equiv ((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \wedge p$	distributive law
	$\equiv ((\neg p \wedge \neg q) \vee F) \wedge p$	negation law
	$\equiv (\neg p \wedge \neg q) \wedge p$	identity law
	$\equiv (\neg q \wedge \neg p) \wedge p$	commutative law
	$\equiv \neg q \wedge (\neg p \wedge p)$	associative law
	$\equiv \neg q \wedge F$	negation law
	$\equiv F$ (contradiction)	domination law

Show that this statement is Contradiction

$(p \wedge q) \wedge \neg(p \vee q)$ (تناقض)

$(p \wedge q) \wedge \neg(p \vee q) \equiv (p \wedge q) \wedge (\neg p \wedge \neg q)$ De Morgan law

$\equiv \mathbf{p \wedge q \wedge \neg p \wedge \neg q}$ associative law

$\equiv \mathbf{p \wedge \neg p \wedge q \wedge \neg q}$ commutative law

$\equiv \mathbf{(p \wedge \neg p) \wedge (q \wedge \neg q)}$ associative law

$\equiv$ F ∧ F negation law

$\equiv F$ (contradiction) idempotent law

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Determine if the following expressions are equivalent by using truth table

$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r) \equiv q \longrightarrow r$ (?)

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$p \rightarrow r$	$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r)$	$q \longrightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F
T	F	T	T	T	T	T	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	F
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

من جدول الحقيقة نجد عدم تطابق في قيم الحقيقة بين العبارتين في السطرين الرابع و السادس بالتالي العبارتين غير متكافئتين منطقياً

Determine if the following expressions are equivalent by using logical Equivalences

$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r) \equiv q \longrightarrow r \quad (?)$

$$(p \longrightarrow q \vee r) \wedge (p \longrightarrow r) \equiv (\neg p \vee (q \vee r)) \wedge (\neg p \vee r) \quad \text{conditional law}$$

$$\equiv (\neg p \vee ((q \vee r) \wedge r) \quad \text{distributive law}$$

$$\equiv \neg p \vee r \quad \text{absorption law}$$

$$\equiv p \longrightarrow r \quad \text{conditional law}$$

$$\not\equiv q \longrightarrow r$$

العبارتين غير متكافئتين منطقياً

Use De Morgan 's law to write negations for the statements :

- Tom is 6 feet tall and he weighs at least 60 kg.
- Negation:Tom is not 6 feet tall or he weighs less than 60 kg.
- The bus was late or Tom's watch was slow.
- Negation:The bus was not late and Tom's watch was not slow.
- $1<x\leq 4$
- Negation: $1\geq x$ or $x>4$

Write (inverse ,converse,contrapositive) for this conditional sentence:

- If (x > 0) and (y > 0) then (x+y > 0)
- **Inverse:** if ($x \leq 0$) or ($y \leq 0$) then ($x+y \leq 0$)
- **Converse:** if (x+y > 0) then (x > 0) and (y > 0)
- **Contrapositive:** if ($x+y \leq 0$) then ($x \leq 0$) or ($y \leq 0$)

homework

Verify this logical equivalence:

$$\neg\,(p \lor \neg q) \lor (\neg p \land \neg q) \equiv \neg p$$

$$(\mathbf{q} \land \mathbf{p}) \lor \neg(\mathbf{q} \longrightarrow \mathbf{p}) \equiv q$$