Steel Structures 2
Summer Sem
2023-2024

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Lecture 5-6

Flexural Members

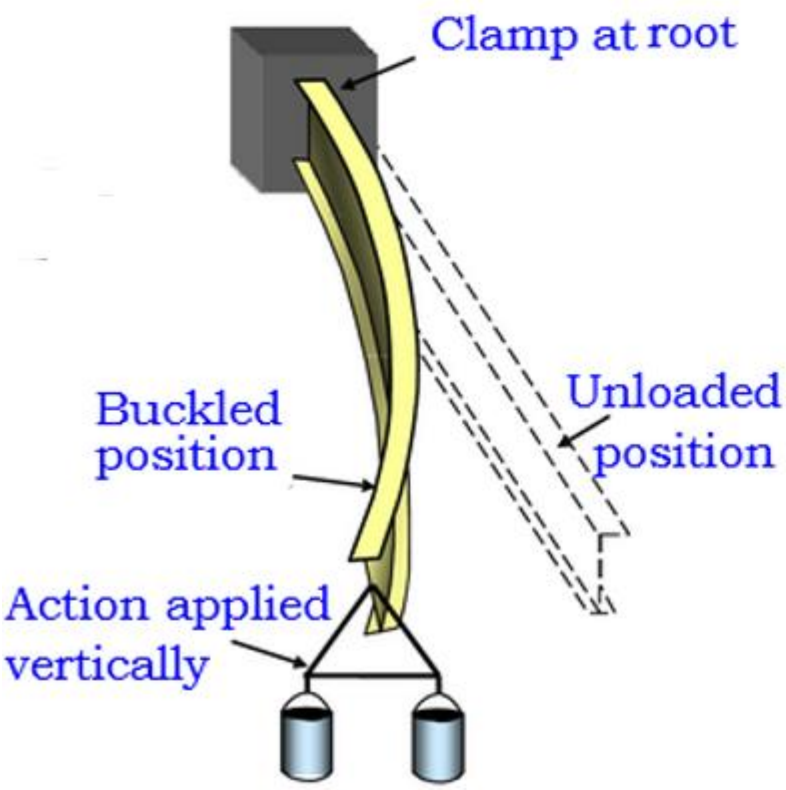
- ✓ -I- Laterally Restrained Beams
- ✓ II- Laterally Unrestrained Beams

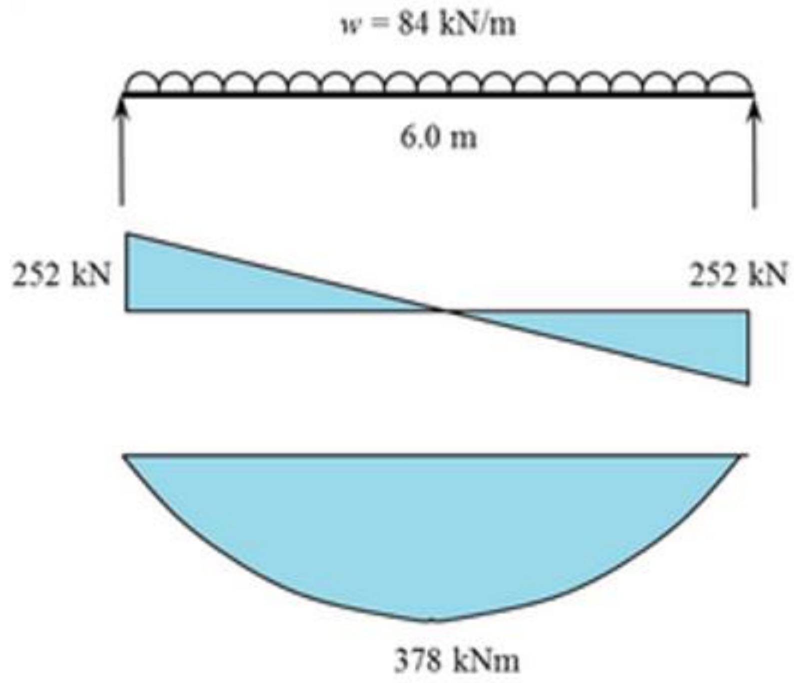
Flexural Members

-II- Laterally Unrestrained Beams

Introduction: Beams, Response to loads

A beam is a structural member which is subjected to transverse loads, and accordingly must be designed to withstand predominantly shear and moment, Generally, it will be bent about its major axis..





Slender structural elements loaded in a stiff plane tend to fail by buckling in a more flexible plane (out-of-plane buckling)

Introduction: Unrestrained Beams

- **this lecture covers the design of unrestrained beams that are prone to lateral torsional buckling.**
- **Beams without continuous lateral restraint are prone to buckling about their major axis, this mode of buckling is called lateral torsional buckling (LTB).**

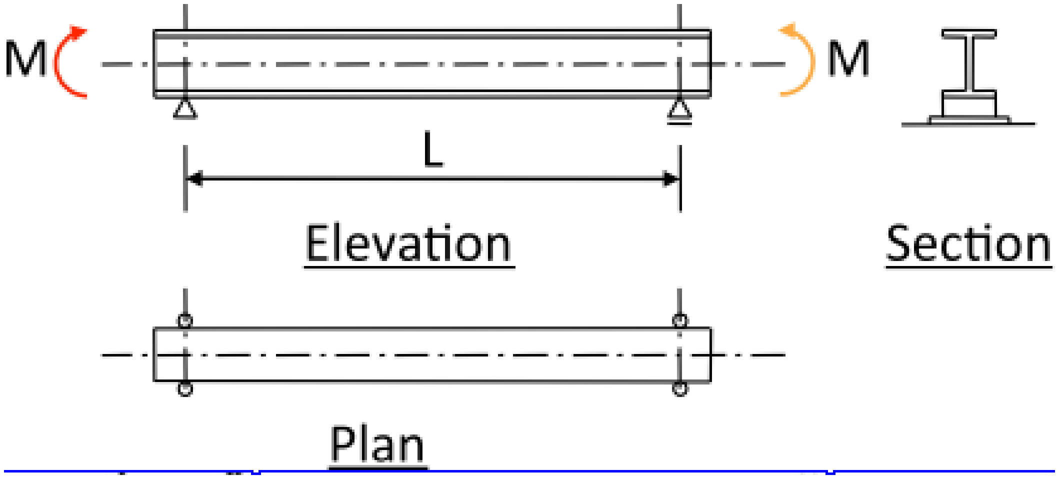
Lateral torsional buckling can be discounted when:

- **The section is bent about its minor axis**
- **Full lateral restraint is provided**
- **Closely spaced bracing is provided making the slenderness of the weakaxis low**
- **The compressive flange is restrained again torsion**
- **The section has a high torsional and lateral bending stiffness**

Introduction: Unrestrained Beams

Behaviour

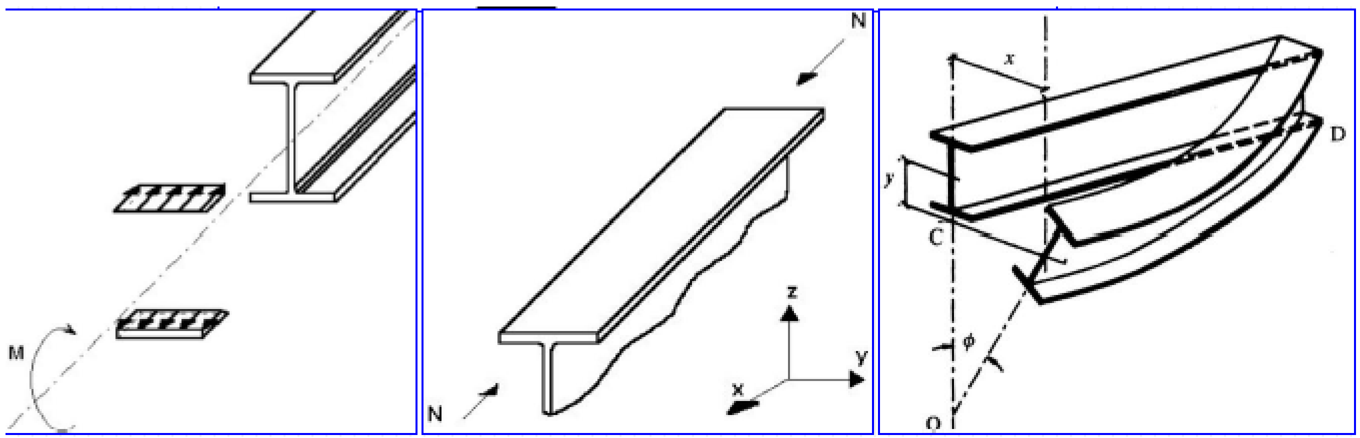
Beam is Perfectly elastic, initially straight, loaded by equal and opposite end moments about its major axis.



- ✓ **Beam is Unrestricted along its length.**
- ✓ **End Supports**
  - ✓**Twisting and lateral deflection prevented.**
  - ✓**Free to rotate both in the plane of the web and on plan.**

Introduction: Unrestrained Beams

Beam is Perfectly elastic, initially straight, loaded by equal and opposite end moments about its major axis.



Three components of displacement are observed i.e

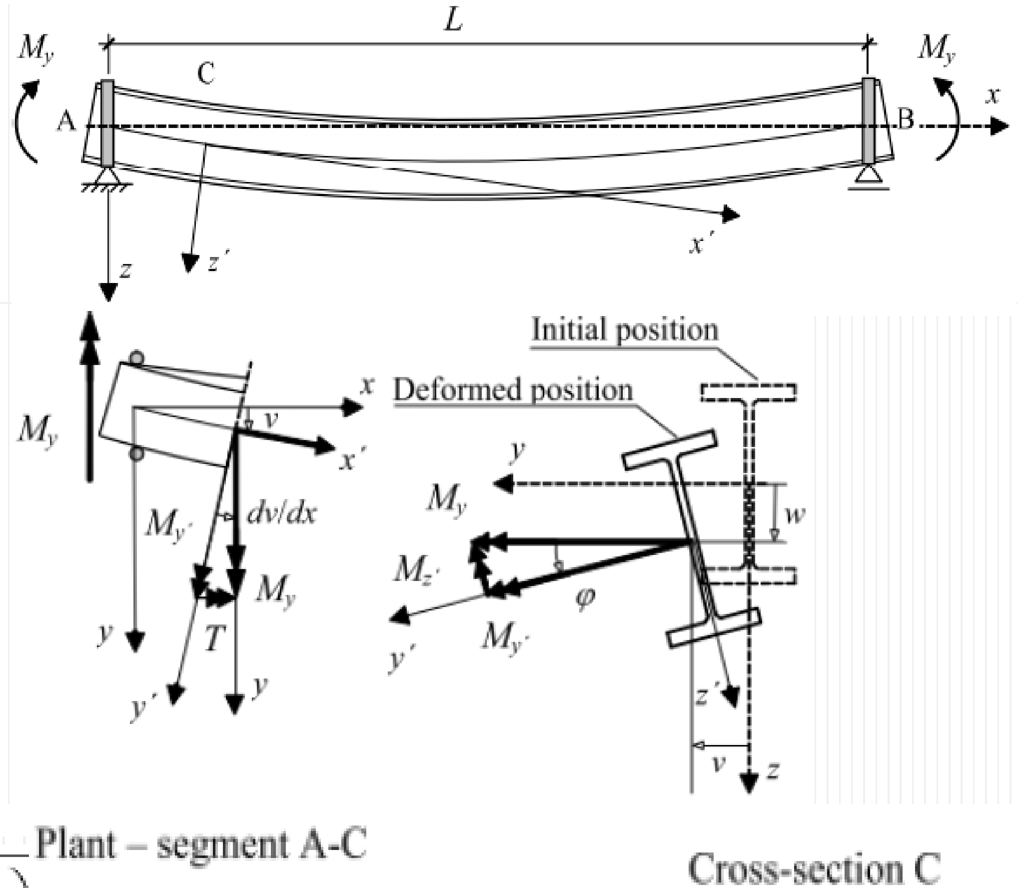
- **Vertical (y)**
- **Horizontal (x)**
- **and torsional (ϕ) displacement**

Elastic critical moment

Consider the following assumptions:

- **Perfect beam**, without any type of imperfections (geometrical or material);
- **Doubly** symmetric cross section;
- Material with **linear** elastic behavior;
- **Small** displacements (cos(ϕ)=1 ; sin(ϕ) = ϕ)

The critical value of the moment about the major axis My , denoted as M^E_{cr} (critical moment of the "standard case") resulting in lateral torsional buckling is obtained:



$$M^E_{cr} = \frac{\pi}{L}\sqrt{G\,I_T\,E\,I_z\left(1+\frac{\pi^2\,E\,I_W}{L^2\,G\,I_T}\right)},$$

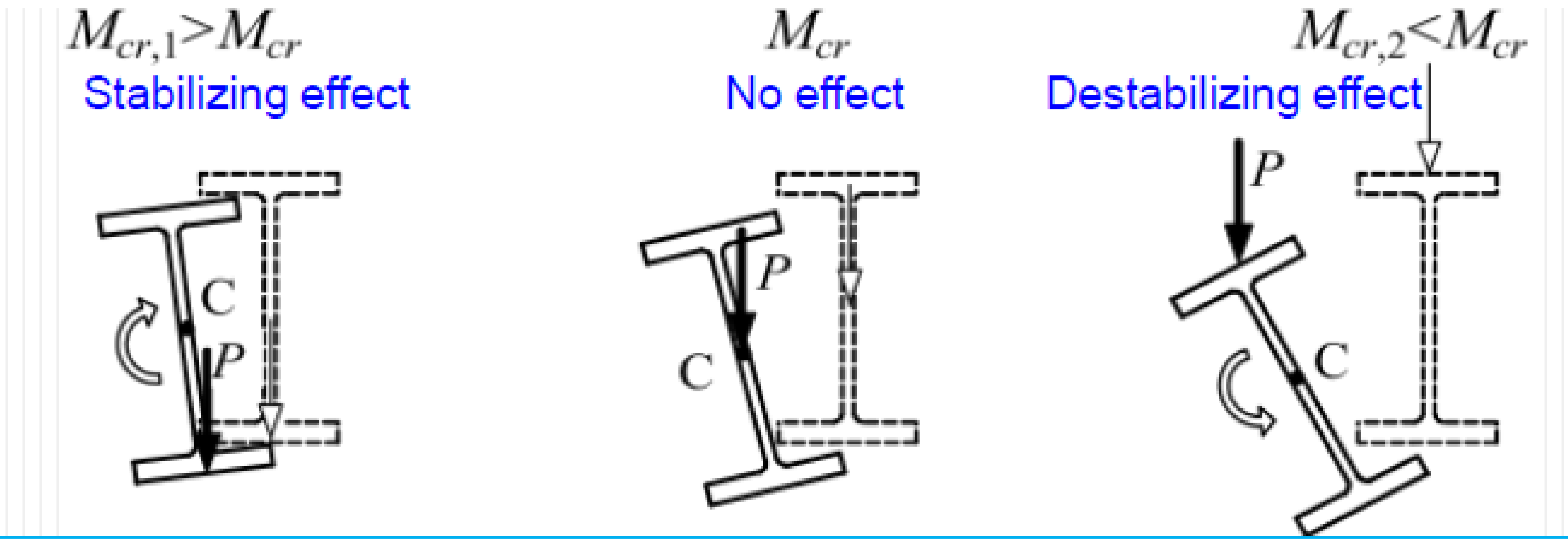
Introduction: Unrestrained Beams-Elastic Critical Moment

Elastic critical moment

It can be observed that the critical moment of a member under bending depends on several factors, such as:

- **loading (shape of the bending moment diagram);**
- **support conditions;**
- **length of the member between laterally braced cross sections;**
- **lateral bending stiffness; torsion stiffness; warping stiffness.**

Besides these factors, the point of application of the loading also has a .directinfluence on the elastic critical moment of a beam



Real Steel Beams

- In reality beams are not free from imperfection, not purely elastic, not always simply supported, not always loaded with only a constant flexure and are not of a doubly symmetric sections, consequently, subject to different bending moment diagrams.

- The derivation of an exact expression for the critical moment for each case of real beams is not practical, as this implies the computation of differential equations of some complexity.

- Therefore, in practical applications approximate formulae are used, which are applicable to a wide set of situations.

Real Steel Beams

As an alternative to some of the expressions, the elastic critical moment can be estimated using expression below proposed by Clark and Hill (1960) and Galea (1981) . It is applicable to members subject to bending about the strong axis, with cross sections mono-symmetric about the weak z axis, for several support conditions and types of loading.

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w} \right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + (C_2 z_g - C_3 z_j)^2 \right]^{0.5} - (C_2 z_g - C_3 z_j) \right\},$$

L	is the distance between points of lateral restraint (L_{cr})
E	is the Young's Modulus = 210000 N/mm²
G	is the shear modulus = 80770 N/mm²
I_z	is the second moment of area about the weak axis
I_t	is the torsion constant
I_w	is the warping constant
k_z	is an effective length factor related to rotations at the end section about the weak axis z (can be conservatively taken as 1.0)
k_w	is an effective length factor related to warping restriction in the same cross sections (can be conservatively taken as 1.0)

Real Steel Beams

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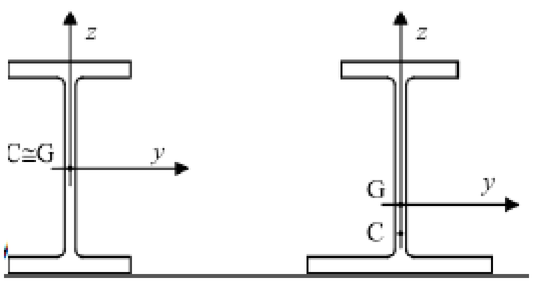

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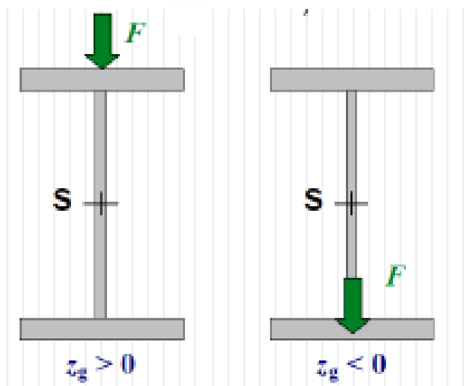


$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \left[\left(\frac{k_z}{k_w}\right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + (C_2 z_g - C_3 z_j)^2 \right]^{0.5} - (C_2 z_g - C_3 z_j) \right\},$$

z_j is a parameter that reflects degree of asymmetry of the cross section in relation to the y axis.

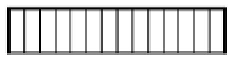
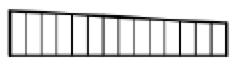
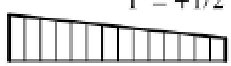
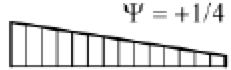
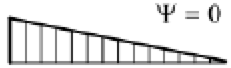
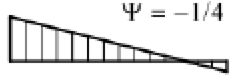
$$z_j = z_s - \left(0.5 \int_A (y^2 + z^2)(z/I_y)\, dA \right)$$

z_g is the distance between the point of load application and the shear center. The value will be positive or negative depending on where the load is applied as shown in the figure.

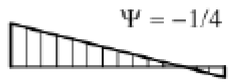
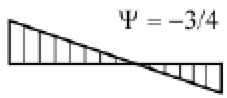
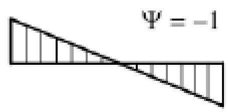
$z_j = 0$ ، for beams with doubly symmetric cross section (such as Ior H cross sections with equal flanges)

Introduction: Unrestrained Beams- Behavior of Real Steel Beams

C1, C2, andC3 are coefficients depending on the shape of the bending moment diagram and on support conditions،

Loading and support conditions	Diagram of moments	k_z	C_1	C_3	
				$\psi_f \leq 0$	$\psi_f > 0$
	$\Psi = +1$ 	1.0	1.00	1.000	
		0.5	1.05	1.019	
	$\Psi = +3/4$ 	1.0	1.14	1.000	
		0.5	1.19	1.017	
	$\Psi = +1/2$ 	1.0	1.31	1.000	
		0.5	1.37	1.000	
	$\Psi = +1/4$ 	1.0	1.52	1.000	
		0.5	1.60	1.000	
	$\Psi = 0$ 	1.0	1.77	1.000	
		0.5	1.86	1.000	
	$\Psi = -1/4$ 	1.0	2.06	1.000	0.850
		0.5	2.15	1.000	0.650

C1, C2, andC3 are coefficients depending on the shape of the bending moment diagram and on support conditions،

Loading and support conditions	Diagram of moments	k_z	C_1	C_3	
				$\psi_f \le 0$	$\psi_f > 0$
		1.0	2.06	1.000	0.850
		0.5	2.15	1.000	0.650
		1.0	2.35	1.000	$1.3-1.2\psi_f$
		0.5	2.42	0.950	$0.77-\psi_f$
		1.0	2.60	1.000	$0.55-\psi_f$
		0.5	2.45	0.850	$0.35-\psi_f$
		1.0	2.60	$-\psi_f$	$-\psi_f$
		0.5	2.45	$-0.125-0.7\psi_f$	$-0.125-0.7\psi_f$

- In beams subject to end moments, by definition $C_2 z_g = 0$.
- $\psi_f = \dfrac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$, where I_{fc} and I_{ft} are the second moments of area of the compression and tension flanges respectively, relative to the weak axis of the section (z axis);
- C_1 must be divided by 1.05 when $\dfrac{\pi}{k_w L}\sqrt{\dfrac{E I_W}{G I_T}} \le 1.0$, but $C_1 \ge 1.0$.

Introduction: Unrestrained Beams- Behavior of Real Steel Beams

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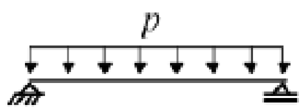
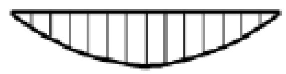
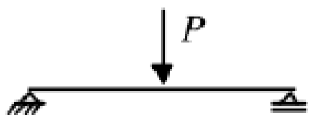
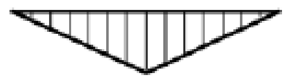
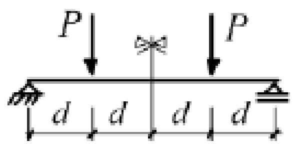
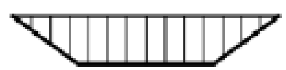
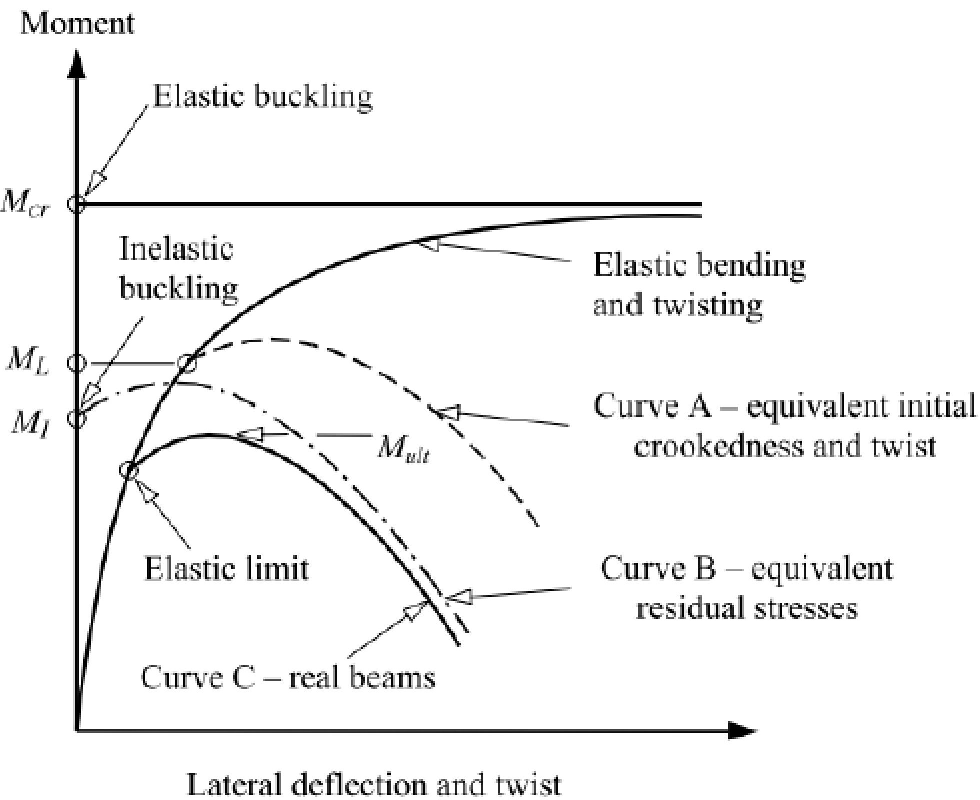
Loading and support conditions	Diagram of moments	k_z	C_1	C_2	C_3
		1.0 0.5	1.12 0.97	0.45 0.36	0.525 0.478
		1.0 0.5	1.35 1.05	0.59 0.48	0.411 0.338
		1.0 0.5	1.04 0.95	0.42 0.31	0.562 0.539

In case of mono-symmetric I or H cross sections, the tables can be used if the following condition is verified

$$-0.9 \leq \psi \leq 0.9$$

Resistance of Real Steel Beams

Real beams differ from an ideal beams in much the same way as do real compression members.



- Thus any small imperfections such as initial crookedness, twist,eccentricity of load, or horizontal load components cause thebeam to behave as if it had an equivalent initial crookedness and twist, as shown by curve A
- Imperfections such as residual stresses or variations in materialproperties cause the beam to behave as shown by curve B.
- The behavior of real beams having both types of imperfection isindicated by curve C.
- Curve C shows a transition from the elastic behaviour of a beam with curvature and twist to the inelastic post-buckling behaviour ofa beam with residual stresses.

The influence of Slenderness

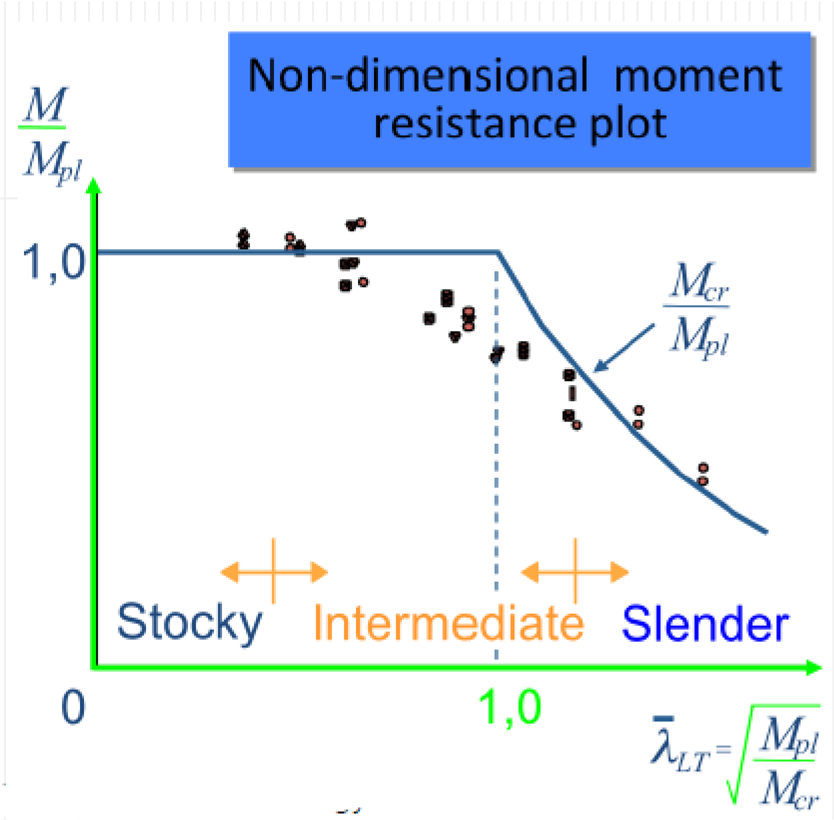
Considering the analogy between Ncr and Mcr, the lateral-torsional behavior of beams in bending is similar to a compressed column. Therefore:

- **The resistance of short/stocky members depends on the value ofthe cross section bending resistance (plastic or elastic bending moment resistance, depending of its cross section class).**
- **The resistance of slender members depends on the value of the critical moment (Mcr), associated with lateral-torsional buckling.**
- **The resistance of members with intermediate slenderness depends on the interaction between plasticity and instability**

Introduction: Unrestrained Beams- Behavior of Real Steel Beams

The influence of Slenderness

Non-dimensional plot permits results from different test series to be compared.



- **Stocky beams** (λ_{LT}<0.4) unaffected by lateral torsional buckling
- **Slender beams** (λ_{LT}>1.2) resistance close to elastic critical moment M_{cr}.
- **Intermediate slenderness** adversely affected by inelasticity and geometric imperfections.
- **EC3 uses a reduction factor χ_{LT} on plastic resistance moment to cover the whole slenderness range..**

Introduction: Unrestrained Beams- Behavior of Real Steel Beams

The influence of Slenderness

Summary of factors to consider influence of Slenderness

Warping: is the distortion of the elements of a steel section out of the plane perpendicular to the axis of the member under twisting/torsion.

Restraining this effects will have a favorable impact in avoiding lateral torsional buckling

End Constraints: Restraints have a major influence on the occurrence of instability and can be utilized to enhance the load carrying capacity of the beam whenever instability is likely to occur.

The stiffness in the minor axis Vs stiffness in the major axis: Section with relatively equal stiffness about both axis are almost never likelyto experience LTB.

Bracing: Lateral bracing of beams is the common measure to overcome the occurrence of LTB

Point of Load application: In relation to the shear center of the section the point of load application may have a favorable/stabilizing or unfavorable/destabilizing effect

Design According to EC3: Unrestrained Beams

Lateral-Torsional Buckling Resistance

The verification of resistance to lateral-torsional buckling of a prismatic member consists of the verification of the following condition (clause6.3.2.1(1)):

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0 ,$$

$M_{b,Rd}$ is the design buckling resistance, given by (clause6.3.2.1(3))

where : $W_y = W_{pl,y}$ for class 1 and 2 cross sections;
$W_y = W_{el,y}$ for class 3 cross sections;
$W_y = W_{eff,y}$ for class 4 cross sections;
χ_{LT} is the reduction factor for lateral-torsional buckling.

In EC3-1-1 two methods for the calculation of the reduction coefficient χ_{LT} in prismatic members are proposed:

A General Method that can be applied to any type of cross section (more conservative)

Alternative Method that can be applied to rolled cross sections or equivalent welded sections.

Design According to EC3: Unrestrained Beams

A General Method-Any section

$$\chi_{LT} = \frac{1}{\phi_{LT} + \left(\phi_{LT}{}^{2} - \bar{\lambda}_{LT}^{2}\right)^{0.5}}, \quad \text{but } \chi_{LT} \leq 1.0,$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^{2}\right];$$

$$\bar{\lambda}_{LT} = \left[W_y\, f_y / M_{cr}\right]^{0.5}$$

α_{LT} is the imperfection factor, which depends on the buckling curve 0.21, 0.34, 0.49 and 0.76 for curves a, b, c and d

M_{cr} is the elastic critical moment.

The buckling curves to be adopted depend on the geometry of the cross section of the member

Section	Limits	Buckling curve
I or H sections rolled	$h/b \leq 2$	*a*
	$h/b > 2$	*b*
I or H sections welded	$h/b \leq 2$	*c*
	$h/b > 2$	*d*
Other sections	---	*d*

Design According to EC3: Unrestrained Beams

Alternative Method-Rolled or equivalent welded sections

Students are highly advised to read more on this topic. The discussion of this method presented in "*Design of Steel Structures Eurocode 3, 2010, by da Silva L.S.* " is recommended as a starting literature.

Deflection Resistance

- **Deflections of flexural members must be limited to avoid damage to finishes, ceilings and partitions, and should be calculated under SLS loads.**

- **EC3 states that limits for vertical deflections should be specified for each project and agreed with the client. The UK National Annex to EC3 suggests:**

NA.2.23 Vertical deflections [BS EN 1993-1-1:2005, 7.2.1(1)B]

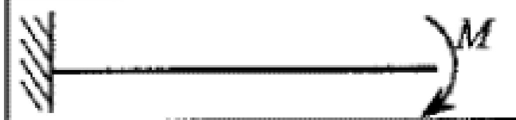
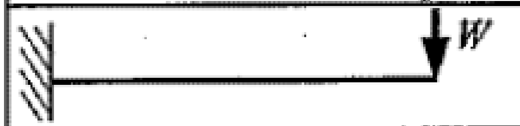
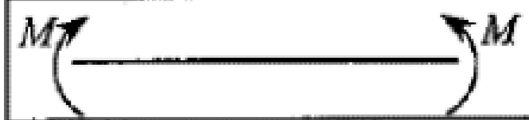
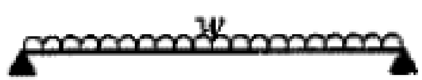
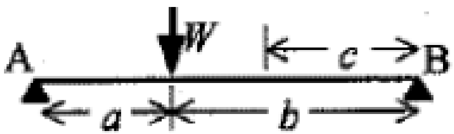
The following table gives suggested limits for calculated vertical deflections of certain members under the characteristic load combination due to variable loads and should not include permanent loads. Circumstances may arise where greater or lesser values would be more appropriate. Other members may also need deflection limits.

On low pitch and flat roofs the possibility of ponding should be investigated.

Vertical deflection	
Cantilevers	Length/180
Beams carrying plaster or other brittle finish	Span/360
Other beams (except purlins and sheeting rails)	Span/200
Purlins and sheeting rails	To suit the characteristics of particular cladding

Standard rules for maximum deflection:

BEAM BENDING

L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \le b, \quad c = \sqrt{\frac{1}{3} b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

Deflection Resistance Summary

1. Define Service loads (Actions)
2. Define Section and beam prosperities
3. Draw the bending moment diagram
4. Determine Maximum deflection of beam
5. Determine Deflection limits
6. Compare Maximum deflection of beam with Deflection limits

Design According to EC3: Unrestrained Beams

Conditions for ignoring the lateral-torsional buckling verification

The verification of lateral-torsional buckling for a member in bending may be ignored if at least one of the following conditions is verified:

$$\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,0} \text{ or } M_{Ed}/M_{cr} \leq \bar{\lambda}_{LT,0}^{\ 2}$$

Where; $\bar{\lambda}_{LT,0} = 0{,}4$ (maximum value)

Improving the lateral torsional buckling resistance

In practical situations, for given geometrical conditions, support conditions and assumed loading, the lateral-torsional buckling behaviour of a member can be improved in two ways:

- by increasing the lateral bending and/or torsional stiffness, by increasing the section or changing from IPE profiles to HEA or HEB or to closed hollow sections (square, rectangular orcircular);
- by laterally bracing along the member the compressed part of the section (the compressed flange in the case of I or H sections). This is more economical, although sometimes it is not feasible.

Design According to EC3: Unrestrained Beams

Bending Moment Resistance Summary:

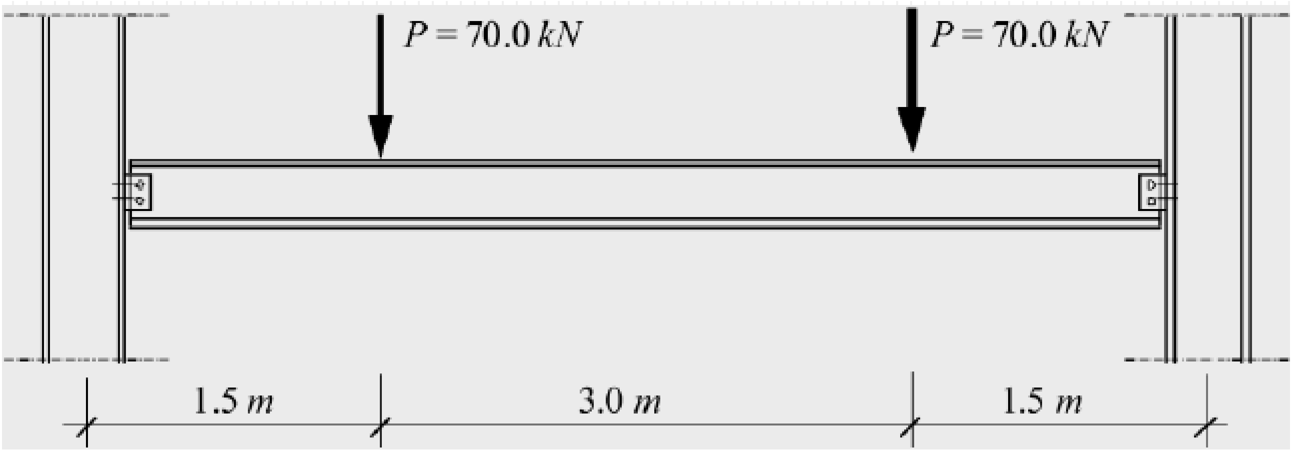
1. Draw the bending moment diagram to obtain the value of the maximum bending moment, M_{Ed}.
2. Determine fy and calculate the class of the section. Once you know the class of the section then you will know which value of the section modulus you will need to use in the equation for $M_{b,Rd}$.
3. Work out the effective length, L_{cr}.
4. Work out the value of M_{cr}, the critical moment.
5. Work out the lateral torsional slenderness ratio using either the general case or alternative expression.
6. Work out Φ_{LT} using either the general case or alternative expression.
7. Work out χ_{LT} using either the general case or alternative expression.
8. Calculate the design buckling resistance $M_{c,Rd}$.
9. Carry out the buckling resistance $\mathbf{M_{c,Rd} > M_{Ed}}$.

Worked Example: Example on cross-section resistance in bending

Example4.4.

Consider the beam, supported by web cleats and loaded by two concentrated loads, P=70.0kN (design loads). Design the beam usinga HEA profile, inS235 steel (E=210GPa and G=81GPa), according to EC3-1-1. Consider free rotation at the supports with respect to the y-axis and the z-axis. Also assume free warping at the supports but consider that the web cleats donot allow rotation around the axis of the beam (x axis). Assume:

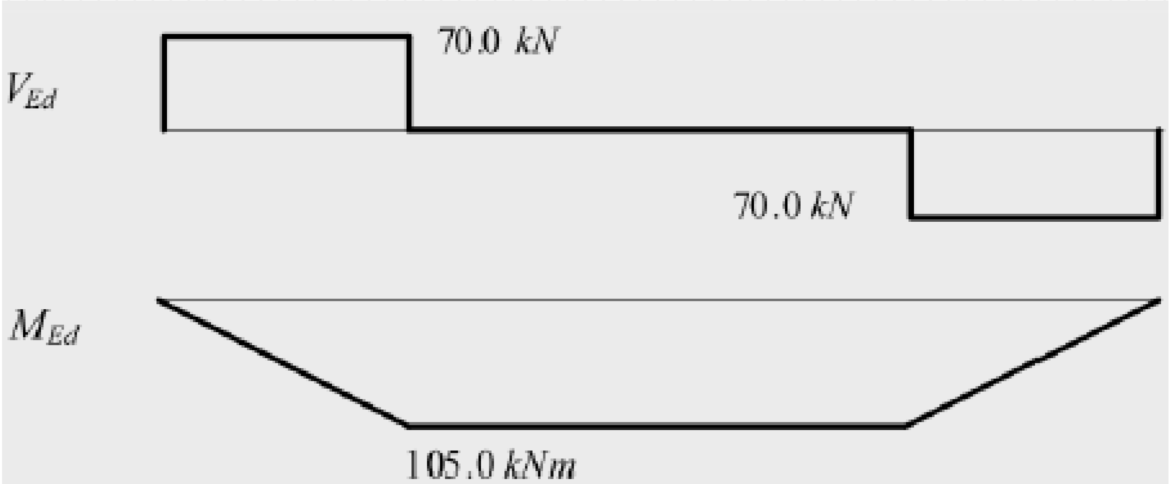
a) Unbraced beam;
b) Beam is braced at points of application of the concentrated loads.



Worked Example: Example on cross-section resistance in bending

Solution :a

Step1: Draw the internal action diagrams to get M_{Ed} & V_{Ed}.



Step2: Select a trial section and carryout the section classification.

Considering a HEA 240 profile.

Worked Example: Example on cross-section resistance in bending

The cross section class of a HEA 240 is obtained as follows

Web in bending, $\frac{c}{t} = \frac{164}{7.5} = 21.9 < 72\varepsilon = 72 \times 1 = 72.0$

Flange in compression,

$$\frac{c}{t} = \frac{240/2 - 7.5/2 - 21}{12} = 7.9 < 9\varepsilon = 9 \times 1 = 9$$

The HEA 240 is class 1, confirming the use of $W_{pl,y}$

S 235 for t≤16mm

Material Properties:

- f_y = 235 MPa
- f_u = 390 MPa
- E = 210 Gpa
- G= 81 GPa

HEA 240

- $W_{pl,y}$=744.6cm³
- I_y=7763cm⁴
- I_z=2769cm⁴
- I_T=41.55cm⁴
- I_w=328.5 X 10³ cm⁶

Step3: Check for Lateral-torsional buckling without intermediate bracing [a].

$$\frac{M_{Ed}}{M_{b,Rd}} \le 1.0,$$

Step3.1: Compute the buckling resistance

$$M_{b,Rd} = \chi_{LT} W_y f_y / \gamma_{M1},$$

$W_y = W_{pl,y}$ for class 1= 744.6cm³

Worked Example: Example on cross-section resistance in bending

$$\chi_{LT} = \frac{1}{\phi_{LT} + \left(\phi_{LT}^{\,2} - \bar{\lambda}_{LT}^{2}\right)^{0.5}}, \quad \text{but } \chi_{LT} \le 1.0,$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^{2}\right];$$

$$\bar{\lambda}_{LT} = \left[W_y \, f_y / M_{cr}\right]^{0.5}$$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2}\left\{\left[\left(\frac{k_z}{k_w}\right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j\right)^2\right]^{0.5} - \left(C_2 z_g - C_3 z_j\right)\right\},$$

4

3

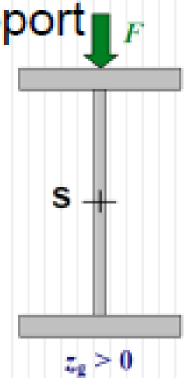
2

1

$L = 6.00$ m

$k_z = k_w = 1.0$, as the standard case support

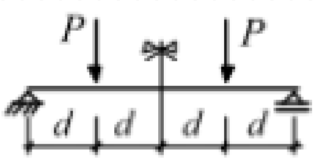
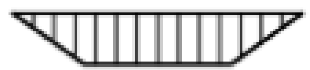
$z_g = 115$ mm



Worked Example: Example on cross-section resistance in bending

z_j = 0 for beams with doubly symmetric cross section (such as I or H cross sections with equal flanges)

C_1 = 1.04, C_2 = 0.42 and C_3 = 0.562

Loading and support conditions	Diagram of moments	k_z	C_1	C_2	C_3
		1.0 0.5	1.04 0.95	0.42 0.31	0.562 0.539

1 $M_{cr} = 231.5\ kNm \Rightarrow \bar{\lambda}_{LT} = 0.87$. 2

Since $\alpha_{LT} = 0.21$ (H rolled section, with $h/b \leq 2$)

3 $\phi_{LT} = 0.95 \Rightarrow \chi_{LT} = 0.75$. 4

Compute the buckling resistance

$$M_{b,Rd} = \chi_{LT} W_y f_y / \gamma_{M1},$$

$$M_{b,Rd} = 0.75 \times 744.6 \times 10^{-6} \times \frac{235 \times 10^3}{1.0} = 131.2\ kNm > M_{Ed} = 105.0\ kNm$$ **O.K.**

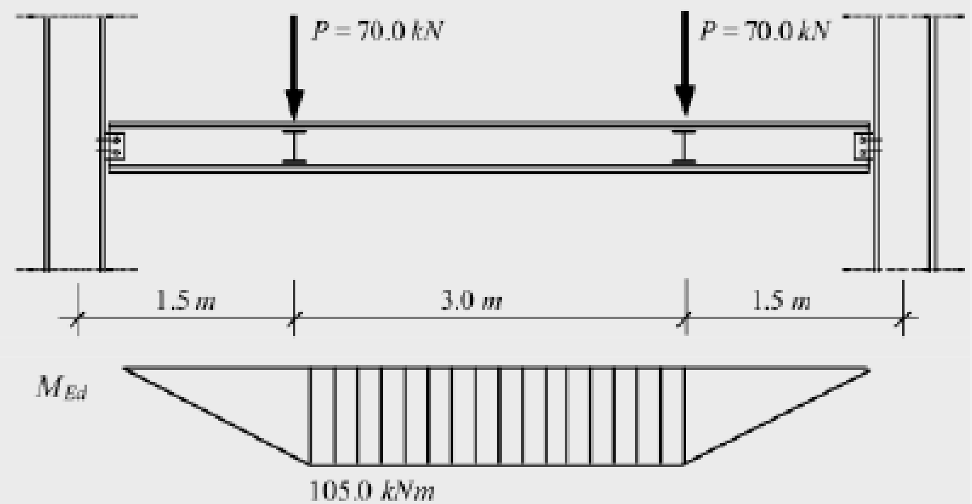
solution :b

Step4: Check for Lateral-torsional buckling with intermediate restraints [b].

Step4.1: Compute the buckling resistance

If the beam is laterally braced at the points of application of the loads, the lateral-torsional buckling behavior is improved.

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0 ,$$



Here a lesser profile of HEA 240 is selected which is checked to be of class 1, confirming the use of $W_{pl,y}$

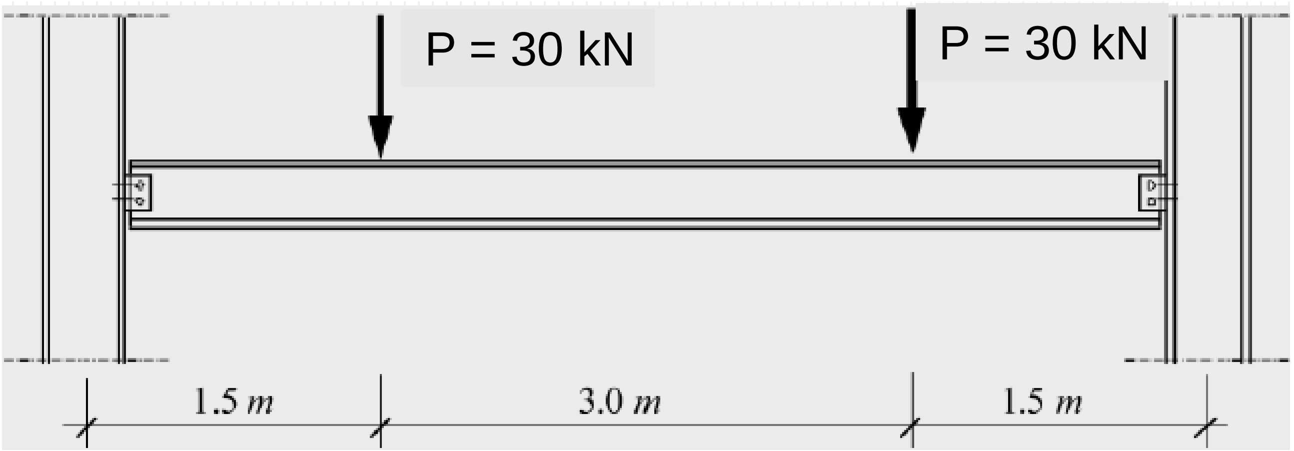
HEA 220

- $W_{pl,y}$=568.5cm³ ► I_T= 28.46cm⁴
- I_z=1955cm⁴ ► I_W= 193.3 X 10³ cm⁶

Worked Example: Example on cross-section resistance in bending

Deflection Verification: SLS unfactored imposed actions.

Unfactored variable loads are shown below



Consider max deflection $\delta = \frac{waL^2}{12EI}$

W = 30 kN, a=1.5 m, L=6m, E= 210000 N/mm^2, I=7763 10^4 mm^4

$$\delta = \frac{waL^2}{12EI} = \frac{30000x1500x6000^2}{12x210000x77630000} = 8.28 \text{ mm}$$

Vertical deflection limit:

$$\frac{L}{360} = \frac{6000}{360} = 16.7\ mm \quad \rightarrow\rightarrow 16.67 \text{ mm} > 8.28 \text{ mm O.K.}$$

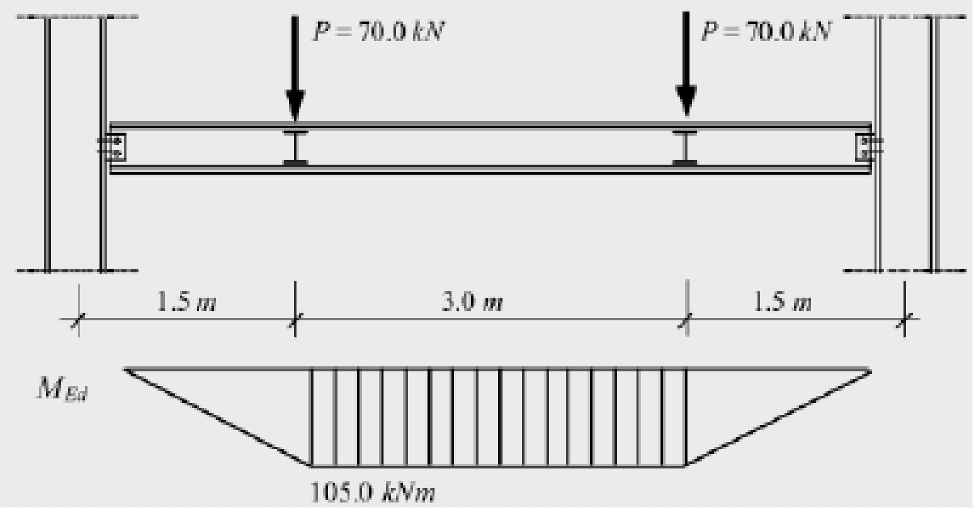
Worked Example: Example on cross-section resistance in bending

Step4: Check for Lateral-torsional buckling with intermediate restraints [b].

Step4.1: Compute the buckling resistance

If the beam is laterally braced at the points of application of the loads, the lateral-torsional buckling behavior is improved.

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0 ,$$



Here a lesser profile of HEA 240 is selected which is checked to be of class 1, confirming the use of $W_{pl,y}$

HEA 220

- $W_{pl,y}$=568.5cm^3
- I_z=1955cm^4
- I_T= 28.46cm^4
- I_W= 193.3 X 10^3 cm^6

Worked Example: Example on cross-section resistance in bending

$$\chi_{LT} = \frac{1}{\phi_{LT} + \left(\phi_{LT}^{\,2} - \overline{\lambda}_{LT}^{2}\right)^{0.5}}, \quad \text{but } \chi_{LT} \le 1.0,$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^{2}\right];$$

$$\overline{\lambda}_{LT} = \left[W_y\, f_y / M_{cr}\right]^{0.5}$$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2}\left\{\left[\left(\frac{k_z}{k_w}\right)^2 \frac{I_W}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j\right)^2\right]^{0.5} - \left(C_2 z_g - C_3 z_j\right)\right\},$$

L = 3.00 m

$k_z = k_w = 1.0$, as the standard case support

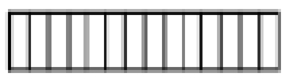
$z_g = 0$, The elastic critical moment of the beam is not aggravated by the fact that the loads are applied at the upper flange, because these are applied at sections that are laterally restrained.

$W_y = W_{pl,y}$ for class 1= 568.5cm^3

Worked Example: Example on cross-section resistance in bending

z_j = 0 for beams with doubly symmetric cross section (such as I or H cross sections with equal flanges)

C_1 = 1.00, C_2 = not important as Z_g=0 and C_3 = 1.0

Loading and support conditions	Diagram of moments	k_z	C_1	C_3	
				$\psi_f \le 0$	$\psi_f > 0$
		1.0 0.5	1.00 1.05	1.000 1.019	

$$M_{cr} = 551.3\ kNm \quad \Rightarrow \quad \bar{\lambda}_{LT} = 0.49\,.$$

As $\alpha_{LT} = 0.21$ (rolled H section, with $h/b \le 2$),

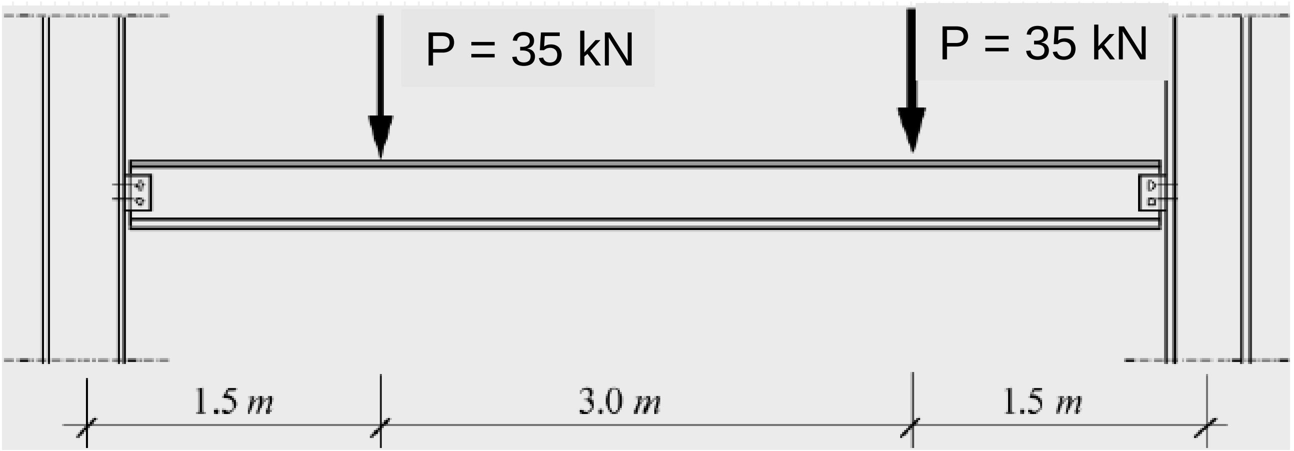
$$\phi_{LT} = 0.65 \quad \Rightarrow \quad \chi_{LT} = 0.93\,.$$

$$M_{b,Rd} = 0.93 \times 568.5 \times 10^{-6} \times \frac{235 \times 10^{3}}{1.0} = 124.2\,kNm > M_{Ed} = 105.0\,kNm$$ **O.K.**

Worked Example: Example on cross-section resistance in bending

Deflection Verification: SLS unfactored imposed actions.

Unfactored variable loads are shown below



Notes:

- **Imposed (variable) Loads must be determined**
- **Max deflection must be calculated.**

Consider max deflection $\delta = \frac{waL^2}{12EI}$

W = 35 kN, a=1.5 m, L=6m, E= 210000 N/mm^2, I=54100000 10^4 mm^4

$$\delta = \frac{waL^2}{12EI} = \frac{35000x1500x6000^2}{12x210000x54100000} = 13.86 \text{ mm}$$

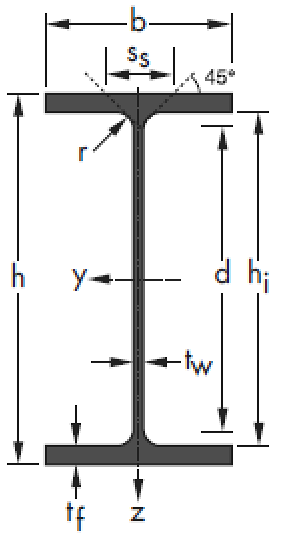
Vertical deflection limit:

$$\frac{L}{360} = \frac{6000}{360} = 16.7\ mm \quad \rightarrow \rightarrow 16.67 \text{ mm} > 13.86 \text{ mm O.K.}$$

Worked Example: Example on cross-section resistance in bending

Summary

Criteria	Unbraced beam	Braced beam
LTB (General method)	HEA 240	HEA 220



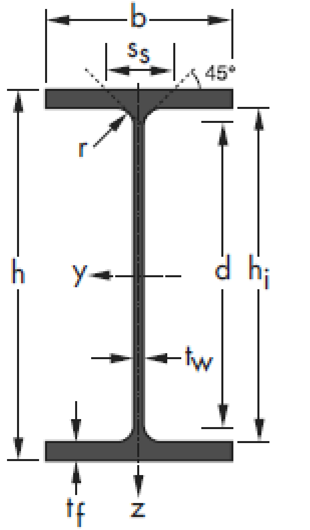
	G kg/m	h mm	b mm	t_w mm	t_f mm	r mm	A mm^2	h_i mm	d mm
							x 10^2		
HE 240 AA*	47,4	224	240	6,5	9	21	60,4	206	164
HE 240 A	60,3	230	240	7,5	12	21	76,8	206	164

	G kg/m	I_y mm^4	$W_{el.y}$ mm^3	$W_{pl.y}$♦ mm^3	i_y mm	A_{vz} mm^2	I_z mm^4	$W_{el.z}$ mm^3	$W_{pl.z}$♦ mm^3	i_z mm	s_s mm	I_t mm^4	I_w mm^6
		x 10^4	x 10^3	x 10^3	x 10	x 10^2	x 10^4	x 10^3	x 10^3	x 10		x 10^4	x 10^9
HE 240 AA	47,4	5835	521,0	570,6	9,83	21,54	2077	173,1	264,4	5,87	49,10	22,98	239,6
HE 240 A	60,3	7763	675,1	744,6	10,05	25,18	2769	230,7	351,7	6,00	56,10	41,55	328,5

Worked Example: Example on cross-section resistance in bending

Summary

Criteria	Unbraced beam	Braced beam
LTB (General method)	HEA 240	HEA 220



	G kg/m	h mm	b mm	t_w mm	t_f mm	r mm	A mm^2 x 10^2	h_i mm	d mm
HE 220 A	50,5	210	220	7	11	18	64,3	188	152

	G kg/m	I_y mm^4 x 10^4	$W_{el.y}$ mm^3 x 10^3	$W_{pl.y}$♦ mm^3 x 10^3	i_y mm x 10	A_{vz} mm^2 x 10^2	I_z mm^4 x 10^4	$W_{el.z}$ mm^3 x 10^3	$W_{pl.z}$♦ mm^3 x 10^3	i_z mm x 10	s_s mm	I_t mm^4 x 10^4	I_w mm^6 x 10^9
HE 220 A	50,5	5410	515,2	568,5	9,17	20,67	1955	177,7	270,6	5,51	50,09	28,46	193,3

جَامعة المَنارة
MANARA UNIVERSITY

Steel Structures 2
Summer Sem.
2023-2024

أ.د. نايل محمد حسن

Lecture 7-8

- Flexural Members

- ✓ -I- Laterally Restrained Beams
- ✓ II- Laterally Unrestrained Beams

- Beam-Column Members

Beam-Column Members

Introduction: Beam-Column Members

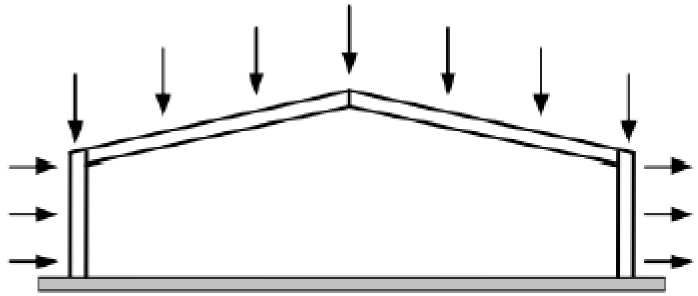
- **Axial force members are, in practice, subjected to axial load as well as bending in either or both the axis of the cross section.**

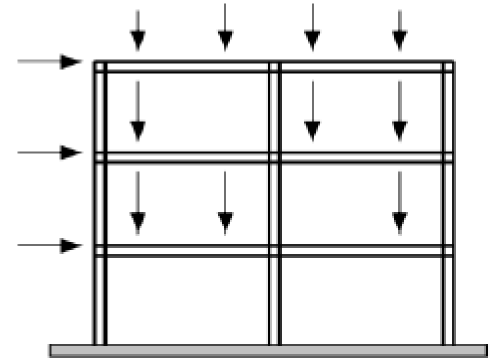
- **Similarly flexural members may also be subjected to axial load.**

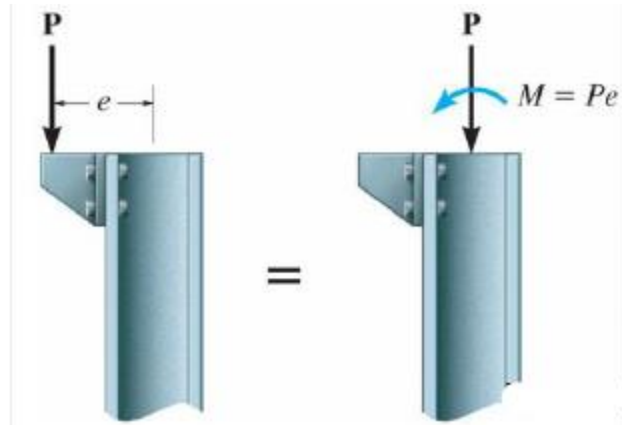
- **In either case, a member subjected to both significant axial and bending stresses is termed as Beam-Column Members.**

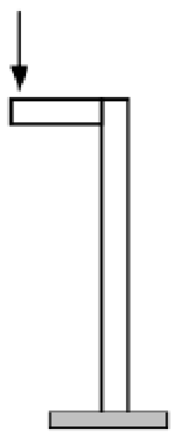
- **The behavior of such members results from the combination of both effects and varies with slenderness.**

Introduction: Beam-Column Members









A member subjected to both significant axial and bending stresses is termed as Beam-Column Members.

Introduction: Beam-Column Members

- At low slenderness, the cross sectional resistance dominates.
- With increasing slenderness, pronounced second-order effects appear, significantly influenced by both geometrical imperfections and residual stresses.
- At high slenderness range, buckling is dominated by elastic behavior, failure tending to occur by flexural buckling (typical of members in pure compression) or by lateral-torsional buckling (typical of members in bending).
  - The behavior of a member under bending and axial force results from the interaction between instability and plasticity and is influenced by geometrical and material imperfections. Therefore very complex.

Introduction: Beam-Column Members

The verification of the safety of members subject to bending and axial force is made in two steps:

- **Verification of the resistance of cross sections .**
- **Verification of the member buckling resistance (in general governed by flexural or lateral-torsional buckling).**

Cross section resistance

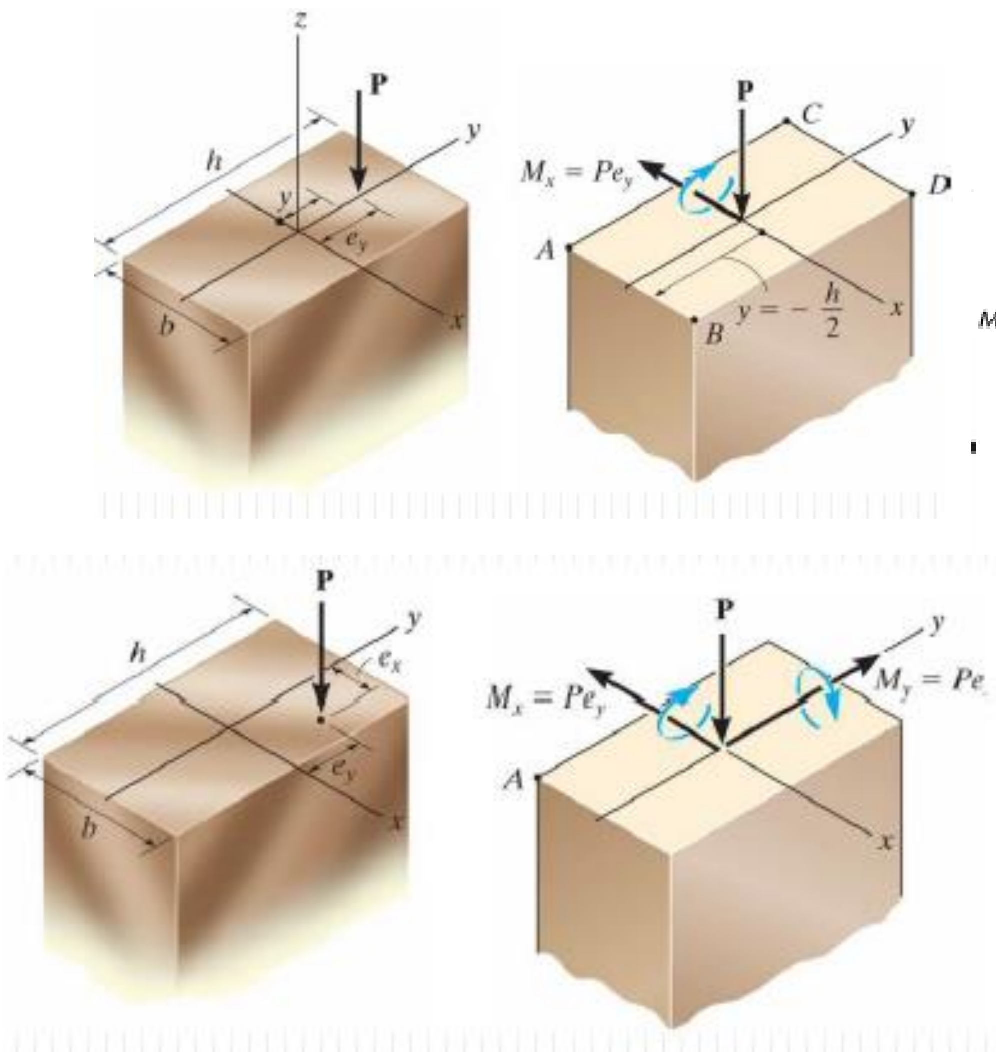
The cross section resistance is based;

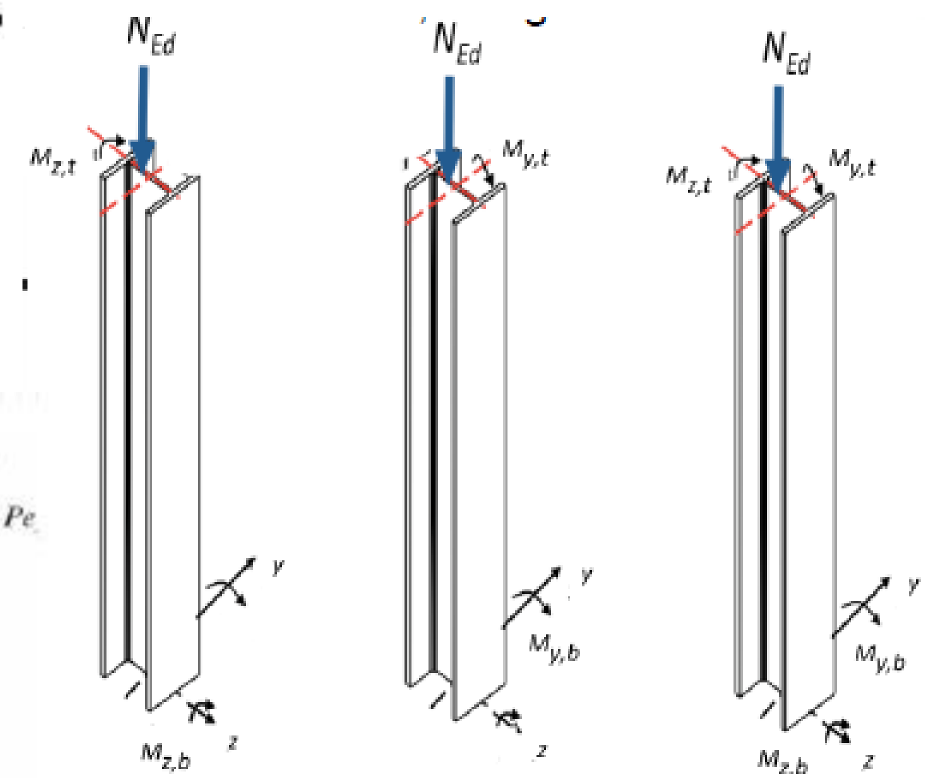
- **on its plastic capacity (class 1 or 2 sections) or**
- **on its elastic capacity (class 3 or 4 cross sections).**

When a cross section is subjected to bending moment and axial force ($N + M_y$, $N + M_z$ or even $N + M_y + M_z$),

the bending moment resistance should be reduced, using interaction formulas.

Cross Section Resistance : M-N interaction

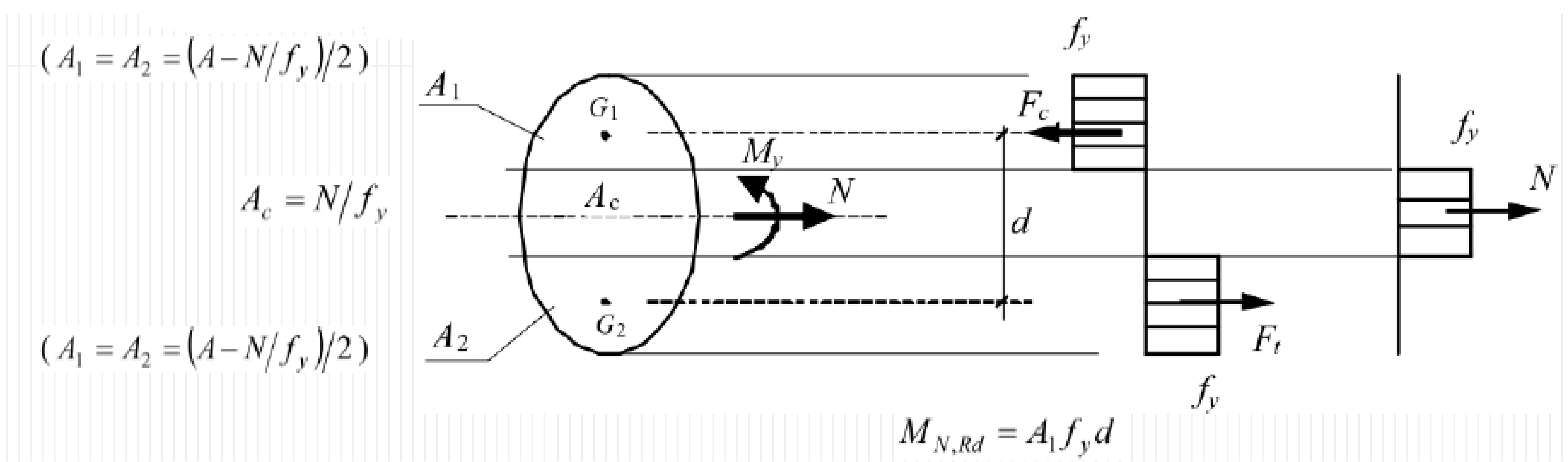




Cross Section Resistance : M-N interaction

- The interaction formulae to evaluate the elastic cross section capacity are the well known formulae of simple beam theory, valid for any type of cross section.
- However, the formulae to evaluate the plastic cross section capacity are specific for each cross section shape.
- For a cross section subjected to N + M, a general procedure may be established to evaluate the plastic bending moment resistance $M_{N.Rd}$, reduced by the presence of an axial force N.

Cross Section Resistance : M-N interaction



- **Although the interaction formulae are easy to obtain by applying the general method, the resulting formulae differ for each cross sectional shape and are often not straightforward to manipulate.**

Cross Section Resistance : M-N interaction

- **Historically, several approximate formulae have been developed, and, Villette (2004) proposed an accurate general formula, applicable to most standard cross sections. with an axis of symmetry with respect to the axis of bending, given by:**

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(\frac{N_{Ed}}{N_{pl,Rd}}\right)^{\alpha,plan} = 1.0 \qquad \alpha_{plan} = 1.0 + 1.82\sqrt{\left(\frac{k}{w_{pl}} - 1.01\right)\frac{k-1}{w_{pl}-1}}\,.$$

- **$w_{pl} = W_{pl}/W_e$ is the ratio between the plastic bending modulus and the elastic modulus,**
- **$k = v/i$ is the ratio between the maximum distance v from an extreme fiber to the elastic neutral axis and the radius of gyration i of the section about the axis of bending.**

Cross Section Resistance : M-N interaction

- **For a circular hollow section, the following exact expression may be established (Lescouarc'h, 1977): :**

$$M_{N,Rd} = M_{pl,Rd} \sin \frac{\pi(1-n)}{2}$$ where, $$n = N_{Ed} / N_{pl,Rd}$$

- **Interaction formulae for axial force and bi-axial bending have usually the following general format:**

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} = 1$$

For I or H cross sections subjected to N + My + Mz,

$$\alpha = \left(1.0 - 0.5\sqrt{n}\right)\alpha_{y,plan}$$

$$\beta = \frac{1+n}{1.0 - n^{(\alpha_{z,plan} - 0.5)}},$$

For RHS cross subjected to N + My + Mz,

$$\alpha = \beta = \frac{1.7}{1 - 1.13\,n^2} \qquad (\text{if } n < 0.8);$$

$$\alpha = \beta = 6 \qquad (\text{if } n \geq 0.8).$$

Cross Section Resistance : Design Resistance

EC1993-1-1 Provisions

Clause 6.2.9 provides several interaction formulae between bending moment and axial force, in the plastic range and in the elastic range. These are applicable to most cross sections. But in all case the following shall be satisfied;

Class 1 or 2 sections

$$M_{Ed} \leq M_{N,Rd}$$

M_{Ed} is the design bending moment and $M_{N.Rd}$ represents the design plastic moment resistance reduced due to the axial force N_{Ed}

For rectangular solid sections under uni-axial bending and axial force, $M_{N,Rdis}$ given by

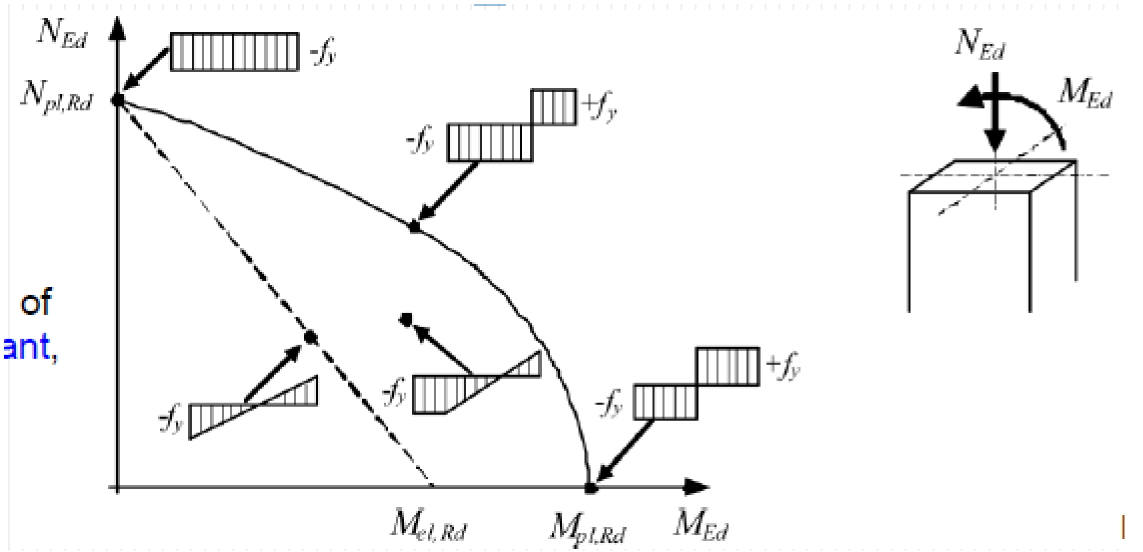
Cross Section Resistance : Design Resistance

For rectangular solid sections under uni-axial bending and axial force, $M_{N,Rd}$ is given by

$$M_{N,Rd} = M_{pl,Rd}\left[1-\left(\frac{N_{Ed}}{N_{pl,Rd}}\right)^2\right]$$

of

ant,



For low values of axial force, the reduction of the plastic moment resistance is not significant, as can be seen.

Cross Section Resistance : Design Resistance

For doubly symmetric I or H sections,

- It is not necessary to reduce the plastic moment resistance about y if the two following conditions are satisfied:

$$N_{Ed} \le 0.25\, N_{pl,Rd} \quad \text{and} \quad N_{Ed} \le 0.5\, h_w\, t_w\, f_y / \gamma_{M0}$$

- It is not necessary to reduce the plastic moment resistance about z if the following condition is verified:

$$N_{Ed} \le h_w\, t_w\, f_y / \gamma_{M0}$$

For I or H sections, rolled or welded, with equal flanges and where fastener holes are not to be accounted for,

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5\,a} \quad \text{but} \quad M_{N,y,Rd} \le M_{pl,y,Rd} \, ;$$

$$M_{N,z,Rd} = M_{pl,z,Rd} \qquad \text{if } n \le a \, ;$$

$$M_{N,z,Rd} = M_{pl,z,Rd} \left[1 - \left(\frac{n-a}{1-a} \right)^2 \right] \qquad \text{if } n > a \, ,$$

where, $a = (A - 2bt_f)/A$, but $a \le 0.5$.

For circular hollow sections,

$$M_{N,Rd} = M_{pl,Rd} \left(1 - n^{1.7}\right)$$

Cross Section Resistance : Design Resistance

For RHS of uniform thickness and for welded box sections with equal flanges and equal webs and where fastener holes are not to be accounted for,

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5\,a_w} \quad \text{but} \quad M_{N,y,Rd} \le M_{pl,y,Rd}$$

$$M_{N,z,Rd} = M_{pl,z,Rd} \frac{1-n}{1-0.5\,a_f} \quad \text{but} \quad M_{N,z,Rd} \le M_{pl,z,Rd}$$

where $a_w \le 0.5$ and $a_f \le 0.5$ are the ratios between the area of the webs and of the flanges, respectively, and the gross area of the cross section.

In a cross section under bi-axial bending and axial force, the N + M_y + M_z interaction can be checked by the following condition:

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} \le 1.0$$

where

α and β are parameters that depend on the shape of the cross section

I or H sections	$\alpha = 2;\ \beta = 5\,n$, but $\beta \ge 1$;
circular hollow sections	$\alpha = \beta = 2$;
rectangular hollow sections	$\alpha = \beta = \frac{1.66}{1-1.13\,n^2}$, but $\alpha = \beta \le 6$.

Cross Section Resistance : Design Resistance

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Class 3 or 4 cross sections

In class 3 or 4 cross sections, the interaction between bending and axial force requires that the following condition be checked:.

$$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}}$$

where

$\sigma_{x,Ed}$ is the design value of the local longitudinal stress due to bending moment and axial force, taking into account the fastener holes where relevant. It is calculated based on the gross cross section for class 3 cross sections, and on a reduced effective cross section for class 4 sections.

Interaction of bending, axial and shear force

The interaction between bending, axial and shear force should be checked as follows :

- When $V_{Ed} \leq$ % 50 of the design plastic shear resistance $V_{Pl,Rd}$, no reduction need be made in the bendin and axial force resistances

- When $V_{Ed} >$ % 50 of the design plastic shear resistance $V_{Pl,Rd}$, then the design resistance to the combination of bending moment and axial force should be calculated using a reduced yield strength for the shea area. This reduced strength is given by $(1-\rho)f_y$, where $\rho=(2\,V_{Ed} / V_{Pl,Rd} -1)^2$

Cross Section Resistance : Design Resistance

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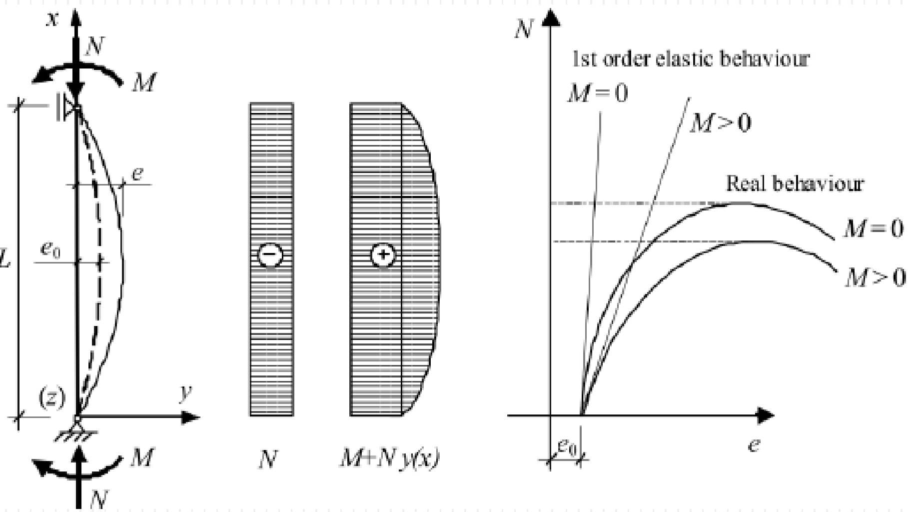
Buckling Resistance: Introduction

For a member under bending and compression, besides the first-order moments and displacements (obtained based on the undeformed configuration), additional second-order moments and displacements exist (“P-δ” effects); these should be taken into account.



- In the past, various interaction formulae have been propo
to represent this situation over the full slenderness range.

- The present approach of EC3-1-1 is based on a linear-addi
interaction formula, illustrated by expression:

$$f\left(\frac{N}{N_u}, \frac{M_y}{M_{uy}}, \frac{M_z}{M_{uz}}\right) \leq 1.0$$

Where,
N, M_y and M_z are the applied forces and
N_u , M_{uy} and M_{uz} are the design resistances, that take in due acco
the associated instability phenomena.

Buckling Resistance: Design Resistance

The development of the design rules, and in particular those adopted by EC3-1-1, is quite complex, as they have to incorporate;

- two instability modes, flexural buckling and lateral-torsional buckling (or a combination of both),
- different cross sectional shapes and several shapes of bending moment diagram, among other aspects.
- several common concepts, such as that of equivalent moment, the definition of buckling length and the concept of amplification.

Several procedures provided in EC3-1-1 were described for the verification of the global stability of a steel structure, including the different ways of considering the second order effects (local P-δ effects and global P-Δ effects).

This topic is solely focused on dealing with the second order effect arising from local P-δ effects.

Buckling Resistance: Design Resistance

Local P-δ effects are generally taken into account according to the procedures given in clause 6.3 of EC3-1-1

Clause 6.3.3(1) considers two distinct situations

Members not susceptible to torsional deformation,

such as members of circular hollow section or other sections restrained from torsion.

Here, flexural buckling is the relevant instability mode.

Members that are susceptible to torsional deformations,

such as members of open section (I or H sections) that are not restrained from torsion.

Here, lateral torsional buckling tends to be the relevant instability mode.

Buckling Resistance: Design Resistance

Members which are subjected to combined bending and axial compression should satisfy the following condition given in clause 6.3.3 of EC3-1-1

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

About major axis y-y,

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

About minor axis z-z,

Where,

N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$	are the design values of the compression force and the maximum moments about the y-y and z-z axis along the member, respectively
$\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$	are the moments due to the shift of the centroidal axis on a reduced effective class 4 cross section
χ_y and χ_z	are the reduction factors due to flexural buckling
χ_{LT}	is the reduction factor due to lateral torsional buckling
k_{yy}, k_{yz}, k_{zy}, k_{zz}	are the interaction factors

Buckling Resistance: Design Resistance

Members which are subjected to combined bending and axial compression should satisfy the following condition given in clause 6.3.3 of EC3-1-1

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

About major axis y-y,

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

About minor axis z-z,

Where,

Values for $N_{Rk} = f_y A_i$, $M_{i,Rk} = f_y W_i$ and $\Delta M_{i,Ed}$

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	0	0	0	$e_{N,y}\, N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z}\, N_{Ed}$

Buckling Resistance: Design Resistance-interaction factors

In EC3-1-1 two methods are given for the calculation of the interaction factors k_{yy} , k_{yz} , k_{zy} and k_{zz}.

Regardless of the method to be applied;

- In members that are not susceptible to torsional deformation, it is assumed that there is no risk of lateral torsional buckling (χ_{LT} = 1.0). And calculating the interaction factors k_{yy} , k_{yz} , k_{zy} and k_{zz} for a member not susceptible to torsional deformation.

Method 1, developed by a group of French and Belgian researchers,

According to this method , a member is not susceptible to torsional deformations if

- $I_T \geq I_Y$, or
- Incase $I_T < I_Y$, but the following condition is satisfied. $\bar{\lambda}_0 \leq 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$,

Where,

C_1 is a coefficient that depends on the shape of the bending moment diagram between laterally braced sections

$N_{cr,z}$ and $N_{cr,T}$ represent the elastic critical loads for flexural buckling about z and for torsional buckling, respectively

λ_0 is the non dimensional slenderness coefficient for lateral torsional buckling, assessed for a situation with constant bending moment.

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

Interaction factors	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	$C_{my} C_{mLT} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{yy}}$
k_{yz}	$C_{mz} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{yz}} 0.6\sqrt{\dfrac{w_z}{w_y}}$
k_{zy}	$C_{my} C_{mLT} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{zy}} 0.6\sqrt{\dfrac{w_y}{w_z}}$
k_{zz}	$C_{mz} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{zz}}$

Auxiliary terms:

$$\mu_y = \frac{1-\dfrac{N_{Ed}}{N_{cr,y}}}{1-\chi_y \dfrac{N_{Ed}}{N_{cr,y}}} \; ; \quad \mu_z = \frac{1-\dfrac{N_{Ed}}{N_{cr,z}}}{1-\chi_z \dfrac{N_{Ed}}{N_{cr,z}}} \; ; \quad w_y = \frac{W_{pl,y}}{W_{el,y}} \le 1.5 \; ; \quad w_z = \frac{W_{pl,z}}{W_{el,z}} \le 1.5$$

$$n_{pl} = \frac{N_{Ed}}{N_{Rk}/\gamma_{M1}} \; ; \quad a_{LT} = 1-\frac{I_T}{I_y} \ge 0$$

; C_{my} and C_{mz} are factors of equivalent uniform moment, determined by the table on the slide # 26,

For class 3 or 4, consider $w_y = w_z = 1.0$.

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

$$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{W_{el,y}}{W_{pl,y}},$$

where $b_{LT} = 0.5\, a_{LT}\, \bar{\lambda}_0^2 \dfrac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \dfrac{M_{z,Ed}}{M_{pl,z,Rd}}$.

$$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq 0.6 \sqrt{\frac{w_z}{w_y}} \frac{W_{el,z}}{W_{pl,z}},$$

where $c_{LT} = 10\, a_{LT} \dfrac{\bar{\lambda}_0^2}{5 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT} M_{pl,y,Rd}}$.

$$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0.6 \sqrt{\frac{w_y}{w_z}} \frac{W_{el,y}}{W_{pl,y}},$$

where $d_{LT} = 2\, a_{LT} \dfrac{\bar{\lambda}_0}{0.1 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT} M_{pl,y,Rd}} \dfrac{M_{z,Ed}}{C_{mz} M_{pl,z,Rd}}$.

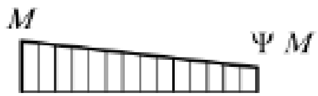
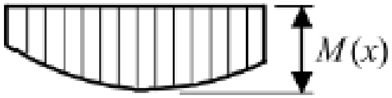
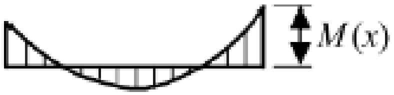
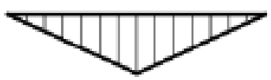
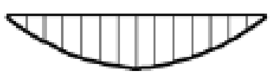
$$C_{zz} = 1 + (w_z - 1)\left[\left(2 - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max} - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max}^2\right) - e_{LT}\right] n_{pl} \geq \frac{W_{el,z}}{W_{pl,z}}, \quad 4$$

where $e_{LT} = 1.7\, a_{LT} \dfrac{\bar{\lambda}_0}{0.1 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT} M_{pl,y,Rd}}$.

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

Diagram of moments	$C_{mi,0}$
	$C_{mi,0} = 0.79 + 0.21\Psi_i + 0.36(\Psi_i - 0.33)\frac{N_{Ed}}{N_{cr,i}}$
 	$C_{mi,0} = 1 + \left(\frac{\pi^2 E I_i \lvert\delta_x\rvert}{L^2 \lvert M_{i,Ed}(x)\rvert} - 1 \right) \frac{N_{Ed}}{N_{cr,i}}$ $M_{i,Ed}(x)$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ according to the first order analyses $\lvert\delta_x\rvert$ is the maximum lateral deflection δ_z (due to $M_{y,Ed}$) or δ_y (due to $M_{z,Ed}$) along the member
	$C_{mi,0} = 1 - 0.18\frac{N_{Ed}}{N_{cr,i}}$
	$C_{mi,0} = 1 + 0.03\frac{N_{Ed}}{N_{cr,i}}$

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

Auxiliary terms (continuation):

$\bar{\lambda}_{max} = \max(\bar{\lambda}_y, \bar{\lambda}_z)$;

$\bar{\lambda}_0 =$ non dimensional slenderness for lateral torsional buckling due to uniform bending moment, that is, taking $\Psi_y = 1.0$ in Table 3.15;

$\bar{\lambda}_{LT} =$ non dimensional slenderness for lateral torsional buckling;

If $\bar{\lambda}_0 \leq 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$: $C_{my} = C_{my,0}$; $C_{mz} = C_{mz,0}$; $C_{mLT} = 1.0$;

If $\bar{\lambda}_0 > 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$: $C_{my} = C_{my,0} + (1-C_{my,0})\frac{\sqrt{\varepsilon_y}\, a_{LT}}{1+\sqrt{\varepsilon_y}\, a_{LT}}$;

$$C_{mz} = C_{mz,0};\ C_{mLT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}} \geq 1;$$

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A}{W_{el,y}}$ for class 1, 2 or 3 cross sections;

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A_{eff}}{W_{eff,y}}$ for class 4 cross sections;

$N_{cr,y}$ is the elastic critical load for flexural buckling about y;

$N_{cr,z}$ is the elastic critical load for flexural buckling about z;

$N_{cr,T}$ is the critical load for torsional buckling;

I_T is the constant of uniform torsion or St. Venant's torsion;

I_y is the second moment of area about y;

$C_1 = \left(\frac{1}{k_c}\right)^2$ where k_c is taken from Table 3.10.

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

According to Method 2, the following members may be considered as not susceptible to torsional deformation:

- members with circular hollow sections (CHS).
- members with rectangular hollow sections (RHS) (there is widly argued exception to this rule presented in (l
- members with open cross section, provided that they are torsionally and laterally restrained.

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Interaction factors k_{ij} in members not susceptible to torsional deformations according to Method 2

Interaction factors	Typeof section	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	I or H sections and rectangular hollow sections	$C_{my}\left(1+0.6\,\bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0.6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+(\bar{\lambda}_y-0.2)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0.8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I or H sections and rectangular hollow sections	k_{zz}	$0.6\,k_{zz}$

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Interaction factors k_{ij} in members not susceptible to torsional deformations according to Method 2

k_{zz}	I or H sections	$C_{mz}\left(1+0.6\bar{\lambda}_z\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(2\bar{\lambda}_z-0.6\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+1.4\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
	rectangular hollow sections	$\leq C_{mz}\left(1+0.6\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(\bar{\lambda}_z-0.2\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+0.8\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$

In I or H sections and rectangular hollow sections under axial compression and uniaxial bending ($M_{y,Ed}$), k_{zy} may be taken as zero.

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Interaction factors k_{ij} in members susceptible to torsional deformations according to Method 2

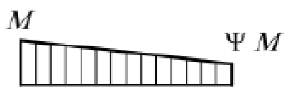
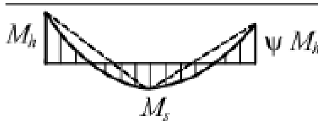
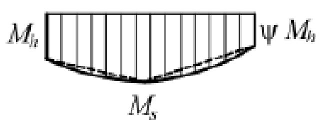
Interaction factors	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	k_{yy} of Table 3.16	k_{yy} of Table 3.16
k_{yz}	k_{yz} of Table 3.16	k_{yz} of Table 3.16
k_{zy}	$\left[1-\frac{0.05\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] \geq \left[1-\frac{0.05}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$	$\left[1-\frac{0.1\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] \geq \left[1-\frac{0.1}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ for $\bar{\lambda}_z < 0.4$: $k_{zy} = 0.6+\bar{\lambda}_z \leq 1-\frac{0.1\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}$
k_{zz}	k_{zz} of Table 3.16	k_{zz} of Table 3.16

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Equivalent factors of uniform moment C_{mi} according to Method 2

Diagram of moments	Range		C_{my}, C_{mz} and C_{mLT}	
			Uniform loading	Concentrated load
	$-1 \le \Psi \le 1$		$0.6+0.4\Psi \ge 0.4$	
 $\alpha_s = M_s/M_h$	$0 \le \alpha_s \le 1$	$-1 \le \Psi \le 1$	$0.2+0.8\alpha_s \ge 0.4$	$0.2+0.8\alpha_s \ge 0.4$
	$-1 \le \alpha_s < 0$	$0 \le \Psi \le 1$	$0.1-0.8\alpha_s \ge 0.4$	$-0.8\alpha_s \ge 0.4$
		$-1 \le \Psi < 0$	$0.1(1-\Psi)-0.8\alpha_s \ge 0.4$	$0.2(-\Psi)-0.8\alpha_s \ge 0.4$
 $\alpha_h = M_h/M_s$	$0 \le \alpha_h \le 1$	$-1 \le \Psi \le 1$	$0.95+0.05\alpha_h$	$0.90+0.10\alpha_h$
	$-1 \le \alpha_h < 0$	$0 \le \Psi \le 1$	$0.95+0.05\alpha_h$	$0.90+0.10\alpha_h$
		$-1 \le \Psi < 0$	$0.95+0.05\alpha_h(1+2\Psi)$	$0.90+0.10\alpha_h(1+2\Psi)$
In the calculation of α_s or α_h parameters, a hogging moment should be taken as negative and a sagging moment should be taken as positive.				

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

For members with sway buckling mode, the equivalent uniform moment factor should be taken as $C_{my} = 0.9$ or $C_{mz} = 0.9$, respectively.

Factors C_{my}, C_{mz} and C_{mLT} should be obtained from the diagram of bending moments between the relevant braced sections, according to the following:

Moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

Equivalent factors of uniform moment C_{mi} according to Method 2

Design According to EC3:
Section classification for sections under bending and axial force

According to EC3, the classification of a cross section is based on its maximum resistance to the type of applied internal forces, independent from their values.

- This procedure is straightforward to apply for cross sections subjected to either bending or compression.
- However, the presence of both the compression and bending moment on the cross-section member, generates a stress distribution between that related to pure compression and that associated with the presence of the sole bending moment.
- Bearing in mind this additional complexity, simplified procedures are often adopted, such as:

i. to consider the cross section subjected to compression only, being the most unfavourable situation (too conservative in some cases)

ii. to classify the cross section based on an estimate of the position of the neutral axis based on the applied internal forces.

- In the later case the neutral axis depth depends on whether the section can plastify, the bending axis, the section profile.

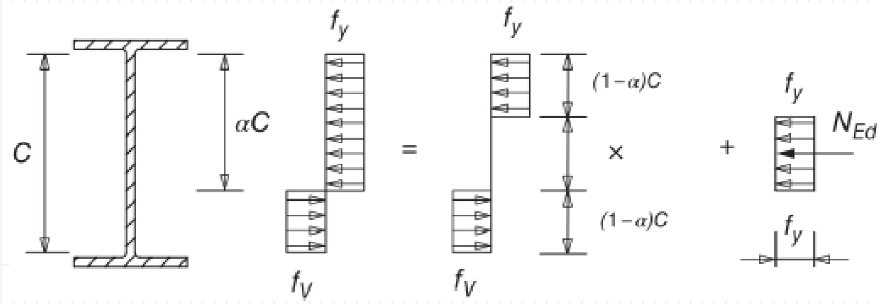
Design According to EC3:
Section classification for sections under bending and axial force

For Bending and Compression about a strong Axis (y-y).

Normal stress distribution on the web depends on the value of the design axial load by means of parameter α for profiles able to resist in the plastic range (classes 1 and 2).

Applying Section Equilibrium and Super positioning

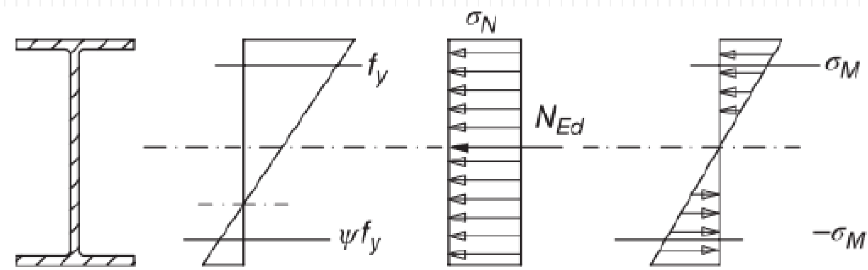
$$\alpha = \frac{1}{2}\left(1 + \frac{1}{c} \cdot \frac{N_{Ed}}{t_w f_y}\right)$$



in case of elastic normal stress distribution, reference has to be made to parameter ψ (classes 3 and 4).

Applying Section Equilibrium and Super positioning

$$\psi = 2\frac{N_{Ed}}{A\, f_y} - 1$$



With reference to the case of a neutral axis located in the web, α ranges between 0.5 (bending) and 1 (compression) and ψ ranges between −1 (bending) and 1 (compression).

Once the stress distribution is assumed and the values of α and ψ can be used to classify the section using tables 5.2 (sheet1 through 3)