جامعة المنارة

كلية: الهندسة

اسم المقرر: الرياضيات المتقطعة

رقم الجلسة (الثالثة)

عنوان الجلسة

الحجج

Arguments

Use truth table to prove that the following argument is valid:

1. $p \longrightarrow (q \vee r)$
2. $\neg q$

$\therefore\ p \longrightarrow r$

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$\neg q$	$p \longrightarrow r$
T	T	T	T	T	F	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	F	T	T	T

Argument is valid (الحجة صالحة)

Use truth table to prove that the following argument is valid:

1. $p \vee r$
2. $(p \longrightarrow q) \wedge (q \longrightarrow r)$

$\therefore \neg q$

p	q	r	$p \vee r$	$p \to q$	$q \to r$	$(p \longrightarrow q) \wedge (q \longrightarrow r)$	$\neg q$
T	T	T	T	T	T	T	F
T	T	F	T	T	F	F	F
T	F	T	T	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	F
F	T	F	F	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

استناداً إلى السطر الأول أو السطر الخامس نجد أن الحجة غير صالحة (Argument is invalid)

Use truth table to prove that the following argument is valid:

1. $p \lor \neg q$
2. $\neg p \lor q$

$\therefore\ p \longrightarrow q$

p	q	¬p	¬q	p∨¬q	¬p ∨ q	p ⟶ q
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Argument is valid (الحجة صالحة)

Valid argument Forms

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Modus Ponens	$p \to q$ p $\therefore q$	
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	
Generalization	**a.** p $\therefore p \vee q$	**b.** q $\therefore p \vee q$
Specialization	**a.** $p \wedge q$ $\therefore p$	**b.** $p \wedge q$ $\therefore q$
Conjunction	p q $\therefore p \wedge q$	
Elimination	**a.** $p \vee q$ $\sim q$ $\therefore p$	**b.** $p \vee q$ $\sim p$ $\therefore q$
Transitivity	$p \to q$ $q \to r$ $\therefore p \to r$	

prove that the following argument is valid:

argument

a. $\neg p \wedge q$

b. $r \longrightarrow p$

c. $\neg r \longrightarrow s$

d. $s \longrightarrow t$

$\therefore t$

solution

1. $\neg p \wedge q$ — from (a)
   $\therefore \neg p$ — by specialization
2. $r \longrightarrow p$ — from (b)
   $\neg p$ — from (1)
   $\therefore \neg r$ — by modus tollens
3. $\neg r \longrightarrow s$ — from (c)
   $\neg r$ — from (2)
   $\therefore s$ — by modus ponens
4. $s \longrightarrow t$ — from (d)
   s — from (3)
   $\therefore t$ — by modus ponens

Argument is valid

prove that the following argument is valid:

solution

argument

a. $p \longrightarrow q$

b. $q \vee s \longrightarrow t$

c. $\neg t$

$\therefore \neg p \wedge \neg q$

1. $q \vee s \longrightarrow t$	from (b)
$\neg t$	from (c)
$\therefore \neg(q \vee s)$	by modus tollens
$\neg(q \vee s) \equiv \neg q \wedge \neg s$	by De morgan law
2. $\neg q \wedge \neg s$	from (1)
$\therefore \neg q$	by specialization
3. $p \longrightarrow q$	from (a)
$\neg q$	from (2)
$\therefore \neg p$	by modus tollens
4. $\neg p$	from (3)
$\neg q$	from (2)
$\therefore \neg p \wedge \neg q$	by conjunction

Argument is valid

prove that the following argument is valid:

solution

argument
a. $p \longrightarrow \neg q$
b. $r \longrightarrow q$
c. r
$\therefore \neg p$

1. $r \longrightarrow q$	from (b)
r	from (c)
$\therefore q$	by modus ponens
2. $p \longrightarrow \neg q$	from (a)
q	from (1)
$\therefore \neg p$	by modus tollens

Argument is valid

prove that the following argument is valid:

argument

a. p
b. $p \longrightarrow q$
c. $S \vee r$
d. $r \longrightarrow \neg q$
$\therefore s$

solution

1.	$p \longrightarrow q$	from (b)
	p	from(a)
	$\therefore q$	by modus ponens
2.	$r \longrightarrow \neg q$	from (d)
	q	from (1)
	$\therefore \neg r$	by modus tollens
3.	$S \vee r$	from (c)
	$\neg r$	from (2)
	$\therefore s$	by elimination

Argument is valid

prove that the following argument is valid:

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solution

argument

a. $p \vee (q \vee (r \vee s))$

b. $\neg p$

c. $\neg q$

d. $\neg s$

$\therefore r$

1. $p \vee (q \vee (r \vee s))$	from (a)
$\equiv (p \vee q) \vee (r \vee s)$	by associative law
2. $\neg p$	from (b)
$\neg q$	from (c)
$\therefore \neg p \wedge \neg q$	by conjunction
$\equiv \neg (p \vee q)$	by De morgan law
3. $(p \vee q) \vee (r \vee s)$	from (1)
$\neg (p \vee q)$	from (2)
$\therefore (r \vee s)$	by elimination
4. $(r \vee s)$	from (3)
$\neg s$	from (d)
$\therefore r$	by elimination

Argument is valid

prove that the following argument is valid:

argument

a. $\neg p \lor (q \longrightarrow r)$

b. $\neg r$

$\therefore \neg (p \land q)$

solution

1. $\neg p \lor (q \longrightarrow r) \equiv \neg p \lor (\neg q \lor r)$ from (a) by conditional law

$\equiv (\neg p \lor \neg q) \lor r$ by associative law

$\equiv \neg (p \land q) \lor r$ by De morgan law

2. $\neg (p \land q) \lor r$ from (1)

$\neg r$ from(b)

$\therefore \neg (p \land q)$ by elimination

Argument is valid

prove that the following argument is valid:

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solution

argument

a. $p \longrightarrow q$

b. $s \longrightarrow q$

c. $\neg q$

$\therefore \neg p \wedge \neg s$

1.	$p \longrightarrow q$	from (a)
	$\neg q$	from(c)
	$\therefore \neg p$	by modus tollens
2.	$s \longrightarrow q$	from (b)
	$\neg q$	from(c)
	$\therefore \neg s$	by modus tollens
3.	$\neg p$	from (1)
	$\neg s$	from (2)
	$\therefore \neg p \wedge \neg s$	by conjunction

Argument is valid

prove that the following argument is valid:

argument

a. $\neg p \longrightarrow r \wedge \neg s$
b. $t \longrightarrow s$
c. $u \longrightarrow \neg p$
d. $\neg w$
e. $u \vee w$
$\therefore \neg t$

solution

1.	$u \vee w$	from (e)
	$\neg w$	from (d)
	$\therefore u$	by elimination
2.	$u \longrightarrow \neg p$	from (c)
	u	from (1)
	$\therefore \neg p$	by modus ponens
3.	$\neg p \longrightarrow r \wedge \neg s$	from (a)
	$\neg p$	from (2)
	$\therefore r \wedge \neg s$	by modus ponens
4.	$r \wedge \neg s$	from (3)
	$\therefore \neg s$	by specialization
5.	$t \longrightarrow s$	from (b)
	$\neg s$	from (4)
	$\therefore \neg t$	by modus tollens

Argument is valid

prove that the following argument is valid:

argument

a. $\neg p \vee q \longrightarrow r$

b. $r \longrightarrow s \vee t$

c. $\neg s \wedge \neg u$

d. $\neg u \longrightarrow \neg t$

$\therefore p$

solution

1. $\neg s \wedge \neg u$ from (c)
   $\therefore \neg u$ by specialization
2. $\neg u \longrightarrow \neg t$ from (d)
   $\neg u$ from(1)
   $\therefore \neg t$ by modus ponens
3. $\neg s \wedge \neg u$ from (c)
   $\therefore \neg s$ by specialization
4. $\neg t$ from (2)
   $\neg s$ from (3)
   $\therefore \neg s \wedge \neg t$ by conjunction
   $\neg s \wedge \neg t \equiv \neg(s \vee t)$ by De morgan law
5. $r \longrightarrow s \vee t$ from (b)
   $\neg(s \vee t)$ from(4)
   $\therefore \neg r$ by modus tollens
6. $\neg p \vee q \longrightarrow r$ from (a)
   $\neg r$ from (5)
   $\therefore \neg(\neg p \vee q)$ by modus tollens
   $\neg(\neg p \vee q) \equiv p \wedge \neg q$ by De morgan law
7. $p \wedge \neg q$ from(6)
   $\therefore p$ by specialization

Argument is valid

homework

prove that the following arguments are valid:

Argument(1)

a. p

b. q

$\therefore (p \wedge q) \vee r$

Argument(2)

a. $(p \longrightarrow q) \wedge (r \longrightarrow s)$

b. p

c. r

$\therefore q \wedge s$

Argument(3)

a. $(p \vee q) \wedge (r \longrightarrow s)$

b. $\neg q$

c. $\neg s$

$\therefore p \wedge \neg r$

Argument(4)

a. $p \wedge q$

b. $(p \vee q) \longrightarrow r$

$\therefore r$