B. Haidar

Structural Mechanics (2)

Week No-03

Slope-Deflection for Beams and Frames

B. Haidar

Slope-Deflection Method for Beams and Frames

- Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.
- Analysis of Continuous Beams.
- Analysis of Frames without Sidesway.
- **Analysis of Frames with Sidesway.**

Member with One End Hinged

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Slope Deflection Equations:

$$M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi) + \mathrm{FE}M_{AB}$$

$$M_{BA} = \mathbf{0} = \frac{2EI}{L}(\theta_A + 2\theta_B - 3\psi) + \mathrm{FE}M_{BA}$$

$$\theta_B = \frac{-\theta_A}{2} + \frac{3}{2}\psi - \frac{L}{4EI}(\mathrm{FE}M_{BA})$$

Modified Slope Deflection Equations:

$$M_{AB} = \frac{3EI}{L}(\theta_A - \psi) + (\mathrm{FE}M_{AB} - \frac{\mathrm{FE}M_{BA}}{2})$$

$$M_{BA} = \mathbf{0}$$

Modified Slope Deflection Equations:

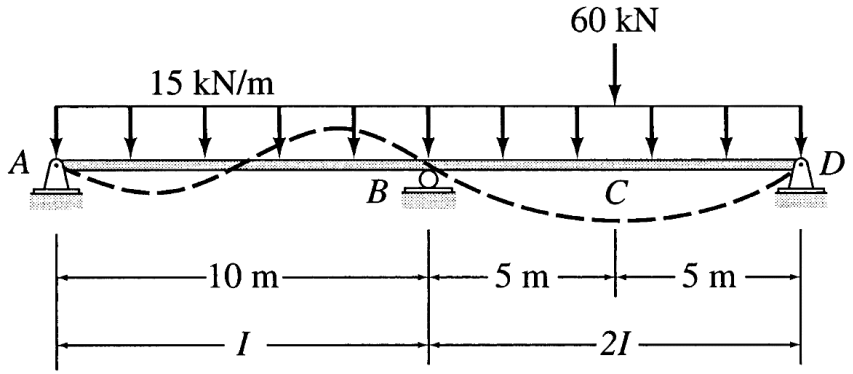
$$\boldsymbol{M_{rh} = \frac{3EI}{L}(\theta_r - \psi) + (\mathrm{FE}M_{rh} - \frac{\mathrm{FE}M_{hr}}{2})}$$

$$\boldsymbol{M_{hr} = 0}$$

r: rigidly connected end
h: hinged end

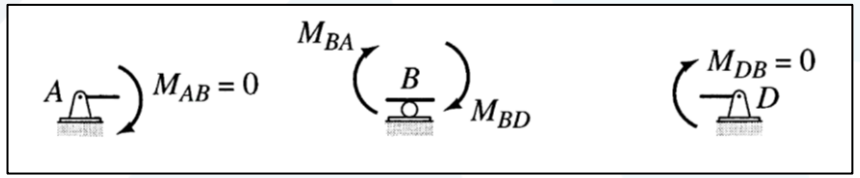
Ex.1: Determine the member end moments and reactions

$$\theta_A = \theta_D = 0$$
$$\&\Delta = 0$$
$$\theta_B \neq 0 \ , \theta_C \neq 0$$



$E = 200$ GPa
$I = 700(10^6)$ mm^4

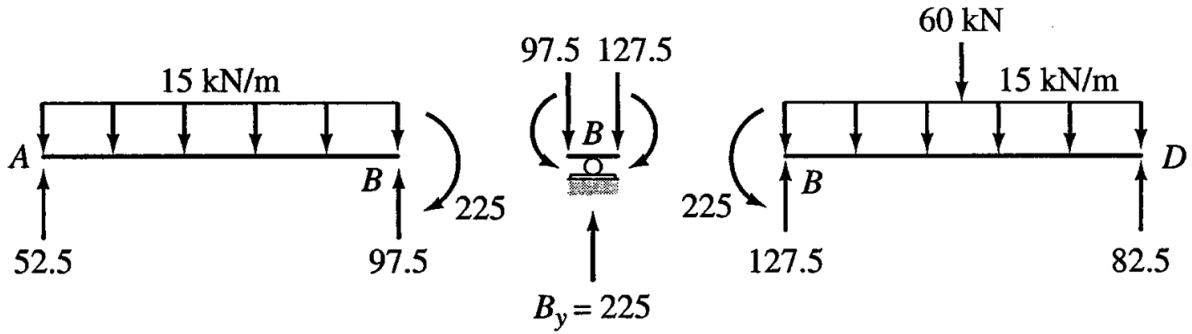
Since the end supports A and D of the beam are **simple supports** at which **no external moment is applied**, the moments at the end A of member AB and at the end D of member BD must be zero. (This can easily be verified by considering moment equilibrium of the free bodies of joints A and D shown in figure. Thus the end A of member AB and the end D of member BD can be considered to be hinged ends, and the modified slope-deflection equations can be used for these members.

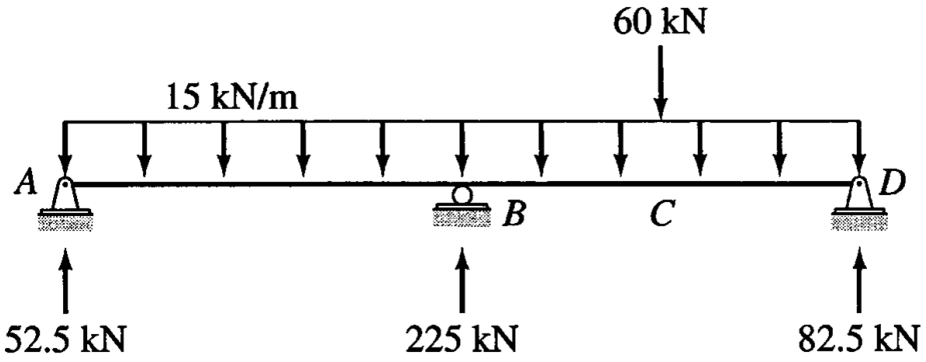


$$\boldsymbol{M_{rh} = \frac{3EI}{L}(\theta_r - \psi) + (\mathrm{FE}M_{rh} - \frac{\mathrm{FE}M_{hr}}{2})}$$

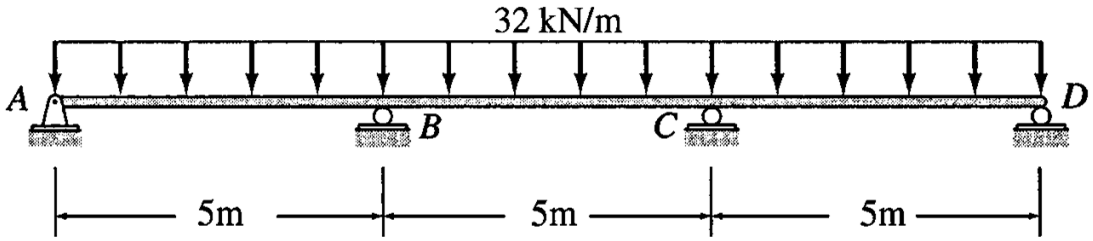
$$\boldsymbol{M_{hr} = 0}$$

Ex.1: Determine the member end moments and reactions





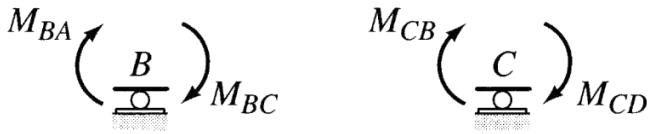
Ex.2: Determine the member end moments and reactions for the continuous beam shown in figure due to the uniformly distributed load and due to the support settlements of 15 mm at B, 36 mm at C, and 18 mm at D.



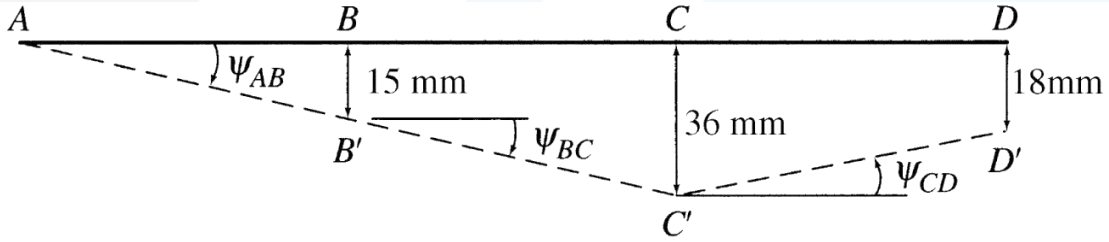
$E = 200$ GPa $\quad I = 1705 \times 10^6$ mm^4

Degrees of freedom:

θ_B & θ_C



Chord rotations:

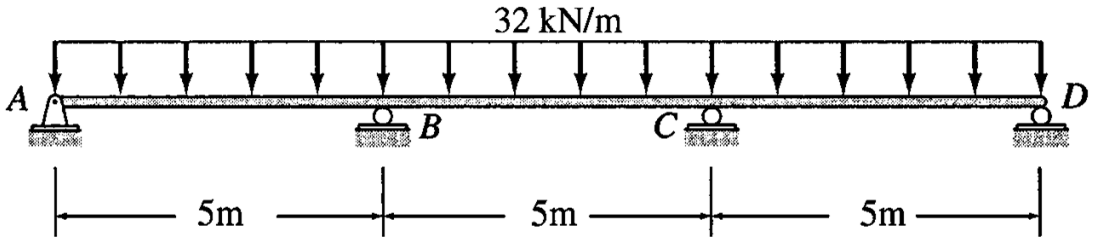


$$\psi_{AB} = -\frac{0.015}{5} = -0.003$$

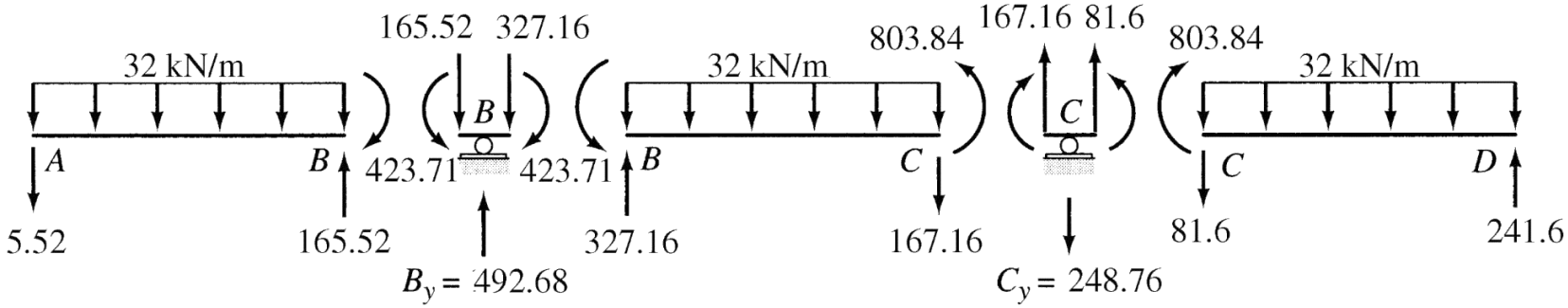
$$\psi_{BC} = -\frac{(0.036 - 0.015)}{5} = -0.0042$$

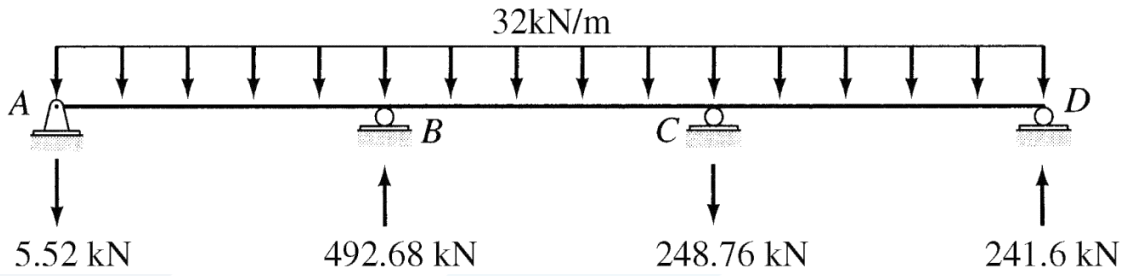
$$\psi_{CD} = \frac{(0.036 - 0.018)}{5} = 0.0036$$

Ex.2: Determine the member end moments and reactions for the continuous beam shown in figure due to the uniformly distributed load and due to the support settlements of 15 mm at B, 36 mm at C, and 18 mm at D.

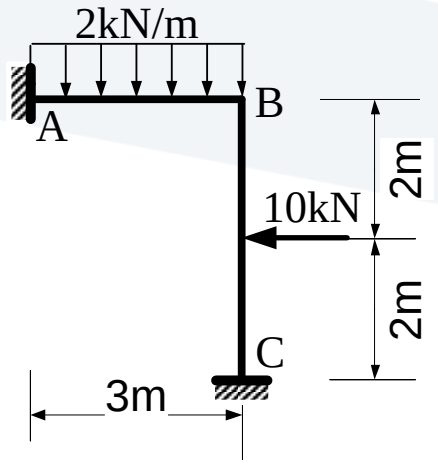


$E = 200$ GPa $I = 1705 \times 10^6$ mm^4





Ex.3: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams. EI is constant.



Ex.4: For the next frame under the given loads, by the slope-deflection method, calculate the reactions.