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Sets

Sets

Recall that $\mathbf{R}$ denotes the set of all real numbers, $\mathbf{Z}$ the set of all integers, and $\mathbf{Z}^+$ the set of all positive integers. Describe each of the following sets.

a. $\{x \in \mathbf{R} \mid -2 < x < 5\}$

b. $\{x \in \mathbf{Z} \mid -2 < x < 5\}$

c. $\{x \in \mathbf{Z}^+ \mid -2 < x < 5\}$

Solution:

a. $]-2,5[$

b. $\{-1,0,1,2,3,4\}$

c. $\{1,2,3,4\}$

Sets

4. Indicate the elements in each set defined in (a)–(f).
   a. $S = \{n \in \mathbf{Z} \mid n = (-1)^k, \text{ for some integer } k\}$.
   b. $T = \{m \in \mathbf{Z} \mid m = 1 + (-1)^i, \text{ for some integer } i\}$.
   c. $U = \{r \in \mathbf{Z} \mid 2 \leq r \leq -2\}$
   d. $V = \{s \in \mathbf{Z} \mid s > 2 \text{ or } s < 3\}$
   e. $W = \{t \in \mathbf{Z} \mid -1 < t < -3\}$
   f. $X = \{u \in \mathbf{Z} \mid u \leq 4 \text{ or } u \geq 1\}$

Solution:

a. $S=\{1,-1\}$

b. $T=\{2,0\}$

c. $U=\{\}=\emptyset$

d. $V=[3,\infty[\cup]-\infty,2]=Z$

e. $W=\{\}=\emptyset$

f. $X=]-\infty,4]\cup[1,\infty[=Z$

Sets

List the members of these sets.

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
b) $\{x \mid x \text{ is a positive integer less than } 12\}$
c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$
d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

Solution:

a. {-1,1}

b. {1,2,3,4,5,6,7,8,9, 10,11}

c. {0,1,4,9,16,25,36,49,64,81}

d. ∅

Set Equality

Which of the following sets are equal?

$$A = \{a, b, c, d\} \qquad B = \{d, e, a, c\}$$
$$C = \{d, b, a, c\} \qquad D = \{a, a, d, e, c, e\}$$

Solution:

A=C

B=D

Which of the following sets are equal?

$$A = \{0, 1, 2\}$$
$$B = \{x \in \mathbf{R} \mid -1 \leq x < 3\}$$
$$C = \{x \in \mathbf{R} \mid -1 < x < 3\}$$
$$D = \{x \in \mathbf{Z} \mid -1 < x < 3\}$$
$$E = \{x \in \mathbf{Z}^+ \mid -1 < x < 3\}$$

Solution:

A=D

Sets

Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$. Answer each of the following questions. Give reasons for your answers.

a. Is $B \subseteq A$? b. Is $C \subseteq A$?

Solution:

a. False , $\exists\ j \in B$ but $j \notin A$

b. True, every element in C exists in A

Which of the following are true statements?

a. $2 \in \{1, 2, 3\}$ b. $\{2\} \in \{1, 2, 3\}$ c. $2 \subseteq \{1, 2, 3\}$

d. $\{2\} \subseteq \{1, 2, 3\}$ e. $\{2\} \subseteq \{\{1\}, \{2\}\}$ f. $\{2\} \in \{\{1\}, \{2\}\}$

Solution:

a. True

b. False

c. False

d. True

e. False

f. True

Sets

For each of the following sets, determine whether 2 is an element of that set.

a) $\{x \in \mathbf{R} \mid x \text{ is an integer greater than } 1\}$

b) $\{x \in \mathbf{R} \mid x \text{ is the square of an integer}\}$

c) $\{2, \{2\}\}$

d) $\{\{2\}, \{\{2\}\}\}$

e) $\{\{2\}, \{2, \{2\}\}\}$

f) $\{\{\{2\}\}\}$

Solution:

a. True

b. False

c. True

d. False

e. False

f. False

Sets

a. Is $3 \in \{1, 2, 3\}$?

b. Is $1 \subseteq \{1\}$?

c. Is $\{2\} \in \{1, 2\}$?

d. Is $\{3\} \in \{1, \{2\}, \{3\}\}$?

e. Is $1 \in \{1\}$?

f. Is $\{2\} \subseteq \{1, \{2\}, \{3\}\}$?

g. Is $\{1\} \subseteq \{1, 2\}$?

h. Is $1 \in \{\{1\}, 2\}$?

i. Is $\{1\} \subseteq \{1, \{2\}\}$?

j. Is $\{1\} \subseteq \{1\}$?

Solution:

a. True

b. False

c. False

d. True

e. True

f. False

g. True

h. False

i. True

j. True

Sets

a. Is the number 0 in $\emptyset$? **b.** Is $\emptyset = \{\emptyset\}$?

c. Is $\emptyset \in \{\emptyset\}$? d. Is $\emptyset \in \emptyset$?

Solution:

a. False.

b. False

c. True

d. False

Proper subsets

- Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$.

a. Is $C \subseteq C$? b. Is C a proper subset of A?

Solution:

a. True

b. True

- Let $A = \{2,4,6\}$ $B = \{0,2,4,6,8\}$

a. Is B a proper subset of B ? b. Is A a proper subset of B ?

Solution:

a. False

b. True

Set cardinality

What is the cardinality of each of these sets?

a) $\{a\}$ **b)** $\{\{a\}\}$

c) $\{a, \{a\}\}$ **d)** $\{a, \{a\}, \{a, \{a\}\}\}$

Solution:

a. 1

b. 1

c. 2

d. 3

What is the cardinality of each of these sets?

a) $\emptyset$ **b)** $\{\emptyset\}$

c) $\{\emptyset, \{\emptyset\}\}$ **d)** $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Solution:

a. 0

b. 1

c. 2

d. 3

Power Sets

Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$ **b)** $\{a, b\}$ **c)** $\{\emptyset, \{\emptyset\}\}$

d) $\{1, 2, 3\}$

Solution:

a. $P(\{a\}) = \{\emptyset, \{a\}\}$

b. $P(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

c. $P(\{\emptyset,\{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

d. $P(\{1,2,3\}) = \{\emptyset,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$

How many elements does each of these sets have where a and b are distinct elements?

a) $P(\{a, b, \{a, b\}\})$

b) $P(\{\emptyset, a, \{a\}, \{\{a\}\}\})$

Solution:

a. $2^3 = 8$

b. $2^4 = 16$

Set Operations

Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9\}$, and $C = \{2, 4, 6, 8\}$.
Find each of the following:

a. $A \cup B$ **b.** $A \cap B$ **c.** $A \cup C$ **d.** $A \cap C$

e. $A - B$ f. $B - A$ g. $B \cup C$ h. $B \cap C$

Solution:

a. {1,3,5,7,9,6}

b. {3,9}

c. {1,2,3,4,5,6,7,8,9}

d. ∅

e. {1,5,7}

f. {6}

g. {3,6,9,2,4,8}

h. {6}

Cartesian Products

Let $A = \{x, y, z, w\}$ and $B = \{a, b\}$. List the elements of each of the following sets:

a. $A \times B$ b. $B \times A$

c. $A \times A$ d. $B \times B$

Solution:

a. {(x,a), (x,b), (y,a), (y,b), (z,a), (z,b), (w,a), (w,b)}

b. {(a,x), (a,y), (a,z), (a,w), (b,x), (b,y), (b,z), (b,w)}

c. {(x,x), (x,y), (x,z), (x,w), (y,x), (y,y), (y,z), (y,w),(z,x), (z,y), (z,z), (z,w), (w,x), (w,y), (w,z), (w,w) }

d. {(a,a), (a,b), (b,a), (b,b)}

Cartesian Products

Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$ b) $C \times B \times A$

Solution:

a. A×B×C={(a,x,0), (a,x,1), (a,y,0), (a,y,1), (b,x,0), (b,x,1), (b,y,0), (b,y,1), (c,x,0,), (c,x,1), (c,y,0), (c,y,1)}

b. C×B×A={(0,x,a), (0,x,b), (0,x,c), (0,y,a), (0,y,b), (0,y,c), (1,x,a), (1,x,b), (1,x,c), (1,y,a), (1,y,b), (1,y,c),}

homework

Determine whether each of these statements is true or false.

a) $x \in \{x\}$ **b)** $\{x\} \subseteq \{x\}$ **c)** $\{x\} \in \{x\}$
d) $\{x\} \in \{\{x\}\}$ **e)** $\emptyset \subseteq \{x\}$ **f)** $\emptyset \in \{x\}$

Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$
b) $\{\{1\}\}, \{1, \{1\}\}$ **c)** $\emptyset, \{\emptyset\}$